Social Identity, Electoral Institutions, and the Number of Candidates

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The existing empirical literature in comparative politics holds that social cleavages affect the number of candidates or parties when electoral institutions are "permissive." However, this literature lacks a theoretical account of the strategic candidate entry and exit decisions that ultimately determine electoral coalitions under different institutions in plural societies. This paper incorporates citizen-candidate social identities into game-theoretic models of electoral competition under both plurality and majority-runoff electoral rules. Our theoretical results indicate that social group demographics *can* affect the equilibrium number of candidates, even in non-permissive systems. Under plurality rule, the relationship between social homogeneity and the effective number of candidates is non-monotonic and, contrary to the prevailing Duvergerian intuition, for some demographic configurations even the *effective* number of candidates cannot be near two. We find empirical patterns in cross-national data on presidential election results that are consistent with key intuitions derived from the formal model.

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1 Introduction

What are the consequences of social cleavages for the structure of electoral competition in democratic polities? Do social divisions determine the number of candidates competing in elections, and if so, by what mechanism? What is the role of specific electoral institutions in mediating this relationship?

Accounting for the number of candidates or parties is a classic question in comparative politics; a large empirical literature explores the implications of social cleavages and political institutions for the effective number of electoral candidates or parties (e.g. Duverger 1954, Ordeshook and Shvetsova 1994, Neto and Cox 1997, Chhibber and Kollman 2004, Clark and Golder 2006, Golder 2006). The current consensus within this literature is that social cleavages leave a direct imprint on the structure of electoral coalitions only when election rules allow them to do so; empirical studies demonstrate a positive relationship between ethnic fractionalization and the number of candidates or parties under so-called "permissive" electoral systems, but little if any relationship when institutions are not "permissive." The literature, however, leaves strategic candidate entry decisions unmodeled; as a result, it lacks a theoretical mechanism accounting for the incentives of candidates to enter or exit campaigns in specific demographic settings and under specific electoral rules.

This paper explores the logic of such a mechanism as it might play out in a demographically simple society by incorporating social identities into a game-theoretic model of candidate entry and competition. Following the empirical literatures on public opinion and voting behavior, along with insights from psychology and sociology, we assume that social identity can provide an important motivation for political behavior, including vote choice and decisions to seek office (or not to). In our model, citizen-candidate utility depends not only on policy outcomes and other familiar factors, such as entry costs and the benefits of office, but also on identity-related payoffs, an innovation that allows us to make a detailed theoretical connection between social group demographics and the equilibrium number of candidates or parties in a polity. We study electoral competition under both plurality rule and majority runoff systems, two common and well-studied institutions.

Our theoretical results resemble insights of the existing literature in some respects, but not in others. The most striking departure is our finding that demographics strongly influence the number of candidates in electoral equilibria even in the highly "unpermissive" plurality system. Specifically, we find that plurality-system equilibria reflecting a Duvergerian election with two (effective) candidates can be sustained either within a socially homogeneous population *or* within a closely-divided one, but *not* over a range of intermediate demographics. This non-monotonicity results from a discontinuity, previously unnoticed in the literature, in the dynamics of electoral competition as the size of the largest social group increases; intuitively, groups of different relative sizes have different "carrying capacities" for the number of candidates that can be supported in an election without risking coordination failure. Our other findings include that: (i) two-candidate equilibria do not exist under a majority runoff system; (ii) single-candidate equilibria do not exist under either plurality or majority runoff rules; and (iii) multi-candidate equilibria exist under both systems but are less likely under plurality rule than under majority runoff rule.

We then explore the potential empirical relevance of these theoretical intuitions by analyzing cross-national data from presidential elections in the 1990s. Consistent with these intuitions, we find that the effective number of candidates actually *increases* in plurality elections as *social homogeneity* increases across a specific threshold. This finding suggests that, contrary to the literature, social group demographics *can* affect the equilibrium number of candidates or parties not only under permissive electoral systems but under more restrictive ones as well. We find that a regression specification explicitly modeling the demographic discontinuity derived in our theoretical section outperforms standard specifications in predicting the effective number of candidates based on social demographics. These empirical findings contribute to the growing empirical literature on the determinants of candidate and party participation in comparative elections, and suggest the usefulness of our theoretical strategy of explicitly modeling candidate entry decisions in divided societies. While our theoretical results were derived under a number of simplifying assumptions, we believe that our model offers a helpful starting point to research on this question, and that future extensions of the model may provoke further empirically relevant insights on electoral competition in heterogeneous societies.

The paper contains five additional sections. Section 2 motivates the analysis by discussing the importance of identity-related considerations in determining the political behavior of citizens. Section 3 presents a model of electoral competition in which citizen candidates who care about policy, office, and their social groups decide whether to enter an election as candidates and how to cast their ballots given the entry and voting decisions of a polity's other citizens. Section 4 describes the implications of the model for the equilibrium number of candidates under simple plurality and majority runoff rules. Section 5 discusses the empirical fit of the model for presidential elections around the world during the 1990s. The final section contains a summary of key findings and a discussion of possible extensions.

2 Identity-Related Behavior and Elections

Virtually all of the seminal empirical work on voting emphasizes the importance of one type of social identity or another for explaining why citizens cast the ballots that they do (e.g. Lazarsfeld, Berelson, and Gaudet 1944; Campbell, Converse, Miller, and Stokes 1960; Lipset and Rokkan 1967). The empirical foundation of such accounts of voting is derived largely from the correlations between social category membership and vote choice found in survey data.

The interpretation of these correlations, however, is highly contested. On this question there are two main schools of thought. The first is that the correlation between social group membership and vote choice simply reflects the extent to which individuals in the same social groups have similar policy interests (e.g. Bates 1974; Rabushka and Shepsle 1974; Chandra 2004). The extreme version of this view is that social identity is epiphenomenal, playing no independent role in motivating behavior once individual policy preferences are taken into account. An alternative perspective holds that individuals develop psychological attachments to social groups (e.g. Horowitz 1985) and that the correlation between social group membership and vote choice is heightened by these attachments. In this view, the act of voting is at least in part expressive rather than instrumental, and identity is a direct and central causal determinant of political behavior.

It is well beyond the scope of this paper to review the theoretical and empirical merits of these two interpretations. In our view, both the rational-choice policy-based and psychological identity-related research traditions contain valuable insights into voter behavior. As such, we develop a model that explores the consequences for party systems if indeed citizens are motivated by both their policy interests and their social identities.

