Coalition Parties versus Coalitions of Parties: How Electoral Agency Shapes the Political Logic of Costs and Benefits

by
Kathleen Bawn
Department of Political Science
UCLA

and
Frances Rosenbluth
Department of Political Science
Yale University

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Abstract

This paper argues that governments formed from post-election coalitions (majority coalition governments in PR systems) and pre-election coalitions (majority parties in SMD systems) aggregate the interests of voters in systematically different ways. We show that the multiple policy dimensional policy space that emerges from PR rules motivate parties in the government coalition to logroll projects among themselves without internalizing the costs of those projects in the same way that a majoritarian party would be forced to do. The size of government should therefore tend to be larger in PR systems. We further show that, although centrifugal electoral incentives dominate in PR systems, some incentives towards coalescence across groups and across parties exist through the greater likelihood that large parties have in becoming a member of a minimal winning coalition of parties.

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Introduction

Democratic government is government by coalition. In many parliamentary systems, governments are explicit multi-party coalitions. Even in cases of single party government, a party that wins a parliamentary majority represents -- almost by definition -- a coalition of interests. The difference seems to be one of sequence. In some cases majority coalitions are formed by parties after elections. In other cases, parties work to forge a majority support base before the election.

In this paper, we ask whether and how the sequence of coalition formation and electoral contest matter for how society's interests are aggregated. What difference, if any, does it make if governments are formed by a single party that represents a coalition of interests, or a by a coalition of parties, each of which represents a single interest? One might think that in either case, a (roughly majority-sized) coalition of interests would control government through their agents.

We argue here that the sequence does matter. Coalitions of parties behave differently than coalition parties, even when the set of interests included in the coalition are identical. The difference stems from the nature of electoral accountability. A single party in government is electorally accountable for all policy decisions it makes. Parties that participate in coalition governments, by contrast, are held responsible only for a subset of policy decisions, for the policy areas in which they have the biggest stake and the biggest impact. Our key assumption is that groups in society know not only the policy position of each party, but also the relative priority given to different dimensions. Voters and interest groups assign responsibility for a policy decision to the party in government that gives that dimension the highest priority. Voters and groups assign more credit and blame for environmental policy to an incumbent Green party, for example, than to its coalition partners. Specifically, voters and groups (1) assume that in multi-party governments, policy decisions are made by the coalition partner that gives the policy the highest priority, and (2) use this assumption to assess strategically the policy consequences of what party to support in the election. The separate electoral accountability of each party in a coalition government causes the coalition to make different policy decisions than would a single party majority government that represented exactly the same coalition of interests in society.

This model of policy-making by multiparty coalitions owes much to Laver and Shepsle's (1994, 1996) model of ministerial government. Their focus on ministerial independence offers one reasonable mechanism by which coalition governments implement logrolls. Our argument is also consistent with the empirical finding that parties in coalition seem to get cabinet positions in rough proportion to their size (Laver and Schofield 1990; Druckman and Wakefield 2001).[1]

[1] At the same time, we don't see a contradiction with Thies's (2001), monitoring model, which suggests that ministers have to work within the "coalitional contract." This kind of oversight is also necessary

Our focus on electoral agency leads us to consider the impact of electoral rules on party systems and election outcomes. This is a natural frame, given the well-documented association between proportional representation (PR) and multiparty government on the one hand and single member districts (SMD) and single party majorities (Duverger 1953, Riker 1982, Cox 1990 and 1997.) Under any type of electoral rules, voters and groups face the same basic choice. One option is support a coalition party that does not perfectly represent the group (because it represents a coalition of interests), but that has a good chance of forming a single-party government. The other option is to support a party that perfectly represents the group's interests, but does not have a chance of forming a government on its own (because of its narrow focus). Our model shows how the advantages and disadvantages of each alternative depend on the electoral rules.

Other scholars have contrasted policy-making by coalition government and single-party government in the context of a unidimensional spatial model. Kalandrakis (2001) and Powell and Vanberg (1998) argue that coalition governments actually come closer than majoritarian systems to representing the preferences of the median voter. Coalition governments, they point out, produce relatively stable outcomes over time whereas majoritarian governments take turns skewing policy towards the interests of their respective constituencies.[2]

The advantages of the unidimensionality assumption are well known: greater tractability and a clear normative benchmark in the form of the median voter's ideal point. Moreover, in some countries, observed political competition does indeed seem to be captured by a single left-right dimension.

Convenience aside, it is hard to justify the assumption that the space of feasible governments is unidimensional on theoretical grounds. For one thing, electoral competition in many countries (PR countries in particular) displays more dimensions of conflict (Rogowski 1987). Even in countries where observed political conflict appears to be unidimensional, this fact should more properly be regarded as a consequence of the choices of politicians, rather than a fundamental constraint (Ferejohn 1993, Hinich and Munger 1994). Governments choose policies from a multidimensional choice set – they can choose, for example, high or low levels of military spending, high and low degrees of protection for domestic industry, more or less regulation of various sectors. The fact that some choices seem to go together empirically is, in our view, of product of the way institutions (Ferejohn 1993) and ideology (Hinich and Munger 1994, Bawn 1999) structure choices.

We use a multi-dimensional model in which the dimensionality of the policy space reflects the structure of society. That is, we assume there are as many true policy dimensions as there are groups in society. The *observed* dimensionality of party competition, however, will depend on the number of electorally viable parties – the more parties, the more dimensions. The number of viable parties depends in turn on the

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because, unless issue dimensions are entirely unrelated, policies implemented in one jurisdiction can undermine policies implemented in another.

[2] See also Alesina and Rosenthal 1989.

electoral system. Following Cox (1990), we find it useful to think of the consequences of electoral systems in terms of "centripetal" and "centrifugal" incentives, though unlike Cox, we conceptualize these multidimensionally. Centripetal incentives are those that lead groups to form coalition parties; centrifugal incentives lead groups to splinter and support narrow, single-interest parties.

In addition to demonstrating the policy consequences of electoral agency, our model gives the following predictions. First, PR countries are likely to have a larger number of parties, each of which will represent a narrower group of interests than their SMD counterparts. This is, of course, not a new insight, but more of a reality check for the basic framework. The second result is more surprising, demonstrating that there is a centripetal incentive to form coalition parties even under PR, albeit a much weaker one than under SMD. The third result is that, this centripetal incentive notwithstanding, PR countries will have a larger public sector than SMD countries.

1. A Model of Electoral Accountability: Basic Assumptions

Our argument has two basic parts. The first is that coalition parties make policy choices differently than coalitions of parties, other things equal. The second is that coalition parties will be present to a much greater extent with SMD than with PR, other things equal. The "other things" we hold equal in our model are the following:

1. The composition of society and the preferences of groups.

2. The information available to groups, and the ways that groups use information in deciding what party to support.

3. The process of government formation and decision-making within governments.

We discuss each aspect of the overall framework in turn.

(a) The composition of society and the preferences of groups

Suppose there are n groups in society and that each group i has a "project"[3] or issue that it cares about. Let x_i denote the scale of the i-th project – the degree of protection for a particular industry, for example, or the level of public benefits targeted to a particular group. A government's policy $X = (x_1, x_2, \ldots x_n)$ simply consists of the scale of every potential project.