To incorporate identity-related political behavior into a model of electoral competition, it is necessary to alter standard formulations of citizen utility in a manner consistent with basic empirical findings about the role identity plays in motivating behavior. We follow Akerlof and Kranton (2000) by adopting a utility function with the following general form:

$$U_i = U_i(\mathbf{a}_i, \mathbf{a}_{-i}, I_i) \tag{1}$$

where individual i's utility depends on her actions, $\mathbf{a}_i$; on the actions of other individuals, $\mathbf{a}_{-i}$; but also, unlike in standard models, on i's identity or self-image, I_i. The Akerlof-Kranton model of identity is based on the assignment of social categories. Individuals place themselves and others in society in some finite set of categories, $\mathbf{C}$. Let $\mathbf{c}_i$ be a mapping for individual i assigning the set of all individuals, $\mathbf{F}$, to categories in $\mathbf{C}$ ($\mathbf{c}_i : \mathbf{F} \to \mathbf{C}$). Crucially, social categories may be associated with *behavioral prescriptions* $\mathbf{P}$, which are sets of actions (or characteristics) deemed appropriate for individuals in given social categories. Finally, individuals are endowed with basic characteristics, ϵ_i, that are not *a priori* assumed to be correlated with social categories. Identity payoffs are then represented as:

$$I_i = I_i(\mathbf{a}_i, \mathbf{a}_{-i}; \mathbf{c}_i, \epsilon_i, \mathbf{P}) \tag{2}$$

In the Akerlof-Kranton framework, a person's identity depends on his or her social categories assigned by $\mathbf{c}_i$, which may be exogenous and fixed or endogenously chosen. Identity is also allowed to be a function of the extent to which an individual's own characteristics, ϵ_i, match any ideal characteristics, defined by $\mathbf{P}$, associated with the social categories to which he or she is assigned. Most relevant for us, identity payoffs may also depend on the extent to which an individual's own actions, $\mathbf{a}_i$, and the actions of others, $\mathbf{a}_{-i}$, correspond to the behavioral prescriptions for social categories, also defined by $\mathbf{P}$. The violation of prescriptions associated with social categories is thought to generate anxiety and thus identity losses.[1]

The model of identity formalized in Equation 2 is based on the key principles of social identity theory (Tajfel and Turner 1979, 1986; Tajfel 1981; Turner 1984). Individuals are understood to have a sense of self or ego that is defined on both an individual and collective basis. The construction of the self involves a process of identification in which one associates oneself with others in one's social categories and differentiates oneself from nonmembers. To the extent to which social rather than personal identity is salient, self-esteem, understood to be a central motivation of behavior, is substantially determined and maintained by individuals' social settings and the categories or roles they fill in that environment.

In section 3, we adapt this framework to the context of voter behavior in the following ways. An individual citizen candidate must decide whether to enter an election as a candidate

[1] We therefore take the position that individuals internalize relevant group prescriptions, and that the identity issues in question are therefore psychological in nature rather than a result of external enforcement.

for office and how to cast her ballot ($\mathbf{a}_i$) given the entry and voting decisions of the other citizen candidates ($\mathbf{a}_{-i}$). With respect to social identity, we assume that the mapping of social categories ($\mathbf{c}_i$) is exogenous and fixed, that it is commonly held, and that it partitions the voter population, so that each member of the public is unambiguously affiliated with a single social group, both in her mind and in the minds of all the other actors.[2] We will also suppose that for each social group there exists a behavioral prescription $\mathbf{P}$ instructing citizen candidates to choose no actions (entry or vote choice) that might harm the electoral performance of the group. Those that violate this prescription will suffer identity losses that reflect psychological anxiety generated by deviating from internalized behavioral prescriptions.

3 The Model

In this section, we define a citizen candidate electoral model (Osborne and Slivinski 1996, Besley and Coate 1997) that incorporates identity-related behavior as discussed in the previous section. Our model adopts all the features of Osborne and Slivinski's citizen candidate model and adds the exogenous assignment of two social identities that partition the population and motivate individual behavior.

We begin our description by formally defining how identity concerns are incorporated into the model. Citizens are associated with exactly one social group which is indexed by j; the set of possible social groups {A,B} partitions the population.[3] Let A and B equal the proportion of citizens from groups A and B respectively. We assume throughout that A is the larger group,

[2] Obviously, the relevant social categories ($\mathbf{C}$) for political competition in the real world are in part endogenous and a matter for contestation. This paper takes a given set of politically relevant identities and examines how demographic characteristics and institutional rules interact to determine salient features of the party system.

[3] This assumption, by which the definitions of salient social groups are exogenous and clearly defined, is nonetheless consistent with the main claim of constructivist scholars of ethnicity. Although social identity categories are clearly constructed, they are often persistent and difficult to reconstruct, particularly over the relatively short time frames of a single election as examined here. See e.g. discussions in Van Evera (2001) and Darden (2004). Obviously, in some contexts treating social groups as exogenous and clearly defined will not be warranted but for many polities, the social groups salient for electoral competition remain stable over substantial periods of time consistent with our assumptions.

so that $A > B$ and $A \in (\frac{1}{2}, 1)$. A citizen i who is a member of group j has a utility function

$$U_i = -|x - x_i^*| + \gamma_i + g_i(j) - c_i - I_i(j) \tag{3}$$

and must decide whether to enter (E) or not to enter (N) an election as a candidate for office.

The first term represents actors' policy interests. The set of possible policy outcomes is represented by a one-dimensional space $\mathbf{X}$ with real elements x. Each citizen i has a policy ideal point x_i^* at which her policy utility would be maximized, and single-peaked preferences over the set of policy positions. The first term in the utility function specifies the policy utility as a function of the distance between the policy outcome x and i's ideal point. This policy outcome term is operative whether or not i decides to become a candidate in the election. For groups A and B respectively, the distribution functions of citizens' ideal points are given by F_A and F_B. We assume both of these to be continuous, and we assume that both have unique medians. The distributions F_A and F_B may be the same or they may be different; their supports may also be the same or may be different.

The following terms relate more specifically to i's entry decision. γ_i is an indicator variable equalling $\gamma > 0$ if i enters as a candidate and wins the election, but equalling 0 if i either enters but loses the election, or if i does not enter as a candidate. As such, γ represents the size of the reward associated with the benefits of winning an election. If a candidate wins an election with probability p, her expected utility from winning will therefore be γp.

The next term describes an alternative electoral benefit that a losing candidate can receive: the status that comes from being the most electorally popular candidate from her own group, though losing the election itself. While such benefits are not institutionalized in nature, leading a campaign and receiving a stronger endorsement from one's own group members than other candidates may bestow a certain level of credibility that can be useful in other parts of the political process or in future political campaigns. Such status may of course also provide consumption value to candidates. $g_i(j)$ takes the form of an indicator variable that equals $g(j) > 0$ if i enters the race and loses it, but is the most successful candidate in group j. Otherwise, $g_i(j)$ is equal to 0. That is, $g_i(j) = 0$ if i enters the race and wins it; if i enters the race, loses, and is also not the most successful candidate from group j; or if i does not enter the race at all. It is of course intuitive to think of the "consolation prize" g as being substantially smaller than γ for two reasons. First, overall winners of elections are also the most successful candidates from *their* own groups, so that γ implictly includes benefits from group leadership in the election as well as from the benefits of office. And second, the institutionalized benefits from office would seem likely to be substantially stronger than the status that could be gained from losing a good fight in almost all settings. To reflect this, we will assume throughout that $\gamma > 2g(j)$ for all permitted values of j. We also note that we assume $g(.)$ to be strictly increasing in the size of the group, in particular that $g(A) > g(B)$. If a number of losing candidates from a given group tie, we assume the benefits of group leadership to be divided evenly among them.

The next term, c_i, represents the cost of entry. c_i is an indicator variable taking on the value of $c > 0$ for citizen i if that citizen becomes a candidate in the election, but the value 0 if the citizen chooses not to enter. We assume throughout that unambiguously winning an election or receiving the largest vote share of in-group support is always worthwhile, so that $\frac{\gamma}{2} > g(j) > c$.

The final term, $I_i(j)$, represents the identity-related payoffs that are attached to the acts of voting and candidacy. We specify I in the following way. If a citizen or citizen-candidate takes no action that harms the electoral performance of her group, $I = 0$. If, on the other hand, a citizen or citizen-candidate does take such an action, $I_i(j) = k_j > 0$, so that a utility loss occurs from violating the behavioral prescriptions associated with group membership. Specifically, a voter will be considered to act against her group's interests if she casts a vote in favor of a candidate from a group not her own; otherwise, she will not be considered to act against her group's interests. A citizen who has decided to enter (exit) as a candidate will be considered to act against her group's interests if this act of entry (exit) reduces the group's overall vote share or aids the victory of a candidate from another group; otherwise, she will not be considered to act against her group's interests. For clarity of analysis we will take the k_j to be effectively infinite so that no voter or candidate will ever act against her group.

This assumption defines citizens, in their roles as voters and candidates, as having lexicographic preferences over the social identity of their political representatives. As noted earlier, large empirical literatures exist demonstrating the importance of identity concerns in motivating individuals' behavior both in politics and more generally. It is important to note, however, that lexicographic identity preferences do not necessarily follow from either the theoretical literature on social identity or the existing empirical literature referred to in Section 2. Our discussion in that section simply claimed that identity concerns were an important motivation for behavior. The assumption of lexicographic preferences depends on an additional claim that the magnitude of these concerns relative to other considerations is large, at least in the realm of electoral politics in plural societies. We believe that this claim is quite plausible in the context of many empirical cases – in part, because political elites often have the incentives as well as the capacity to increase the salience of social identities (Dickson and Scheve 2006) – while it is more debatable in others. While some polities exhibit strong tendencies towards "census elections," crossover voting obviously occurs to some extent in many elections, and to a considerable extent in some. Our simplifying assumption that preferences are lexicographic is meant, from a theoretical standpoint, to draw a natural contrast to Osborne and Slivinski's citizen-candidate model, in which identity considerations are assumed to be zero. We believe that there is much to be gained from theoretically fleshing out the polar-opposite case. However, if rates of crossover voting from one social group to the other can be assumed, across the idiosyncracies of different cases, to average out, then the basic qualitative forces at play in our theoretical results can also be expected to emerge in empirical data, though perhaps with some quantitative differences.

The sequence of events in the election game follows Osborne and Slivinski (1996). Citizens choose to enter the election (E) or not (N). If a citizen i enters, she proposes her policy ideal point x_i^*; she is assumed not to be able to credibly commit to a different position. After citizens make their simultaneous entry decisions, they cast their votes. Voting, as in Osborne and Slivinski (1996), is taken to be sincere, with each voter casting her ballot for the candidate yielding the highest utility as determined by Equation 3. Our assumption that the k_j are very large means that sincere voting is consistent with adhering to the behavioral prescriptions of group membership.

We consider two different electoral systems: simple plurality and majority runoff. Under simple plurality rule, the candidate who garners the most votes wins. If two or more candidates tie for first place, then each wins with equal probability (ties among candidates within the same identity group are also resolved by lottery). Under the majority runoff rule, a candidate who receives a majority of votes in the initial election wins. If there is no such candidate, a second election is held between the candidates with the two highest vote totals in the first round. In this case, the candidate who receives a majority of votes in the second ballot wins. Ties in either round are resolved randomly. The solution concept for the model is Nash equilibrium, which we refer to simply as equilibrium or entry equilibrium.

Using this framework, we derive a variety of existence and non-existence results for our model of citizen candidates for a range of different demographics under the two different electoral institutions. We will refer to various configurations of candidates using the notation (y, z), indicating the presence of y candidates from group A (the majority group) and z candidates from group B (the minority group). We present our findings using the following terminology:

Definition. Possible. We say that (y, z) is *possible* if there exist values of c, $g(A)$, $g(B)$, and γ such that a configuration (y, z) constitutes an entry equilibrium.

For both electoral systems, we consider all possible (y, z) configurations containing up to four entered candidates, and demonstrate which are possible and which are not.

4 Equilibrium Number of Candidates

Simple Plurality Elections

We begin with simple plurality elections. The proofs for each proposition can be found in the Appendix; we limit our discussion in the text to establishing the general logic behind each result and considering its empirical implications. The first proposition eliminates the possibility of equilibria in which no members of a given identity group enter the contest as candidates.

> **Proposition 1.** $(0, n)$ is not possible for any n for any $A \in (\frac{1}{2}, 1)$. $(n, 0)$ is not possible for any n for any $A \in (\frac{1}{2}, 1)$.

The intuition for this result is straightforward. If a group does not have a candidate, it fails to win as many votes as it could, and by not entering, its citizens have violated the behavioral prescription by not furthering the group's electoral performance. As such, in equilibrium at least one citizen from the group must always enter, and $(0, n)$ and $(n, 0)$ cannot be equilibria. This result precludes single-candidate elections under plurality rule.

The next proposition also eliminates a set of entry equilibria. Its logic highlights the important role that *minority* group entry decisions play in assisting *majority* group candidates to deter entry by other *majority* group members.

> **Proposition 2.** For $n > 1$, $(1, n)$ is not possible for any $A \in (\frac{1}{2}, 1)$.

In any $(1, n)$ configuration, the A candidate as the single representative from the majority group would clearly win. Further, the policies and entry decisions chosen by the B candidates would not affect the incentives of the A candidate when it comes to policy choice. So the only payoffs earned by B candidates in such an equilibrium would come through leadership of the B group. In particular, if $(1, n)$ is to be an equilibrium, there must be an n-way tie among the n candidates of group B. The existence of more than one B candidate, however, has a major effect on the incentives of potential candidates from group A. Suppose there is an A citizen who happens to be at the ideal point of the A candidate already entered. Such a citizen by entering would split the A vote with the A incumbent, earning vote share $\frac{A}{2}$; and, because $A > B$, this vote share must exceed the vote share $\frac{B}{n}$ earned by each group B candidate. Therefore, such a citizen would tie the election at her ideal policy, and would have an incentive to enter because $\frac{\gamma}{2} > c$. Thus, we do not expect to observe plurality elections in which a single candidate from a majority identity group competes with multiple candidates from the minority group.