Let the benefits of project i, $B_i(x_i)$, accrue only to the group i, while the costs $C_i(x_i)$ are born by all groups, so that each group's cost share of project i is $\frac{C_i(x_i)}{n}$. The goal of each group i is to maximize its net benefits $B_i(x_i) - \frac{1}{n}\sum_{j=1}^{n} C_j(x_j)$. For simplicity, we

[3] The term "project" reflects the historic use of this type of model in studies of pork barrel politics, such as Weingast, Shepsle and Johnsen 1981.

assume that the relationships between project scale and benefits and costs is the same for all projects. We want to focus on cases in which all participants have finite ideal levels for the project (even the farm lobby doesn't want all of GDP to be spent on subsidies.) A simple way to implement this focus is to assume that benefits are linear

$$B_i(x_i) = x_i$$

and that costs are quadratic

$$C_i(x_i) = x_i^2 .$$

for all projects *i*. Note then that the *i*th group's ideal policy is to set $x_j = 0$ when $j \neq i$ and $x_i = \frac{n}{2}$.

We assume that the groups are all the same size, and that virtually all voters belong to one of the groups. There are, however, a small number of unattached voters. These voters cannot be targeted by politicians and their voting behavior is random.[4] Realistically, we can think of the unattached voters as those who pay no attention to politics. For simplicity, we assume here that the number of unattached voters is smaller than the size of any one group. The unattached voters are important for our model because they create electoral uncertainty.

(b) Information Sources and Voting Decisions

We assume that voters are fully strategic in the sense that they accurately anticipate the impact of their vote on policy, including anticipating the consequences of multi-party government. That is, a voter who votes for a party that focuses on representing the interests of her group alone understands that this party will only be able to get into government by compromising with other parties. The group votes in the way that gives it the highest expected utility from policy, given the behavior of other groups.

This kind of strategic voting is not problematic with single party governments. It is only feasible for multi-party coalitions, however, if voters can determine an individual party's marginal contribution to a coalition government's decisions. While in theory this can be a difficult problem (see the discussion in Schwartz 1994), we note that in reality parties work hard to give voters information about their marginal contribution to coalition policies. We assume that parties promulgate platforms that contain their *goals* and *priorities*. Goals indicate the policy positions the party will try to bring about – they are the party's public declaration of its ideal point in the *n*-dimensional space of group projects.

Priorities, as the name implies, indicate which projects or dimensions the party thinks are important. Specifically, they indicate which issues the party is willing to take responsibility for if in government. Formally, party *j*'s priorities are denoted by an *n*-vector $\boldsymbol{\alpha_j} = (\alpha_{1j}, \alpha_{2j}, \ldots \alpha_{nj})$ where $0 \leq \alpha_{ij} \leq 1$ indicates the extent to which party *j* wants

[4] Zaller (2002) shows that unattached voters are primarily responsible for the impact of the economy and other valence issues on elections. In this sense, of course, their behavior is not random. For our model's purpose, however, the important thing is that parties cannot compete for the support of the unattached on the basis of policy.

to associate itself with policy regarding the *i*-th project. In ordinary usage, the word "priority" indicates relative importance, and we preserve that in the model by requiring the elements of any party's priority vector to sum to one. For example, if there are five projects, a party with priority vector (1, 0, 0, 0, 0) associates itself completely with the first project, a party with priority vector (1/5, 1/5, 1/5, 1/5, 1/5) associates itself with all projects equally, and a party with (1/2, 1/3, 1/6, 0, 0) associates itself most strongly with the first project, somewhat less with the second, less still with the third and not at all with the fourth and fifth. The requirement that the priority weights sum to one implies that in order to strengthen its associate with one issue a party must weaken its association with another.

When we say "party *j* represents group *i*" or that "*i* is one of *i*'s constituent groups" we mean that $\alpha_{ij} > 0$. Note however, that when the party decides to represent group *i*, it must accept blame for the costs of project *i* as well as credit for the benefits.

As we will discuss below, we assume that on each policy dimension, a coalition government implements the ideal policy (the platform goal) of the coalition partner who assigned the highest priority to that dimension. For example, suppose that parties A and B are in coalition together, that party A's platform has a goal level of 1 on a dimension to which it gives priority 1/3, and that B has a goal of 2 and a priority of 1 on that same dimension. Because B assigned higher priority to the dimension, the coalition government implements B's goal policy of 2. This assumption is very close in spirit to Laver and Shepsle's model of multiparty government, although we focus on policy outcomes directly, without regard to the allocation of portfolios.

By observing the parties' platforms, groups can thus precisely anticipate the policies that will be implemented by any possible coalition government. This anticipation allows them to react strategically to the electoral system.

The unattached voters are not strategic – they are too poorly informed to be so. Although unattached voters in reality undoubtedly react systematically to some aspects of the political environment – incumbency, scandal, the economy – we model them here as a complete wild card. We assume that unattached voters vote with equal probability for any viable party. In order for a party to be viable, it must get the support of at least one organized group. Note that the unattached voters vote as a block, consistent with evidence that they react to the economy and other valence issues.

(c) Government Formation and Policy-Making

We recognize that coalition governments can occur in majoritarian countries and that single party majorities can emerge in proportional countries. The empirical regularity whereby single party governments are associated with majoritarian electoral systems and coalition governments with PR is a result, not an assumption, of our model.

We assume that governments are minimal winning coalitions (MWC) in Leiserson's (1966) fairly strong sense: Governments will be composed of the minimum number of parties needed to make a majority. That is, if a two-party majority is possible, we will

not see three-party coalitions; not even those that would not involve superfluous parties. A weaker assumption would simply be that there would be no superfluous parties in coalitions (i.e. no party that could withdraw and still leave the coalition with a majority), or that any majority or supermajority coalition could form. A stronger assumption would be that the smallest coalition in terms of seats would form (Riker's minimum winning coalition), or the cheapest one.[5]

We assume that all MWC's, in Leiserson's sense, are equally likely. We do not model any formateur effect whereby the largest, or median, party, would be asked to propose the coalition.[6] Our assumption that coalitions include the minimum number of parties needed for a majority does, however, advantage the largest party. This comports with empirical evidence (Martin and Stevenson, 2000) and contributes to our unorthodox claim that centripetal incentives exist even in proportional systems.

As we discussed above, the party in the government with highest priority on each policy dimension makes decision on that dimension. Parties do not deviate from their platform goals. This assumption is key for the ability of voters to make fully strategic decisions.

(d) Electoral rules

The above discussion has established the assumptions of our model which apply to both SMD and PR electoral systems. A word now is in order about how we model the systems themselves. Our model of PR is straightforward. The party's fraction of seats is the same as its fraction of votes. We ignore real world complications like minimum vote thresholds, district magnitude and allocation formulae.

Under SMD, we assume that groups, and the unattached voters, are evenly distributed across districts. This is a significant departure from reality. By ignoring differences among districts, we ignore a potential source of coalition government in SMD systems. Under SMD, of course, the seat in each district is won by the party that gets a plurality of votes. Our assumption that all districts are the same implies that the same party will win all districts.[7] This is obviously unrealistic, and we hope to consider the impact of heterogeneous districts in future work.

2. The Logic of Electoral Agency: An Example

We begin by offering a loose, intuitive version of our overall argument. When the electoral system is proportional, groups have an incentive to support the party that best champions the group's interest, both in the sense of espousing the group's ideal point, and in the sense of giving highest priority to the group's projects. Groups support parties that cater to their own narrow interests, rather than coalition parties. The key argument

[5] Note that by ruling out coalitions that are composed of more parties than necessary, we do, in effect, systematically rule out more expensive coalitions.

[6] Druckman and Warwick's (2001) empirical work shows that the formateur party is actually somewhat undercompensated in cabinet assignments.