The following proposition demonstrates that two-candidate elections are possible in plurality systems but the size of the identity groups must not be too dissimilar for this outcome to exist.

> **Proposition 3.** $(1, 1)$ is possible for any $A \in (\frac{1}{2}, \frac{2}{3})$. But $(1, 1)$ is not possible for any $A \in (\frac{2}{3}, 1)$. In any $(1, 1)$ equilibrium, the sole candidate from the larger group receives vote share A and wins the election, while the sole candidate from the smaller group receives vote share $1 - A$ and loses the election.

The intuition for this result depends on establishing that (i) the A candidate must not wish to drop out; (ii) the B candidate must not wish to drop out; (iii) no other A candidate must wish to enter; and (iv) no other B candidate must wish to enter. For the cases in which a polity's majority group is not too large, $A \in (\frac{1}{2}, \frac{2}{3})$, the first two conditions are obviously met as exit by either candidate would reduce the group's respective vote shares, violating the behavioral prescription and generating identity losses. Now consider whether the incumbent candidates from each group can deter entry. Imagine an A candidate who shares the median A voter's ideal point. Then a potential A entrant could receive no more than half of the A vote; this would result in the B candidate winning the election, since $B > \frac{A}{2}$. As such, it clearly is possible for an A candidate to deter entry by potential A entrants. Similarly, now suppose that the B candidate has the same ideal point as the median B voter; then a potential B entrant could receive no more than half of the B vote. This would result in the B candidates splitting in-group support while leaving A's electoral supremacy unchanged. If $c > \frac{g(B)}{2}$, such a potential B entrant would not find entry worthwhile. This condition does not conflict with any others necessary for equilibrium and so entry deterrence is possible. Consequently, equilibria with one candidate from each identity group are possible for plurality elections if the majority group is not too large, specifically if $A \in (\frac{1}{2}, \frac{2}{3})$.

For polities with very large majorities ($A \in (\frac{2}{3}, 1)$), such an equilibrium is not possible because another A candidate would wish to enter and there are no actions available to the A incumbent to deter entry. This fact is immediately apparent as an A citizen who shared the pre-existing A candidate's policy preference would be able to tie that opponent with $\frac{A}{2}$ of the vote and generate a tie for first place since $\frac{A}{2} > B$.

Thus, although two-candidate elections are generally associated in the literature with simple plurality electoral systems (Duverger 1954), our model suggests that such outcomes depend crucially on the relative size of social identity groups in a polity. We address this prediction in the empirical analysis below.

The next result suggests that the possibility of elections with two majority group candidates and one minority group candidate also depends on the relative group sizes.

> **Proposition 4.** $(2, 1)$ is not possible for any $A \in (\frac{1}{2}, \frac{2}{3})$. But $(2, 1)$ is possible for any $A \in (\frac{2}{3}, 1)$. In any $(2, 1)$ equilibrium, the two candidates from the larger group receive the same vote share $\frac{A}{2}$ and tie for the win in the election, while the sole candidate from the smaller group receives vote share $1 - A$ and loses the election.

The reasoning why $(2, 1)$ equilibria are not possible when $A \in (\frac{1}{2}, \frac{2}{3})$ builds on the intuition for the previous proposition. For a $(2, 1)$ configuration, one must consider a case in which the two candidates from group A equally split the support of A voters, and a case in which they do not. If they do, each has a vote share equal to $\frac{A}{2}$. For $A \in (\frac{1}{2}, \frac{2}{3})$, $\frac{A}{2} < B$, so the B candidate wins the race. As such, either A candidate has effectively thrown the election to the B candidate by entering; if either A candidate were to drop out, the other would win. Consequently, $(2, 1)$ cannot be an equilibrium in such a setting, because there would be an incentive for an A candidate to exit for identity reasons. In the other case, when the A candidates do not equally split the A vote share, the trailing A candidate pays the costs of entry without experiencing any benefits from winning, and either does not affect policy (if the other A candidate wins) or experiences identity losses (if the B candidate wins). As such, the lagging A candidate would wish to exit the race. Combining these two cases, clearly $(2, 1)$ cannot be an equilibrium when the majority group is not too large ($A \in (\frac{1}{2}, \frac{2}{3})$).

When the majority group is larger ($A \in (\frac{2}{3}, 1)$) such equilibria do become feasible. Suppose the two A candidates have equal vote shares (if they do not, no equilibria exist); for an equilibrium to exist, the four conditions discussed for Proposition 3 must hold with the slight alteration that both A candidates must wish to stay in the race. The first condition is met because with equal vote shares, the two A candidates would each win vote share $\frac{A}{2}$ by the actors' lexicographic identity preferences, and $\frac{A}{2} > B$ since $A \in (\frac{2}{3}, 1)$. So the two A candidates tie for the win in this electoral setting, and both clearly have an incentive to stay in the contest so long as $\frac{\gamma}{2} > c$, which is true by assumption. The only B candidate will not want to exit because of the identity considerations discussed above. While entry deterrence for the two group A candidates is not possible if they have identical policy positions, it is feasible if they are symmetrically spaced around the median voter of group A. Further, a potential B entrant would be deterred so long as $c > g(B)/2$. Thus, in some settings, equilibria are possible with two candidates from the majority group and a single candidate from the minority group.

The next result considers the possibility of equilibria with two candidates from the majority group as in the previous proposition but with more than one candidate from the minority group.

Proposition 5. $(2, 2)$ is possible for any $A \in (\frac{2}{3}, 1)$. In any $(2, 2)$ equilibrium, the two candidates from the larger group receive the same vote share $\frac{A}{2}$ and tie for the win in the election, while the two candidates from the smaller group receive the same vote share $\frac{1-A}{2}$ and lose the election.

The result here is similar to that in Proposition 4. $(2, 1)$ was not possible for $A \in (\frac{1}{2}, \frac{2}{3})$ because the entry of two A candidates threw the election to the B candidate. For $(2, 2)$, an analogous logic applies to exit incentives for the B candidates—as long as the majority group is not too large ($A \in (\frac{1}{2}, \frac{2}{3})$), a B candidate will wish to exit to ensure victory by her group's other candidate.

The final result for simple plurality rule establishes the possibility of equilibria with three majority candidates joined by a single minority candidate.

> **Proposition 6.** $(3, 1)$ is not possible for any $A \in (\frac{1}{2}, \frac{2}{3})$. But $(3, 1)$ is possible for any $A \in (\frac{2}{3}, 1)$. When $\frac{2}{3} < A < \frac{3}{4}$, two of the candidates from the larger group tie for the win, while the third receives fewer votes; and when $\frac{3}{4} < A < 1$, either two of the candidates from the larger group tie for the win, or else all three of them do. In either case, the sole candidate from the smaller group receives vote share $1 - A$.