[7] See Chhibber and Kollman (1998) on how district heterogeneity impacts party systems. McGillivray (1997, 2002) shows how district heterogeneity with respect to sectoral composition affects the geographic distribution of subsidies and tariff protection; and for Persson and Tabellini (1999) district heterogeneity is a key assumption for concluding that SMD systems under-provide public goods.

is this: *Multiparty government coalitions reflect inefficient logrolls because each dimension is controlled by a party that externalizes a large fraction of the costs of that dimension's project.*

Under SMD, in contrast, coalition parties attract the support of voters who realize that a party with a narrow base of support cannot win a plurality. A coalition party, realizing that it will be accountable for multiple dimensions, espouses policies that are efficient from the point of view of its set of constituent groups. The optimal strategy for a party that needs to win the support of a coalition is to maximize the joint utility of the groups it represents.[8]

We now turn to making this argument more precise, using the framework developed in the previous section. For the remainder of the paper, we will work with an example in which there are six groups in society.

(a) Maximally Fragmented Party System

Define a *maximally fragmented party system* (MFPS) as one in which each group has party that gives all priority to that group's dimension and sets its goal equal to the group's ideal. Note that if Group i could set policy unilaterally, it would set $x_i = 3$ and $x_j = 0$ for $i \neq j$. In our example, the MFPS consists of six parties, each of which sets $x_i = 3$ and $\alpha_i = 1$ for one $i = 1\ldots6$. That is each party espouses the ideal policy of its target group and gives all priority to the dimesion that benefits that group. This platform is the one that maximizes the party's marginal product, that is, it maximizes the party's contribution to the group's welfare (Schwartz 1994.)[9]

In the maximally fragmented party system, each party attracts the support of its own group with probability one and the support of the unattached with probability 1/6. The MWC government will consist of the party that won the support of the unattached and two of the remaining five others (with equal probability). Ex ante, each party faces a .5 probability of being in government.

The utility of a group whose party is in government is thus

$$U_{IN}(MFPS) = 3 - \frac{1}{6}(3 \cdot 3^2) = -\frac{3}{2} .$$

The utility of a group whose party is out is

$$U_{OUT}(MFPS) = -\frac{1}{6}(3 \cdot 3^2) = -\frac{9}{2} .$$

The ex ante expected utility of any group under MFPS is –3. No matter what government forms, three projects are undertaken at a scale of 3 and three dimensions left at zero. The overall cost of the government's policy is $3 \cdot 3^2 = 27$.

[8] In our model, all groups are equally important, and have equal bargaining power. We do not allow for the possibility of "captured groups" (Frymer 2000) who are less able than others to switch their support from one party to another.

[9] To see why putting all priority on the group's own dimension maximizes marginal product, observe that lowering costs on other dimensions produces positive externalities for other groups, while all the positive effects of increasing benefits on the group's own dimension are internalized.

(b) Balanced Two-Party System

Similarly, define a *balanced two-party party system* (BTPS) as one in which there are two parties, each of which represents one half of the groups. A party that wanted to maximize the welfare of groups 1, 2 and 3, giving equal weight to each, would set $x_i = 1$ for i = 1, 2, 3 and $x_j = 0$ for $i = 4, 5, 6$. Let Party A represent groups 1, 2 and 3, giving each equal priority ($\alpha_i = 1/3$ for i =1,2,3), and B represent groups 4, 5, 6 with equal priority. Each party wins a plurality (in every district) with probability .5, and the party that wins forms a single-party government.

The utility of a group whose party is in power is

$$U_{IN}(BTPS) = 1 - \frac{1}{6}(3 \cdot 1^2) = \frac{1}{2} \ .$$

The utility of a group whose party is out of government is

$$U_{OUT}(BTPS) = -\frac{1}{6}(3 \cdot 1^2) = -\frac{1}{2} .$$

The ex ante expected utility of any group in the BTPS is 0. As with the MFPS, three projects are undertaken, but at a lower scale. The overall cost of the public sector is 3.

(c) How do party systems arise?

These stylized scenarios of party systems support the intuitive argument that coalitions of parties make different policy choices than coalition parties. Single interest parties adopt platforms that externalize a larger share of costs than coalition parties do. In multiparty coalitions, parties are able to implement their platform goals on the dimensions they care most about. Together, these two claims imply that coalitions of parties will create a larger public sector than a single coalition party.

The example also highlights a problem with the intuitive argument, however. All groups are better off when governed by a single coalition party. Given this, why would they support single interest parties? One possible explanation is naiveté – groups support the party whose platform they like best, ignoring the strategic considerations such as the how the process of forging a multi-party coalition will impact policy.

An alternative explanation, which we will pursue here, is that electoral institutions promote or prevent resolution of the collective action problem among groups. The above two stylized party systems correspond roughly to the stereotypes associated with the two canonical types of electoral rules. A BTPS is stereotypically associated with SMD, and a MFPS with PR (Duverger 1954, Downs 1957, Riker 1984, Cox 1990, 1997). Put another way, the incentives from SMD are largely centripetal because winner-take-all races in the district promote the formation of coalition parties. Similarly, incentives from PR are centrifugal because the proportionality promotes splintering of parties.[10]

[10] Cox looks at district size, number of candidates in a race, the right or not of voters to abstain, etc., to see when a candidate's optimal position is at the median or away from it. This is a trenchant insight, one that extends to the multidimensional case we consider here.

Yet the stereotypes are not necessarily empirically accurate. There are proportional systems with relatively few parties, such as the Federal Republic of Germany[11] from the mid-1950's to the early 1980's and SMD systems with relatively many, such as Canada and India.[12]

We now examine the logic of centripetal and centrifugal incentives in the context of our multi-dimensional model. Specifically, we ask what party system can be sustained under different electoral rules.

We proceed by considering the incentives of groups to support coalition parties ("coalesce") or to support parties that focus on their own interests ("splinter"). We assume that groups correctly anticipate the costs and benefits of each strategy, as reflected in the probabilities of different types governments and different policies. The structure of the pay-offs depends on the electoral system.

3. Electoral Systems and Party Systems

We first examine the decision to splinter or coalesce in an SMD system. Here, our model's results are consistent with the conventional wisdom that incentives in SMD systems are primarily centripetal. A BTPS is stable under SMD. Moreover, if groups are able to coordinate, a fragmented party system will coalesce into a BTPS. We then examine how the strategic situation differs under PR. Here the results get more interesting. We confirm our intuition that the BTPS is not stable under PR. We do not find, however, the party system necessarily degenerates into maximal fragmentation. PR creates both centripetal and centrifugal incentives. The incentive to splinter is partly offset by the incentive larger bloc in order to increase chances of being in government.

We focus on the decisions of groups to support coalition parties or splinter parties. We do not treat parties themselves as strategic actors, but simple assume that various types of parties are available. In all cases, we adopt Schwartz's (1994) model of electoral agency and assume that parties choose their platforms to maximize their marginal contribution to their constituents' welfare. Concretely, this means that platform goals are the policies that maximize the joint utility of all constituent groups, and that priority is equally divided among the constituent groups' dimensions.

The argument proceeds by an examination of which configurations of parties can be sustained as Nash equilibria in games in which the pay-offs are the expected benefits from government policy, taking account of electoral uncertainty (due to the unattached voters) and coalitional uncertainty about the composition of government.

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[11] While Germany's electoral system uses single member districts to select 1/2 of the individuals elected to the Bundestag, these seats do not affect party strength. Because seat shares are proportional, the SMD races cannot create a centripetal incentive at the party system level (Bawn 1993).

[12] As Riker pointed out, national majoritarian parties should emerge out of single member districts because of the economies of scale associated with capturing a parliamentary majority. But strong regional differences, such as in India or Canada, can reduce the perceived gains from merger from the standpoint of some districts.

(a) SMD and Centripetal Incentives

Consider first the most straightforward situation. We have SMD and a BTPS in place, so that Party A represents groups 1, 2, and 3 and Party B represents 4, 5, and 6. Consider the problem from the point of view of groups 1, 2, and 3 – given that groups 4, 5, and 6 are coalesced, is there an incentive to splinter? Specifically, let each group face the choice of supporting coalition party A, which offers the coalitionally-efficient policy of $x_i = 1$ for $i = 1,2, 3$ or a splinter party (C, D or E) which puts all priority on the group's dimension and offers the group its ideal policy $x_i = 3$.

Figure 1 depicts this situation as a three-way game (we are holding the choice of groups 4, 5 and 6 constant). The pay-offs are expected utilities that take account of uncertainty about (1) which party the unattached voters will support and (2) which party or parties will form a majority government. The pay-offs to this and all subsequent games are derived in the Appendix. If all three groups coalesce by supporting party A, then the election outcome depends on the choice of the unattached; A wins with probability 1/2; and the expected utility (as discussed above) is 0. If one or more of the groups splinter, Party B wins the election with probability 1, and the pay-offs to groups 1, 2 and 3 are –1/2. Groups 1, 2 and 3 all have a dominant strategy of coalescing. Note that groups 4, 5 and 6 will also have dominant strategies to coalesce, given that 1, 2 and 3 have. The BTPS is a Nash equilibrium in the overall six-way game. There is no centrifugal incentive under SMD.

Suppose we start from a MFPS, however. Does SMD lead groups to coalesce in a fragmented system? Figure 2 (a) depicts the pay-offs from splintering and coalescing on the part of groups 1, 2 and 3, assuming that 4, 5, and 6 each vote for their own parties as described above. The first thing to note in Figure 2(a) is that all three groups supporting coalition party A is a Nash equilibrium. With groups 4, 5 and 6 splintered, if 1, 2 and 3 coalesce, their party wins with probability 1 and they get pay-offs of 1/2.

The second thing to note is that this Nash equilibrium is not unique. Given the splintered opposition, all it takes is for two of the three groups to support Party A in order for it to win a plurality with probability one. Cells (b), (c) and (e), in which two of the three groups support the coalition party give the same pay-offs as when all three groups coalesce. These cells are also Nash equilibria in this three-way game in which the actions of players 4, 5 and 6 are held constant. But they seem implausible as stable party systems for two reasons. First, if Party A only attracts the support of groups 1 and 2, it seems unlikely that it would keep a platform that offers benefits to group 3. Second, the scenario in which two groups coalesce and the other four splinter seems unlikely to be sustainable in the six-way game. Given that groups 1 and 2 have coalesced and group 3 splinters, wouldn't groups 4, 5, 6 coalesce as Party B, thereby guaranteeing themselves a plurality?

Yes, they would, as Figure 2(b) demonstrates. The scenario depicted here assumes that Groups 1 and 2 have coalesced behind a party that maximizes their joint utility (project levels $x_1 = x_2 = 3/2$) and Group 3 has splintered. Figure 2(b) examines the overall strategic scenario from the point of view of groups 4, 5 and 6. All three groups have a

dominant strategy to coalesce. We would thus not expect the kind of "submajority party system" indicated by cells (b), (c) and (e) in Figure 2(a) to be stable under SMD.

Note, however, that MFPS, indicated by cell (h) in Figure 2(a), is also a Nash equilibrium. If every other group splinters, no single group can benefit by unilaterally switching support to a coalition party. However, this equilibrium is Pareto dominated by that in cell (a). If groups can communicate or observe each other's actions, we would expect that the Pareto dominant equilibrium, the BTPS, will obtain.

The main point of Figures 2(a) and 2(b) is that the centripetal incentives of SMD are indeed strong, limited only by the possibility of coordination failure (cell (h)).

(b) PR: Centrifugal and Centripetal Incentives

Splintering and coalescing give rise to different election outcomes under PR, and thereby to different pay-offs. We begin with the same scenario we started with above. Assuming a BTPS exists, do the groups that constitute one of the coalition parties have an incentive to divert their support to splinter parties that focus on that group's issue alone? Is there a centrifugal incentive in this multi-dimensional scenario?

Figure 3 depicts the incentives to splinter under PR, given a unified opposition. They are strong indeed. The game is a three-way Prisoner's Dilemma. Splintering is a dominant strategy for each of the three groups, giving a unique Nash equilibrium (cell h) that is Pareto inferior to the outcome when all groups coalesce (cell a). Consistent with expectations, the BTPS is not sustainable under PR.

Now suppose that the opposition is fragmented, as in Figure 4. The stereotype of PR would suggest that full fragmentation (cell h) would be the unique equilibrium outcome here, but that is not the case. Full fragmentation (MFPS) is a Nash equilibrium, but it is only one of four. Moreover, the MFPS equilibrium is Pareto dominated by the other three in which two groups coalesce and the third splinters. In our analysis of the centripetal incentive in SMD, we argued that groups are likely to coordinate on one of the efficient Nash equilibria. In Figure 4, the Pareto efficient Nash equilibria are cells b, c and e, in which two groups support party A and the third supports a splinter party.[13] There is a centripetal incentive here, albeit a weak one (relative to SMD) that only brings two groups together under a single party, leaving one with a preference to splinter.

What is the source of this centripetal incentive? To understand this, we need to examine the pay-offs in Figure 4 in more detail. Consider the situation in cell (b), where groups 1 and 2 support coalition party A and group 3 supports its own splinter party E. Given that group 3 reacts to the centrifugal incentive to splinter, why don't groups 1 and 2 have an incentive to follow suit?

[13] This three-way coordination game combines features of Assurance and Battle of the Sexes. As in Assurance, some of the Nash equilibria are better for all players than others. But like Battle of Sexes, each player most prefers a different Nash equilibrium.

Look at the decision from group 1's point of view. If group 1 supports A (given that 2 does and 3 does not), the viable parties will be A and E, along with the splinter parties that represent groups 4, 5, and 6 – call them F, G and H. With probability 1/5, A wins the support of the unattached, thereby getting 3/7 of the seats while E-H each win 1/7 each. In this scenario, an MWC consists of A plus any other party. With probability 4/5, the unattached support one of E, F, G or H, giving that party 2/7 of the seats. Party A would also have 2/7; the remaining three would have 3/7 each. The MWC in this case would consist of whichever splinter party received the unattached support plus Party A. The centripetal incentive arises because the largest party has an advantage in terms of the likelihood of being included in a government.

As in Figure 2 above, we need to address two aspects of the situation depicted in Figure 4. First, in any of the Pareto efficient equilibrium, party A gives benefits to a group that supports a different party at the expense of the two groups that support A. Second, the behavior of groups 4, 5 and 6 is held constant. Figures 5, 6 and 7 address these two points.

Given that a coalition party will only attract the support of two groups in equilibrium, its platform should only aim at two. Assuming that groups 3-6 support splinter parties, will groups 1 and 2 support a party that maximizes their joint utility? The optimal platform for such a party is $x_1 = x_2 = 3/2$.

Figure 5 depicts this two-way game. Like Game 2, which illustrated the centripetal incentive of SMD, this game is Assurance. If the parties can communicate, or observe each other's moves, we would expect them to arrive at the Pareto optimal equilibrium in which both support the coalition party. The centripetal incentive here is strategically identical to that in SMD, but the resulting coalition is smaller.

The final question to ask is whether there is an incentive for a second coalition to form. Figure 6 depicts the incentives to form another two-group coalition, and Figure 7 shows the incentive to form a three group coalition. Given that groups 1 and 2 have coalesced to support Party A′, will groups 4 and 5 coalesce to form B′ (Figure 6)? Will groups 4, 5 and 6 coalesce to form B″ (Figure 7?) In either case, supporting the splinter party is a dominant strategy.