Although the reasoning for this proposition follows the general form employed for the other configurations, it involves considering many more cases and thus all of the details are left to the appendix. The most important substantive point is that the existence of these equilibria again depends on the relative size of the identity groups. We only expect to observe $(3, 1)$ equilibria in polities in which the majority identity group is quite large relative to the minority.

This paper began with the question: do social divisions determine the number of candidates competing in elections? The existing literature suggests that the answer is yes, but chiefly under "permissive" electoral systems. In contrast, the theoretical results in this section suggest that social divisions can matter strongly *even* under the non-permissive simple plurality electoral system. This facet of our results is reflected in Figure 1, which summarizes key observable implications of Propositions 1-6 in a compact form.

For each of the (y, z) equilibria detailed in these Propositions, Figure 1 describes the *conditions* under which the equilibrium exists, in terms of the relative size of the largest social group (A), and the *implications* of the equilibrium for the effective number of candidates. The set of demographic conditions under which a given equilibrium is possible is indicated by the extent of the relevant curve or region along the horizontal (A) axis. For example, the fact that a $(1, 1)$ equilibrium is possible for any $A \in (\frac{1}{2}, \frac{2}{3})$ is indicated by fact that the $(1, 1)$ curve extends from $A = \frac{1}{2}$ to $A = \frac{2}{3}$. In the same way, the fact that a $(3, 1)$ equilibrium is possible for any $A \in (\frac{2}{3}, 1)$

is indicated by fact that the $(3, 1)$ region extends from $A = \frac{2}{3}$ to $A = 1$.

The implications of a given equilibrium for the effective number of candidates are communicated by the vertical axis. The effective number of electoral parties or candidates, *ENEP*, is equal to $\frac{1}{\Sigma_i p_i^2}$, where p_i is the ith candidate or party's vote share. This quantity, which weights candidates or parties by their vote shares, is the standard measure of party diversity in the comparative politics empirical literature. Our Propositions predict the specific number of candidates and these candidates' vote shares as a function of social demography; this information can, in turn, be directly translated into the classic *ENEP* measure. Figure 1 plots *possible* *ENEP* as a function of A, for simple plurality systems. Many of the equilibria in our results imply one specific distribution of candidate vote shares; these equilibria appear in Figure 1 as a curve, with one value of *ENEP* corresponding to each value of A for which the equilibrium is possible. The $(3, 1)$ equilibrium, however, is consistent with a range of possible vote share distributions across four candidates; the feasible values of *ENEP* as A varies are represented by the shaded region in the Figure.

Explaining variation in *ENEP* has been a focus of the relevant empirical literature in comparative politics. Strikingly, and contrary to expectations from that literature, Figure 1 suggests that *ENEP* may under some circumstances *increase* as a polity becomes more socially *homogenous* under majority rule. This is the case because the effective number of candidates can be near 2 in fairly evenly-divided societies ($A < \frac{2}{3}$), but not in societies that are somewhat more homogeneous (A somewhat larger than $\frac{2}{3}$). Overall, the Figure suggests the possibility that the relationship between *ENEP* and A may be substantially non-monotonic. In Section 5, we empirically test this and other implications of the theoretical results summarized in Figure 1.

Majority Runoff Elections

In this section, we turn to the results for majority runoff elections. The first proposition again eliminates the possibility of equilibria in which no member of one of the identity groups chooses to enter the contest as a candidate.

Proposition 7. $(0, n)$ is not possible for any n for any $A \in (\frac{1}{2}, 1)$. $(n, 0)$ is not possible for any n for any $A \in (\frac{1}{2}, 1)$.

The logic for this proposition is identical to that for plurality elections. We do not expect to observe elections in either system that do not include candidates from each identity group because of the identity losses associated with failing to optimize the group's electoral performance. Consequently, our model precludes single-candidate elections under both plurality and runoff rules.

The following result also eliminates a set of entry equilibria including two-candidate elections with one candidate from each identity group.

Proposition 8. For all n, $(1, n)$ is not possible for any $A \in (\frac{1}{2}, 1)$.

For $n > 1$, this result is identical to Proposition 2 for plurality elections and depends on the same reasoning—a single A incumbent from the majority group is not able to deter entry by another A candidate when the vote shares among the minority group B voters are diluted among the multiple candidates from B.

What differentiates Proposition 8 from Proposition 2 is that in majority runoff elections, equilibria with a single candidate from each identity group are not possible. This is because a single A candidate cannot deter entry by another A candidate under runoff rules even when there is only one candidate from group B.

Consider the incentives facing a potential entrant from the majority group who shares the same policy ideal point as the incumbent candidate from the majority group. If such an individual does not enter the race, her payoff will be 0. If she does enter the race, she will achieve vote share $\frac{A}{2}$ in the first round of the election. Because $A > \frac{1}{2}$, $\frac{A}{2} > \frac{1}{4}$, so that there are three possibilities in the first round depending upon the value of A: (1) the two A candidates tie for first place; (2) the two A candidates tie for second place; and (3) all three candidates tie for first place. In (1), the two A candidates advance to a runoff, which is also tied; each wins with probability $\frac{1}{2}$. In (2), with probability $\frac{1}{2}$ the A entrant advances to the runoff, which she wins; with remaining probability $\frac{1}{2}$, the incumbent A candidate advances to the runoff and wins. In (3), with probability $\frac{1}{3}$, the two A candidates advance to the runoff, which each wins with equal probability; with probability $\frac{2}{3}$, the B candidate advances to the runoff along with one of the A candidates, the A candidate ultimately winning. In all three cases, the incumbent A candidate wins with probability $\frac{1}{2}$ and the entrant A candidate wins with probability $\frac{1}{2}$. Because an A candidate always wins the election whether or not entry occurs, there are no identity costs or benefits to entry; and because the A candidates share the same policy position, there are no policy costs or benefits to entry. The potential entrant therefore has an incentive to enter so long as $\frac{\gamma}{2} - c > 0$, which is true by assumption. Consequently, we do not expect to observe elections with a single candidate from each identity group under runoff rules.

Note that Propositions 7 and 8 taken together make the strong prediction that we should not observe two-candidate elections in a majority runoff system.

The next result establishes the possibility of equilibria in which two candidates from the majority identity group and one candidate from the minority group contest runoff elections.

> **Proposition 9.** $(2, 1)$ is possible for any $A \in (\frac{1}{2}, 1)$. For $A \in (\frac{1}{2}, \frac{2}{3})$, in the first round, the two candidates from the larger group receive vote shares $\frac{A}{2}$, while the candidate from the smaller group receives vote share $1 - A$. One of the candidates from the larger group then defeats the candidate from the smaller group in the runoff. For $A \in (\frac{2}{3}, 1)$, in the first round, the candidates from the larger group receive vote shares xA and $(1 - x)A$ respectively, where $(1 - x)A \geq 1 - A$ and $\frac{1}{2} \leq x < \frac{2}{3}$, while the candidate from the smaller group receives vote share $1 - A$. In the runoff, either the two candidates from the larger group tie, or else the candidate from the larger group who received more votes in the first round defeats the candidate from the smaller group.