In the six-way game between all the groups, the Pareto efficient equilibria are party systems characterized by a single *submajority coalition party*; that is, a party that represents more than a single group, but is not capable of winning a majority on its own.

Overall, Games 5, 6 and 7 illustrate the limited centripetal incentive that exists in PR. There is an incentive for a submajority coalition party to form, because of the advantage that size plays in forming a minimal winning coalition. Once one coalition party has formed, however, the centrifugal incentives dominate and the remaining groups prefer to support splinter parties.

4. Electoral Systems, Party Systems, the Size of Government and the Dimensionality of the Political Space

The set of cooperation and coordination problems discussed above support the intuitive argument we began with. Coalitions of parties, each of which represents a narrow interest, will be unable to solve collective action problems between groups. This is a direct, albeit perverse, consequence of each party's desire to maximize its own contribution to the welfare of its constituents. The model acknowledges that this fact gives coalition parties an advantage that is diminished, but not completely overwhelmed, by the centrifugal incentives of PR.

Table 1 summarizes the overall findings. Panel (a) shows the likelihood of single party government, as a function of both the party system and the electoral system, and panel (b) shows the expected size of the public sector for each combination. Party systems that cannot be sustained as Nash equilibria are shaded dark gray, and systems that are equilibria but are Pareto dominated by other equilibria are light gray. We expect that party systems will be non-dominated equilibria.

The first thing to note in Table 1 is that multi-party coalition governments produce a larger public sector than governments made up of a single coalition party. Indeed, the more parties in the government, the larger the public sector. The second notable feature is that single party government is promoted by SMD and discouraged by PR. Third, it shows that the centripetal incentive to coalesce is present with PR and that this incentive impacts policy. The public sector is smaller with the submajority coalition party system than with a MFPS. The broader the constituency of a coalition party, the smaller the public sector.

Finally, the model shows how the dimensionality of observed political competition depends on institutions, and on electoral rules in particular.[14] In an SMD country, the true dimensionality of the policy space may be very large, but a BTPS will organize the politically feasible alternatives along a single dimension. Policies that benefit the constituent groups of Party A, for example, may define the conservative agenda; those that benefit the constituents of Party B would correspondingly define the liberal agenda. Ferejohn (1993) went some distance in this direction when he implied that unidimensional policy space should be stable only under SMD, because only there did it make strategic sense for parties to use one-dimensional ideology as an electorally valuable commitment mechanism. Given the centrifugal incentives that operate in PR rules, a party's best niche in that system may not lie along a single dimension at all, and we should expect parties to exploit whatever existing cleavages exist in society, to find the most secure electoral position possible. The consequence, we argue, is that parties in coalition governments resort to expensive logrolls among themselves. Taxes, prices, and

[14] Wright and Schaffner (2002) present evidence that legislative, as well as electoral, institutions affect the observed dimensionality of the policy space. Like us, they contend that parties are the key actors in the process that reduces the high number of fundamental dimensions to the ofter lower number of observed dimensions.

government services should all be higher under PR governments, as the evidence suggests is indeed the case.

The endogeneity of the dimensionality of political competition has wide reaching implications. Our model cautions against trying to fit European politics into the Procrustean bed of Downsian analysis. Rather, unidimensional electoral competition should be thought of as special case, one that has arguably had an exaggerated affect on how the discipline thinks about politics. The hegemonic status of unidimensional models may be one reason why the study of European politics has tended to be less integrated into mainstream electoral analysis, dominated as it was by ill-fitting models imported from across the Atlantic.

As with much of European politics, our model takes as given groups of voters as the primary unit of electoral behavior. Our model implies that groups may be more salient in PR systems simply because they are more likely to be represented by their own party, as opposed to having their identity obscured by a coalition party. Myerson (1993) and Cox (1997) have shown that the calculus of mobilization in PR systems puts a premium on the political usefulness of groups. In SMD systems, by contrast, it makes less sense for voters to organize politically because parties aim at the median voter's interests, not at her preferences (Achen and Bartels, 2002). Both for the median voter, who is going to be targeted anyway, and for voters at the tails, who have limited power to pull policies from the median, there is less value in learning about their political interests and being vocal about them, let alone expending the effort to mobilize with others to achieve them. There is less of a collective action requirement—or possibility--for political representation under SMD.

5. Discussion: Preference Aggregation and Policy Choice Across Systems

Other scholars have tried to explain the systematic differences in tax rates and consumer prices across electoral systems without exploiting an understanding of the difference in bargaining strategies that should operate under unidimensional and multidimensional settings. Rogowski and Kayser (2001) rely on the different vote-seat elasticities under SMD versus PR rules, showing that politicians are motivated to target the median voter in SMD systems but not in PR systems. While reasonable as far as it goes, this analysis fails to show why coalition parties should not then converge around a median coalition party or median legislator, as Kalandrakis (2001) would suggest.

Persson and Tabellini (1999) present a model in which majoritarian systems supply fewer public goods than PR systems, based on an assumption that parties in SMD systems will maximally redistribute income to the marginal district and will not internalize the votes lost in non-marginal districts. Their empirical canvassing of 64 countries shows that majoritarian countries are associated with a somewhat lower supply of public goods—measured as the sum of expenditures on transportation, education, and order and safety, in percent of GDP. Our model suggests an alternative explanation. At least some public goods are provided less in response to the diffuse demand of those who consume them,

and more in response to the intense "demand" of organized groups who reap rents by, for example, contracting to build the bridge, staff the schools and police forces, etc. (Weingast, Shepsle and Johnsen, 1981). By promoting a less efficient logroll among the groups for whom the costs of the public good count as benefits, PR leads to higher levels of these public goods.[15]

David Austen-Smith (2000) moves away from a unidimensional electoral competition model and focuses instead on legislative bargaining among three parties representing three types of economic groups: employers, employees, and voluntarily unemployed. He argues that redistributive tax rates are higher and wage distributions are flatter in PR systems because legislative bargaining targets the pivotal, rather than the median voter, defined as the voter with average employee income, which is endogenous to the chosen tax rate. When the cost of working is not too high, the positive impact on average employee income to due the induced change in the distribution of employee types dominates the negative impact on this income due to the increased tax burden. This characterization makes his evaluation of the policy consequences of multi-party bargaining under PR rules considerably more sanguine than ours. But note that his result rests uncomfortably on the assumption about the importance of legislative, rather than coalitional, bargaining. Provided that it matters, as we suggest, if any particular legislative party is or is not in government, this legislative bargaining model begins to look more dubious.

Our model provides alternative electoral micro-foundations for comparative politics in general, and in particular, for the burgeoning literature on "Varieties of Capitalism."[16] According to that literature, coordinated market economies—invariably PR systems—provide workers with the necessary wage security that enables them to invest in firm- or industry-specific skills. Firms, in turn, are able to accommodate workers' needs because government regulation buffers their profits at least to some degree from the vagaries of market swings. In terms of our analysis, this could be the result of the sorts of coalitional logrolls that we expect to see in PR systems. In our model, PR would allow coalition governments to form that contain parties representing parts of labor (sectors or demographic groups) and parts of business. These cross-factor coalitions would implement income security policies as well as price support policies. In liberal market economies—which are all SMD systems—labor is never big enough to be an electoral majority by itself (Przeworski and Sprague 1986). Majoritarian parties forge broad pre-election coalitions by championing policies that many voters care about somewhat, at the expense of policies that a minority of voters cares about intensely.[17]

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[15] Persson and Tabellini's work challenges us to think more carefully about what happens to our predictions when districts are extremely heterogeneous, as they assume. On the other hand, their 2-party model of PR systems leaves ample room for improvement.

[16] See the book by the same title, edited by Peter Hall and David Soskice, 2000.

[17] The Varieties of Capitalism notion pushes uws to reevaluate an important aspect of our model. We make a clear distinction between societal interests that are and are not represented by a party in government in deriving utilities of these interests. That makes us curious why so many European countries have institutionalized central wage bargaining structures that lock in, to a certain degree, the influence of employers and of unions, apart from their representation in the cabinet. Although it is beyond the scope of

Another future extension we would like to consider is to see how "ideology" influences the choice of coalition partners. In our framework, ideology could make some coalitions of parties more likely to occur than others, along the lines of Axelrod's Minimal Connected Winning Coalitions (1970). The left-right dimension clearly exists in PR systems as an important basis for political competition; all countries have, broadly construed, parties on the right left and on the right. The other dimensions—they are many and varied—tend to support somewhat smaller parties but are nonetheless important enough in the coalitional bargaining process to undermine the usefulness of a single dimension as a way of organizing political competition. The strength of the left-right continuum across types of electoral systems speaks to the importance of retrospective voting on the basis of income as an electoral motive. The environment, religion, race, or social values seem to galvanize smaller groups of voters, all else equal.

Conclusion

We have argued that coalition parties behave differently than coalitions of parties because parties are electorally accountable agents. When a government coalition is a single party, it is accountable for all policy decisions and, if it wants to stay in power, it must appeal to a broad constituency. Parties in coalition governments can associate themselves with some issues and distance themselves from others via the priorities established in platforms and campaigns. When the party system is sufficiently fragmented, splinter parties can maintain a reasonably good chance of being in government merely by appealing to their respective narrow constituencies. Because coalition parties implement more efficient logrolls, voters are overall better off in consolidated party systems, but this kind of party system may not be sustainable with a proportional electoral system. The need to form a MWC does create a centripetal incentive with PR, and this incentive does reduce the size of the public sector, but not to the level that would be achieved with SMD.

Three principal points emerge from this way of conceptualizing coalition politics. First, research on the U.S. and European politics typically assumes either unidimensional or multidimensional issue space, but we show that dimensionality depends on electoral rules. Under SMD, where only one party can win a plurality, it makes strategic sense for parties to situate themselves along a unidimensional continuum as an electorally valuable commitment mechanism. Under PR, by contrast, parties are driven to find an electorally secure niche, and this niche may not lie along a single dimension at all.

Second, despite the centrifugal forces that operate in PR systems, some centripetal incentives nevertheless remain. If larger parties have an advantage in forming a minimal winning coalition, constituency groups should be motivated to cooperate is supporting an amalgamated party. Once one such merger has formed, however, the centrifugal incentives dominate and the remaining constituency groups prefer to support splinter parties.

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this paper, we would like to explore in future research how this institutional feature fits with our model, and how it affects the low social utility that our model assigns to PR.

Third, we show that SMD and PR systems aggregate the interests of voters in systematically different ways. Assuming homogeneous SMD electoral districts, majoritarian parties should generate smaller government budgets than their multi-party counterparts in PR systems because they internalize more of the costs of spending on constituency projects. Multi-party governments preside over more expensive logrolls on behalf of groups of constituents, because each party in the cabinet is electorally accountable to a distinct constituency group.

Our model can be amended and extended in a number of directions, some of which we have already discussed. A primary limitation of the current model is that parties are not truly strategic players. Our plan is to imbed this model of electoral accountability into a model in which entry decisions by new parties are modeled explicitly. A second area that bears further exploration is the impact of the size of the group of unattached voters. The set of advanced parliamentary democracies to which our model applies tend to have well-informed electorates (compared to the U.S), but it may nonetheless be the case that unattached voters are a larger group than we have considered here. The impact of heterogeneous districts in SMD is another important factor that begs explicit consideration.

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Appendix: Deriving Expected Pay-offs given Electoral and Coalitional Uncertainty

The pay-offs in Figure 1-7 are the expected utilities associated with each combination of actions, taking account of uncertainty associated with (a) the unattached voters, and (b) coalition formation. Recall that the unattached voters number slightly less than any single group.

Figure 1: SMD with Unified Opposition.

Groups 4, 5 and 6 have coalesced to form Party B. Party B will win with probability 1 unless groups 1, 2, 3 all support Party A. If Party B wins, each of groups 1, 2, and 3 pays 1/6 of 3 projects, each of which costs 1. So the pay-offs in any cell but (a) are –1/2. If 1, 2, and 3 all support Party A, then the election hinges on the unattached voters. With probability .5, Party A and each group gets 1- 1/6(1+1+1) = 1/2. The expected utility is .5 (1/2) +.5 (-1/2) = 0.

Figure 2a: SMD with Fragmented Opposition

Groups 4, 5 and 6 splinter.

Cell (a): 1, 2 and 3 coalesce. A wins with probability 1. Pay-offs are 1/2 all around.

Cells (b), (c) and (e): Two groups coalesce, one splinters. Again A with probability 1.

Cells (d), (f), (g): One group supports coalition party, other two splinter. Six parties are viable. Whichever one gets the support of the unattached will form a single party government.

TAble A-1 illustrates cell (d) where group 1 is the lone coalescer.

Table A-1

			Platform					
Party	Wins Votes From	Wins w/ prob	x1	x2	x3	x4	x5	x6
A	1	1/6	1	1	1			
D	2	1/6		3				
E	3	1/6			3			
F	4	1/6				3		
G	5	1/6					3	
H	6	1/6						3

$$EU_1 = \frac{1}{6}\left(\frac{1}{2}\right) + \frac{5}{6}\left(-\frac{1}{6}(3^2)\right) = -\frac{7}{6}$$

$$EU_{2-3} = \frac{1}{6}\left(\frac{1}{2}\right) + \frac{1}{6}\left(3 - \frac{1}{6}(3^2)\right) + \frac{2}{3}\left(-\frac{1}{6}(3^2)\right) = -\frac{3}{4}.$$

Cells (f) and (g) have the same form.

Cell (h): Same as in Figure A-1, except that group 1 votes for splinter party C ($x1=3$) rather than coalition party A. The party that gets the unattached forms a single party govt. Each group's expected pay-off is

$$EU(\text{Game 2, cell h}) = \frac{1}{6}\left(3-\frac{1}{6}(3^2)\right)+\frac{5}{6}\left(-\frac{1}{6}(3^2)\right) = -1$$

Figure 2B: **Groups 1and 2 support submajority coalition party A′. Group 3 splinters.**

Cell (a): If all three groups support Party B, it is bigger than the two-group coalition, regardless of the behavior of the unattached. Each group gets 1 – 1/6(1 + 1 + 1) = ½.

Cells (b), (c) and (e): If two of the three groups support Party B, it wins with probability .5 and Party A (x1= 3/2 and x2=3/2) wins with probability .5. (If the unattached vote with one of the splinter parties the election is a tie between A′ and B, in which the final outcome is decided by a fair coin flip.) The expected utility is

$$EU(\text{Game 2b, cell b}) = \frac{1}{2}\left(1-\frac{1}{6}(1^2+1^2+1^2)\right)+\frac{1}{2}\left(-\frac{1}{6}\left(\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2\right)\right) = -\frac{1}{8}$$

Cells (d), (f), (g) and (h): If one or none of the groups votes for B, then A′ wins with probability one. The pay-off is

$$EU(\text{Game 2b, cell h}) = -\frac{1}{6}\left(\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2\right) = -\frac{3}{4}.$$

Figure 3, PR with Unified Opposition

Groups 4, 5 and 6 support coalition party B platform $x4 = x5 = x6 = 1$.