When $A < \frac{2}{3}$, the B candidate receives a vote share of $(1 - A) \in (\frac{1}{3}, \frac{1}{2})$ in the first round. Clearly it is not possible for as many as two A candidates simultaneously to match or do better than this, so that the B candidate must always make it to the runoff in these equilibria. Because all of the other candidates are from group A, the other candidate in the runoff must be an A candidate; and this A candidate will win the runoff. As such, being the best-placed A candidate in the first round is tantamount to election, and the strategic problem facing A candidates in the first round of the runoff system in a divided society is exactly the same as the one they would face in a plurality system in which the A group comprised the entire electorate. But it is well known in this setting (Osborne and Slivinski 1996) that two-candidate equilibria are possible, in which the candidates are symmetrically spaced about the median voter. It remains to examine the strategic logic facing B candidates and entrants. Clearly the existing B candidate in any $(2,1)$ configuration will not wish to exit because this would reduce the group's total vote share, violating the behavioral prescription and thus leading to identity losses. To understand the incentives facing potential B entrants, we must consider two possibilities.

First, a citizen may wish to enter if by so doing she can increase the probability with which her group wins the election. A single B candidate, who will lose any runoff, cannot win an election, but victory by a B candidate may be possible if there are at least two of them running. In particular, a potential entrant will wish to enter for this reason if and only if, by entering, *both* the two B candidates are able to at least tie the top A candidate in the first round. For $(2,1)$ configurations, this is never possible.

Second, we note that an entrant who is unable to affect a change in group B's probability of winning (and therefore a positive chance of winning for herself also) can easily be deterred so long as $\frac{g(B)}{2} < c < g(B)$. Thus, the existing B candidate wishes to remain in the election and potential B entrants can be deterred. As a result, equilibria are possible under the majority runoff system with two majority candidates and a single minority candidate.

For $A > \frac{2}{3}$, a single B candidate cannot tie or beat both of the A candidates. Further, if only one of the A candidates trails B in the first round, she would not make the runoff, and would have an incentive to exit the race. This leaves three possibilities: either the A's are tied, and both beat B; the A's are not tied, and both beat B; or the A's are not tied, one of them beating B and the other tying B. In any of these instances, both A candidates reach the runoff with positive probability; in any equilibrium, both must also win a runoff they enter with positive probability, or there would be an incentive to exit, meaning that a runoff between the two A candidates must be tied.

We now consider incentives for candidates to exit. The B candidate will not wish to exit for the identity considerations discussed above. An A who does not tie B will not wish to exit because $\frac{\gamma}{2} > c$ (by assumption), and an A who does tie B will not wish to exit so long as $\frac{\gamma}{4} > c$ (since such a candidate would have probability $\frac{1}{2}$ of entering the runoff, and then probability $\frac{1}{2}$ of winning it once there). So it is possible that the existing candidates from both groups will want to remain in the election.

Now, we consider whether these candidates can deter further entry. A candidate from group B would wish to enter if this were to result in two B's making the runoff, so that a B candidate could win with positive probability; but because, as above, we have that the B incumbent at best ties the lagging A candidate, this is clearly not possible. As such, the only incentive for entry is to win a share of group leadership; but this can be deterred so long as the B incumbent's policy corresponds to the median of the group and so long as $\frac{g(B)}{2} < c$.

For a majority group citizen to have an incentive to enter, it must be possible for her to make the runoff, and then have a positive probability of winning it. Deterrence of such entry incentives is possible when the A incumbent candidates have first round vote shares that are not too different. For $A \in (\frac{2}{3}, 1)$, all of the equilibrium conditions for $(2, 1)$ can be met, for some preference distributions, when the A candidates are evenly spaced around the overall median voter, and are situated closely enough together.

The final two propositions extend this result to show the possibility of equilibria with multiple candidates from each group and with three candidates from a majority group and a single minority candidate. A key feature of both propositions is that, as with Proposition 9, equilibria exist in polities with majority groups of all sizes. The reasoning supporting these final two results is left to the appendix.

Proposition 10. $(2,2)$ is possible for any $A \in (\frac{1}{2}, 1)$. In the first round, the two candidates from the larger group receive vote shares xA and $(1-x)A$ respectively, where $(1-x)A \geq \frac{1-A}{2}$ and $\frac{1}{2} \leq x < \frac{2}{3}$, while the two candidates from the smaller group receive vote shares $\frac{1-A}{2}$. In the runoff, either the two candidates from the larger group tie, or else the candidate from the larger group who received more votes in the first round defeats a candidate from the smaller group.

Proposition 11. $(3,1)$ is possible for any $A \in (\frac{1}{2}, 1)$. For $A \in (\frac{1}{2}, \frac{3}{5})$, two candidates from the larger group receive vote share xA, while the third trails with $(1-2x)A$ and the candidate from the smaller group receives $1-A$, where $x \in (max(\frac{1}{3}, \frac{1}{2A} - \frac{1}{2}), \frac{1}{2})$. In the runoff, one of the leading candidates from the larger group then defeats the candidate from the smaller group. For $A \in (\frac{3}{5}, \frac{2}{3})$, either the same is true, or else all candidates from the larger group tie with vote share $\frac{A}{3}$, while the candidate from the smaller group receives vote share $1-A$. The candidate from the smaller group is then defeated in the runoff. For $A \in (\frac{2}{3}, 1)$, any set of vote shares can exist in equilibrium in the first round that (1) involves either a two- or three-way tie for second place, or else a three- or four-way tie for first place; and (2) has the most successful candidate from the larger group receiving less than twice the vote share of the second most successful candidate from that group. The runoff matches either two candidates from the larger group or one candidate from each group, but a candidate from the larger group always wins.

Figure 2 summarizes our theoretical results for majority runoff elections (Propositions 7-11) just as Figure 1 did for elections under plurality rule. Figure 2 displays both the ranges of A (the relative size of the largest identity group) over which various (y, z) equilibria are possible, as well as the values of *ENEP* (the effective number of electoral candidates/parties) that can be sustained for each (y, z) equilibrium at a given value of A. A key difference between these majority runoff results and the earlier plurality results is that, under majority runoff rules, a specific (y, z) equilibrium that is possible for *any* value of A is possible for *all* values of A. This finding has an important empirical implication: majority runoff elections do not exhibit the striking potential for a substantially non-monotonic relationship between *ENEP* and social demographics that was demonstrated in Figure 1 for plurality races. Figure 2 also illustrates the prediction that majority runoff systems do not lead to elections with two or fewer candidates, a feature of our model that resonates strongly with Duverger's (1954) hypothesis that majority runoff systems favor multipartism.