Cell (a): Groups 1,2 and 3 all support Party A. Given that 4, 5 and 6 are united behind Party B, what we have here is the BTPS. Whichever party wins the support of the unattached will win a majority, and be able to form a single party government. The ex ante expected pay-offs are zero, the same as under SMD.

Cells (b) (c) and (d). Two groups support A and the third splinters. Consider cell (b). There are three viable parties,

Table A2 – Game 3, cell (b)

		Platform					
Party	Wins Votes From	x1	x2	x3	x4	x5	x6
A	1,2	1	1	1			
E	3			3			
B	4, 5, 6				1	1	1

With probability 1/3, B wins the support of the unattached and is able to form a single party government. Pay-offs to 1, 2, 3 in this case are –1/2. With probability 2/3, the unattached vote for A or E, and in this case, any two parties form a majority. Group 1's and 2's utilities from the possible coaltions are

$$U_{1,2}(AB) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2) = 0$$

$$U_{1,2}(AE) = 1 - \frac{1}{6}(1^2 + 1^2 + 3^2) = -\frac{5}{6}$$

$$U_{1,2}(BE) = -\frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = -2$$

giving an overall expected utility

$$EU_{1,2}(\text{Cell b}) = \frac{1}{3}\left(-\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{3}(0) + \frac{1}{3}\left(-\frac{5}{6}\right) + \frac{1}{3}(-2)\right) = -\frac{43}{54}.$$

Similarly, group three's pay-offs from the possible coalitions are

$$U_3(AB) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2) = 0$$

$$U_3(AE) = 3 - \frac{1}{6}(1^2 + 1^2 + 3^2) = \frac{7}{6}$$

$$U_3(BE) = 3 - \frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = 1$$

which give an expected pay-off of

$$EU_3(\text{Cell b}) = \frac{1}{3}\left(-\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{3}(0) + \frac{1}{3}\left(\frac{7}{6}\right) + \frac{1}{3}(1)\right) = \frac{17}{54}.$$

Cells(d), (f), (g). Two groups splinter, one coalesces. There are four viable parties, each of which gets the support of the unattached with probability ¼

Table A3 – Game 3, cell (d)

		Platform					
Party	Wins Votes From	x1	x2	x3	x4	x5	x6
A	1	1	1	1			
D	2		3				
E	3			3			
B	4, 5, 6				1	1	1

The probability that B forms a single party majority is now ¼, as above pay-off to all three are

$$U_{1,2,3}(\text{B majority}) = -\frac{1}{2}.$$

With probability ¾, the unattached go to A, D, or E, and in any of these cases, the MWC is
B plus one other party (with equal probability). For group 1, the pay-offs are

$$U_1(BE) = U_1(BD) = -\frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = -2$$

$$U_1(BA) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2) = 0$$

giving an overall pay-off of

$$EU_1(\text{Game 3, cell d}) = \frac{1}{4}\left(-\frac{1}{2}\right) + \frac{1}{4}(0) + \frac{2}{4}(-2) = -\frac{9}{8}.$$

For group 2, we have

$$U_2(BA) = 0$$

$$U_2(BE) = -2$$

and

$$U_2(BD) = 3 - \frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = 1$$

which gives an overall payoff

$$EU_2(\text{Game 3, cell d}) = \frac{1}{4}\left(-\frac{1}{2}\right) + \frac{1}{4}(0) + \frac{1}{4}(-2) + \frac{1}{4}(1) = -\frac{3}{8}.$$

Group 3's pay-off is the same as group 2's.

Cell (h). The situation is the same as cell (d) except that party A is replaced by party C.

$$U_1(BC) = 3 - \frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = 1$$

and

$$U_{2,3}(BC) = -\frac{1}{6}(3^2 + 1^2 + 1^2 + 1^2) = -2$$

so that

$$EU_{1,2,3}(\text{Game 3, cell h}) = \frac{1}{4}\left(-\frac{1}{2}\right) + \frac{2}{4}(-2) + \frac{1}{4}(1) = -\frac{7}{8}.$$

Figure 4, PR with Fragmented Opposition. Groups 4, 5 and 6 splinter (supporting splinter parties E, F, G)

Cell (a). Groups 1,2 and 3 all support coalition party A. With probability ¼, the unattached vote for A and it forms a single-party government with payoffs

$$U_{1,2,3}(A) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2) = \frac{1}{2}.$$

With probability ¾, the unattached support a party other than A. In this case, the MWC's consist of A plus one other party. In all cases, the pay-off is

$$U_{1,2,3}(AF) = U_{1,2,3}(AG) = U_{1,2,3}(AH) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2 + 3^2) = -1.$$

The expected utility is

$$EU_{1,2,3}(\text{Game 4, cell a}) = \frac{1}{4}\left(\frac{1}{2}\right) + \frac{3}{4}(-1) = -\frac{5}{8}$$

Cells (b) (c) and (e): Two groups support A and the third splinters. Consider cell (b). There are five viable parties, each of whose support and platforms are displayed in Table A4.

Table A4

		Platform					
Party	Wins Votes From	x1	x2	x3	x4	x5	x6
A	1,2	1	1	1			
E	3			3			
F	4				3		
G	5					3	
H	6						3

The distribution of election and coalition outcomes is

With prob.	Unattach support	MWC's are
1/5	A	AE, AF, AG, AH, each w/ prob ¼
1/5	E	AE
1/5	F	AF
1/5	G	AG
1/5	H	AH

Coalitions AF, AG and AH each occur with probability ¼ and each give the same pay-offs for groups 1-3

$$U_{1,2,3}(AF) = U_{1,2,3}(AG) = U_{1,2,3}(AH) = 1 - \frac{1}{6}(1^2 + 1^2 + 1^2 + 3^2) = -1.$$

Coalition AE occurs with probability ¼ and gives pay-offs

$$U_{1,2}(AE) = 1 - \frac{1}{6}(1^2 + 1^2 + 3^2) = -\frac{5}{6}$$

7

$$U_3(AE) = 3 - \frac{1}{6}(1^2 + 1^2 + 3^2) = \frac{5}{6}.$$

The ex ante pay-offs are

$$EU_{1,2}(\text{Game 4, cell b}) = \frac{3}{4}(-1)+\frac{1}{4}(-\frac{5}{6}) = -\frac{23}{24}$$

$$EU_{3}(\text{Game 4, cell b}) = \frac{3}{4}(-1)+\frac{1}{4}(\frac{7}{6}) = -\frac{11}{24}.$$

Cells (d), (f) and (g): One groups supports coalition party A, the other two splinter. Table A5 depicts the situation for cell (d).

Table A5

		Platform					
Party	Wins Votes From	x1	x2	x3	x4	x5	x6
A	1	1	1	1			
D	2		3				
E	3			3			
F	4				3		
G	5					3	
H	6						3

The MWC will include 3 of the 6 parties, each with equal ex ante probability. There are a total of 20 combinations. First consider group 1's pay-offs. There are 10 MWC's that do not include Party A,

$$U_1(\text{A out}) = -\frac{1}{6}\left(3^2+3^2+3^2\right) = -\frac{9}{2}.$$

Of the remaining 10, one includes both D and E

$$U_1(ADE) = 1-\frac{1}{6}\left(1^2+3^2+3^2\right) = \frac{-13}{6}.$$

Six include either D or E but not both

$$U_1(\text{A plus D or E}) = 1-\frac{1}{6}\left(1^2+1^2+3^2+3^2\right) = \frac{-7}{3}.$$

And three include neither D nor E

$$U_1(\text{A without D or E}) = 1-\frac{1}{6}\left(1^2+1^2+1^2+3^2+3^2\right) = \frac{-5}{2}.$$

So group 1's expected utility is

$$EU_1(\text{Game 4, cell d}) = \frac{1}{20}\left(-\frac{13}{6}\right)+\frac{6}{20}\left(-\frac{7}{3}\right)+\frac{3}{20}\left(-\frac{5}{2}\right)+\frac{1}{2}\left(-\frac{9}{2}\right) = -\frac{103}{30}.$$

From the point of view of group 2, the relevant considerations are (1) whether its party, D, is included in government, (2) whether low-cost party A is in government, and (3) if A is in government, whether E (whose platform overlaps with A) is in government. See Table A6

Table A6

Coalition Type	Probability	Utility for Group 2
ADE	1/20	3 – 19/6 = -1/6
AD + ¬ E	3/20	3 – 20/6 = -1/3
AE + ¬D	3/20	1 – 20/6 = -7/3
A+ ¬D + ¬E	3/20	1 – 21/6 = -15/6 = -5/3
D + ¬A	6/20	3 – 27/6 = -9/6 = -3/2
¬D + A	4/20	-27/6 = -9/2

The expected utility is

$$EU_2(\text{Game 4, cell d}) = \frac{1}{20}\left(-\frac{1}{6}\right)+\frac{3}{20}\left(-\frac{1}{3}\right)+\frac{3}{20}\left(-\frac{7}{3}\right)+\frac{3}{20}\left(-\frac{5}{2}\right)+\frac{6}{20}\left(-\frac{3}{2}\right)+\frac{4}{20}\left(-\frac{9}{2}\right)$$
$$= -\frac{256}{120} = -\frac{32}{15}.$$

Player 3 has the same expected utility, and the pay-offs for cells (f) and (g) are symmetric.

Cell (h) is simply the MFPS with PR, all pay-offs are –3.

The pay-offs to the remaining games are derived in the same fashion.

Table 1: Electoral Systems, Party Systems, Type of Government and Policy Choices

(a) Likelihood of Multi-Party Government

	SMD	PR
BTPS	0	0
MFPS	0	1
Submajority Coalition Parties	0*	1

(b) Size of the Public Sector (cost)

	SMD	PR
BTPS	3	9
MFPS	9	27
Submajority Coalition Parties	9/2*	18

Key:
Not sustainable as a Nash Equilibrium
Pareto dominated by another NE
*Need to add this reasoning to text

Figure 1: GAME 1, SMD with Unified Opposition.
No Centrifugal Incentive.

Group 1: Support A

		Group 3: Support A		Group 3: Support E	
Group 2: Support A		(a) 0 0 0		(b) -1/2 -1/2 -1/2	
Group 2: Support D		(c) -1/2 -1/2 -1/2		(d) -1/2 -1/2 -1/2	

Group 1: Support C

		Group 3: Support A		Group 3: Support E	
Group 2: Support A		(e) -1/2 -1/2 -1/2		(f) -1/2 -1/2 -1/2	
Group 2: Support D		(g) -1/2 -1/2 -1/2		(h) -1/2 -1/2 -1/2	

Nash equilibria: Cell (a) is the unique NE

Note on depiction of three-person games: Group 1 chooses top or bottom panel, group 2 chooses row and group 3 chooses column. Group 1's pay-offs are in the center of the cell, group 2's in the lower left and group 3's in the upper right.

Figure 2(a): GAME 2a, SMD with Fragmented Opposition
Moderately Strong Centripetal Incentive

Group 1: Support A

		Group 3 Support A		Group 3 Support E	
Group 2 Support A		(a) 1/2 1/2 1/2		(b) 1/2 1/2 1/2	
Support D		(c) 1/2 1/2 1/2		(d) -3/4 -7/6 -3/4	

Group 1: Support C

		Group 3 Support A		Group 3 Support E	
Group 2 Support A		(e) 1/2 1/2 1/2		(f) -3/4 -3/4 -7/6	
Support D		(g) -7/6 -3/4 -3/4		(h) -1 -1 -1	

Nash equilibria: Cells a, b, c, e and h.

Figure 2(b): GAME 2b, SMD with Submajority Opposition
Centripetal Incentive Destabilizes Submajority Party

Groups 1and 2 support submajority coalition party A′. Group 3 splinters. Do 4, 5 and 6 coalesce to support Party B?

Group 4: Support B

		Group 6: Support B		Group 6: Support H	
Group 5: Support B		(a) 1/2 1/2 1/2		(b) -1/8 -1/8 -1/8	
Group 5: Support G		(c) -1/8 -1/8 -1/8		(d) -3/4 -3/4 -3/4	

Group 4: Support F

		Group 6: Support B		Group 6: Support H	
Group 5: Support B		(e) -1/8 -1/8 -1/8		(f) -3/4 -3/4 -3/4	
Group 5: Support G		(g) -3/4 -3/4 -3/4		(h) -3/4 -3/4 -3/4	

Nash equilibria: Cell a.

Figure 3: GAME 3, PR with Unified Opposition
Strong Centrifugal Incentive

Group 1: Support A

			Group 3 Support A		Group 3 Support E	
			(a)	0	(b)	17/54
			0		-43/54	
Group 2	Support A	0			-43/54	
			(c)	-43/54	(d)	-3/8
			-43/54		-9/8	
	Support D	17/54			-3/8	

Group 1: Support C

			Group 3 Support A		Group 3 Support E	
			(e)	-43/54	(f)	-3/8
			17/54		-3/8	
Group 2	Support A	-43/54			-9/8	
			(g)	-9/8	(h)	-7/8
			-3/8		-7/8	
	Support D	-3/8			-7/8	

Nash equilbria: Cell h.

Figure 4: GAME 4, PR with Fragmented Opposition
Moderate Centripetal Incentive

Group 1: Support A

		Group 3 Support A		Group 3 Support E	
Group 2 Support A		(a) -5/8 -5/8 -5/8		(b) -11/24 -23/24 -23/24	
Support D		(c) -23/24 -23/24 -11/24		(d) -32/15 -103/30 -32/15	

Group 1: Support C

		Group 3 Support A		Group 3 Support E	
Group 2 Support A		(e) -23/24 -11/24 -23/24		(f) -32/15 -32/15 -103/30	
Support D		(g) -103/30 -32/15 -32/15		(h) -3 -3 -3	

Nash equilbria: Cells b, c, e and h.

Figure 5: GAME 5, PR with Fragmented Opposition
Centripetal Incentive for a Two-Party Coalition

		Group 2	
		Support A′	Support D
Group1	Support A′	(a) -3/4 -3/4	(b) -27/20 -33/10
	Support C	(c) -33/10 -27/20	(d) -3 -3

Nash equilbria: Cells a and d.

Figure 6: GAME 6, PR with Partly Coalesced Opposition
No incentive to form a second two-party coalition.

		Group 5 Coalesce		Group 5 Splinter	
Group 4 Coalesce		(a) -3/2		(b) -9/16	
		-3/2		-27/16	
Group 4 Splinter		(c) -27/16		(d) -3/2	
		-9/16		-3/2	

Nash equilbria: Cell d.

Figure 7: GAME 7, PR with Party Coalesced Opposition
No incentive to form a three-party coalition.

Group 4: Coalesce

		Group 6 Coalesce		Group 6 Splinter	
Group 5	Coalesce	(a) -11/18 -11/18 -11/18		(b) -11/36 -41/36 -41/36	
	Splinter	(c) -41/36 -41/36 -11/36		(d) -1 -7/4 -1	

Group 4: Splinter

		Group 6 Coalesce		Group 6 Splinter	
Group 5	Coalesce	(e) -11/36 -41/36 -41/36		(f) -1 -1 -7/4	
	Splinter	(g) -7/4 -1 -1		(h) -3/2 -3/2 -3/2	

Nash equilbrium: Cell h.