In sum, our theoretical findings reproduce a number of familiar intuitions from the theoretical and empirical literatures on electoral competition, while also making several novel empirical predictions. By incorporating a model of social identities within Osborne and Slivinski's (1996) citizen-candidate framework, and explicitly modeling the dynamics of electoral competition within societies of different demographic configurations, we are able to identify social pressures that shape citizen-candidates' incentives, and to specify the relationship between demographics and electoral configurations in a more nuanced manner than previous literature.

5 The Effective Number of Presidential Candidates

We now use cross-national data from recent presidential elections in exploring the potential empirical relevance of our theoretical findings. Our formal model evaluates the incentives of citizen-candidates to enter or exit electoral contests under plurality and runoff rules in a single district. Presidential elections fit this assumption well. This fit of presidential elections in a single national district is further bolstered because good demographic data is often available for an entire country but not for subnational electoral districts such as those for legislative elections and/or elections for subnational executives such as governors or mayors. Thus, presidential elections make for a good initial test of the main observable implications of the model. Future research could usefully evaluate the model using cross-regional variation in the number of candidates running for subnational executive positions.[4]

[4]We are less optimistic about using legislative elections to evaluate the model empirically. Legislative contests under plurality and majority runoff rules typically take place in multiple geographical districts whose demographic compositions vary from the nation as a whole and from one another but the policymaking function of the legislature depends on coalitions at the national level. Consequently, how entry and exit decisions are determined in these elections likely depends on considerations at the district and national level and thus departs substantially from our model.

Empirical Hypotheses

Our model offers a number of theoretical intuitions; however, we highlight those which differ most from existing literature by focusing our analysis on the relationship between the relative size of the largest ethnic group (A) and the effective number of presidential electoral candidates (*ENPRES*). Specifically, we distill the key intuitions evident in Figures 1 and 2 into four testable hypotheses:[5]

> **Hypothesis 1.** *ENPRES* is greater under majority runoff rule than under plurality rule if $\frac{1}{2} < A < \frac{2}{3}$.
>
> **Hypothesis 2.** *ENPRES* does not vary significantly between majority runoff rule and plurality rule if $\frac{2}{3} < A < 1$.
>
> **Hypothesis 3.** *ENPRES* is greater for $\frac{2}{3} < A < 1$ than for $\frac{1}{2} < A < \frac{2}{3}$ under plurality rule.
>
> **Hypothesis 4.** *ENPRES* is less (or does not vary significantly) for $\frac{2}{3} < A < 1$ than for $\frac{1}{2} < A < \frac{2}{3}$ under majority runoff rule.

Hypotheses 1 and 2 predict differences in *ENPRES* across electoral institutions, and claim that such differences are a function of social demographics. Hypothesis 3 predicts that, in plurality systems, *ENPRES* is larger for a range of A corresponding to more *homogeneous* societies, in sharp contrast to received intuitions from the literature. Hypothesis 4 predicts that this is however not the case under majority runoff rule. It is worth reiterating that although the model yields quite sharp hypotheses about specific patterns of candidate entry and exit that are amenable to empirical analysis, the larger theoretical insight is that groups of different relative sizes have different "carrying capacities" for the number of candidates that can be supported in an election without risking coordination failure in a given institutional context. This idea seems likely to be a useful one in thinking about the consequences of social cleavages and institutions for patterns of electoral competition beyond the specific hypotheses we investigate here.

[5] For many values of A, multiple equilibria exist and multiple values of *ENPRES* are feasible. Because there is no obvious a priori principle for selecting equilibria here, the model cannot in general predict *specific* values of *ENPRES* for a specific case. The hypotheses reflect reasonable possibilities for what overall empirical patterns can be expected to look like given the logic of the model and the intuitions contained within the Figures.

Data and Econometric Model

To evaluate the hypotheses, we examine the number of candidates for president in all democracies that held presidential elections during the 1990s under either plurality or majority runoff rules. We use ethnic demographic data from Alesina et al (2003), who compiled a cross-national dataset on the ethnic group compositions of nations for the 1990s. Our presidential elections data comes from Golder (2005).

Our analysis is based on four samples of elections for the 1990s. We began by identifying all democracies which held a presidential election under plurality or majority runoff rule during this decade.[6] The first two samples that we analyze are cross-sectional data sets constructed via a procedure similar to Cox (1997, p. 208). For each country, we then selected one 1990s election closest in time to 1995, in order to maximize fit with the Alesina et al demographic data. We report regression results corresponding to this whole sample of elections, as well as to a subsample of those elections which took place within "presidential regimes," that is, countries in which the government serves at the pleasure of an elected president. We report results separately for both contexts because the policy role of the president, assumed to be significant in our model, may be stronger under presidential regimes than under other systems. The second two samples simply pool all presidential elections under plurality or majority runoff rule for the 1990s and examine this entire set of elections and the subsample of these elections that are presidential regimes as defined above.

For each election, we used the Golder dataset to obtain the standard effective number of presidential candidates measure, $ENPRES = \frac{1}{\Sigma_i p_i^2}$, where p_i equals the vote share of the ith candidate. We use $ENPRES$ as the dependent variable in our analysis. We also constructed a dichotomous institutional variable *Runoff* using the Golder dataset, which equaled one for presidential elections held under a majority runoff rule and zero for presidential elections held

[6] Countries with a fused vote for presidential and legislative elections were not included in the sample.

under plurality rule. Finally, we used the Alesina et al data to construct ethnic demographic variables. We took A, the relative size of the largest ethnic group, to be the largest group's share of that part of the population composed of the two largest ethnic groups.[7] We then used this measure to construct a dichotomous indicator variable *AGT2/3*, equal to one if $A > \frac{2}{3}$ and equal to zero if $\frac{1}{2} < A < \frac{2}{3}$.[8]

We model the correlates of the effective number of presidential candidates as follows:

$$ENPRES_i = \beta_0 + \beta_1 Runoff_i + \beta_2 AGT2/3_i + \beta_3 Runoff_i * AGT2/3_i + \epsilon_i \qquad (4)$$

where *ENPRES*, *Runoff*, *AGT2/3*, are the variables defined above, β_0, β_1, β_2, and β_3 are parameters to be estimated, and ϵ_i is a mean zero error term that reflects unobserved factors associated with the effective number of candidates in presidential elections. We estimate Equation 4 by ordinary least squares and report heteroskedastic robust standard errors for the cross-sectional samples (country-clustered robust standard errors for the pooled samples).

The four hypotheses have direct implications for the expected signs of the coefficients in Equation 4. Hypothesis 1 states that *ENPRES* is greater under majority runoff than plurality rule if A is between one-half and two-thirds. The coefficient β_1 indicates this difference and so is expected to be greater than zero. The second hypothesis also focuses on the difference between majority runoff and plurality systems but when A is between two-thirds and one. The sum of β_1 and β_3 indicates this difference and is expected to be approximately equal to zero. Because β_1 is expected to be positive, this implies that β_3 should be negative and of similar

[7] This operationalization assumes that the two largest groups determine the key strategic dynamics in electoral coalition formation. This assumption is of course more plausible, the larger the proportion of citzens that belong to the two largest groups. Below, we discuss ways to incorporate this idea into our empirical analysis. As we discuss in the conclusion, a potentially interesting extension of our theoretical model is to expand the analysis to three or more social groups.

[8] Our cross-sectional sample of elections comes from 49 countries, 23 of which are presidential regimes. The mean (standard deviation) for *ENPRES* is 3.132 (1.259), for *Runoff* is 0.714 (0.456), for A is 0.806 (0.156), and for *AGT2/3* is 0.796 (0.407). For comparison to previous studies using standard ethnic fractionalization measures, the mean fractionalization score for this sample is 0.410 with a standard deviation of 0.258. The summary statistics for the pooled sample of 101 elections, of which 51 are presidential regimes, are extremely similar.

absolute magnitude. Hypothesis 3 indicates that under plurality rule, *ENPRES* is expected to be greater when the size of the largest group is between two-thirds and one than when it is between one-half and two-thirds. This comparison is reflected in the coefficient β_2 which should therefore be positive. Finally, the fourth hypothesis indicates that either *ENPRES* should be less for A between two-thirds and one or it should be the same under majority runoff systems. This ambiguity is due to the possibility of multiple equilibria as indicated in Figure 2. This difference is captured in the sum of β_2 and β_3 which should then be less than or equal to zero. Given the prediction for β_2, this also implies that β_3 is negative. All four of these hypotheses are summarized in Table 1.

It is essential to note how these implications of our model relate to the existing empirical literature on the role of social cleavages and political institutions in explaining the effective number of electoral candidates or parties in elections (e.g. Ordeshook and Shvetsova 1994, Neto and Cox 1997, Clark and Golder 2006, Golder 2006). This literature typically examines a very similar econometric specification to the one suggested here, though one with a different measure of demographic diversity, and focuses attention on evaluating the marginal effect of ethnic heterogeneity which is equal to $\beta_2 + \beta_3 * Runoff$.[9] The findings of this literature generally suggest that this quantity is negative and that separately $\beta_2 = 0$ and $\beta_3 < 0$.[10] These results are consistent with the common claim in the literature that there is a positive relationship between ethnic fractionalization and the number of parties when electoral institutions are "permissive," but little, if any, relationship when institutions are not. The hypotheses suggested by our model differ from current treatments in two main ways. First, the prediction that the number of candidates is sensitive to ethnic heterogeneity even under restrictive electoral systems (more specifically that $\beta_2 > 0$) is not suggested in the existing literature. Second, our measures of

[9] See specifically Golder (2006, p. 43) Equation 2. The important difference is that our measure of social heterogeneity derived from our model is increasing in *homogeneity* while Golder's is increasing in *heterogeneity*. Consequently, the expected signs of the coefficients must be appropriately adjusted.

[10] It might be more precise to say that the existing literature's expectation is that $\beta_2 \leq 0$. This possibility does not change the contrast with our model discussed below.

ethnic heterogeneity (both A, the relative size of the largest group, and *AGT2/3*) are derived directly from our theoretical model in contrast to commonly used fractionalization measures.

Results

Table 2 reports the ordinary least squares coefficient estimates for Equation 4 across the full 1990s cross-section and across those countries that are presidential regimes. Table 3 reports the same estimates for the pooled 1990s samples. For all of these specifications, there is strong evidence consistent with the partial correlations predicted by all four hypotheses. Given the similarity of estimates, the initial discussion of the main findings focuses on the results for the full cross-sectional sample.

The coefficient estimate for the majority runoff variable (β_1) is equal to 2.201 in the full cross-sectional sample and has a relatively small standard error, indicating that the effective number of presidential candidates is greater by a bit over 2 candidates in majority runoff systems compared to plurality systems if A is between one-half and two-thirds. In other words, more permissive electoral institutions are associated with more candidates in relatively more heterogenous polities. This is consistent with Hypothesis 1 and with much of the existing empirical literature.

The coefficient estimate for the interaction term between *Runoff* and *AGT2/3* (β_3) in the full cross-sectional sample is negative and statistically significant. Further, a joint hypothesis test for the sum of β_1 and β_3 indicates that we cannot reject the null hypothesis that the sum of the two coefficients is equal to zero at conventional levels of statistical significance. These results suggest that the effective number of presidential candidates does not vary significantly between majority runoff and plurality electoral systems in relatively homogenous countries (specifically those for which A is between two-thirds and one-half). This result is consistent with the prediction in Hypothesis 2.

The estimate for the A greater than two-thirds variable (β_2) is equal to 1.315 for the full cross-sectional sample and has a relatively small standard error. This implies that in plurality systems, greater homogeneity (specifically cases with A greater than two-thirds) is associated with *more* candidates. This result, consistent with Hypothesis 3, is important because it provides empirical evidence consistent with the main argument of this paper—that demographic composition affects the number of candidates even in polities with restrictive electoral institutions. Further, it indicates that greater social *homogeneity* can be associated with more candidates, contradicting the usual emphasis in the literature on whether social heterogeneity increases the number of candidates or parties.

In evaluating Hypothesis 4, the relevant quantity is the sum of β_2 and β_3 and the expectation is that this quantity is less than or equal to zero. The coefficient estimates are consistent with this hypothesis—the sum is negative and statistically significant at conventional levels. This quantity is also equal to the marginal effect of ethnic homogeneity for those countries with a majority runoff rule and thus, consistent with the literature, indicates a negative correlation between homogeneity and the number of candidates in the more permissive runoff system.

Overall, these regression results are quite consistent with the four empirical hypotheses that summarize the key observable implications of our theoretical model. The results are quite similar across all the samples. Further, we estimated equivalent specifications for two alternative measures of ethnic heterogeneity. The first is simply A; the second is the more traditional fractionalization score. Neither of these more standard measures incorporate the discontinuity at $A = \frac{2}{3}$ that is predicted by our model. Consistent with our model, the specifications employing *AGT2/3* generally explain more variation in the dependent variable *ENPRES* than either of the other two measures of ethnic heterogeneity.

For example, the R^2 for the standard specification in the literature using the ethnic fractionalization measure is 0.011 for the full cross-sectional sample and 0.034 for the presidential regimes cross-section while the equivalent R^2 statistics for our specifications using the *AGT2/3* variable are 0.166 and 0.387. This evidence indicates that the specification suggested by our model fits the data substantially better. Similarly, the R^2 statistic in the specification with A, the relative size of the largest group, as the measure of heterogeneity is 0.037 for the full cross-sectional sample and 0.210 for presidential regimes. This measure does a better job in explaining variation in the effective number of presidential candidates than does the fractionalization score, but it does not do as well as the specification with *AGT2/3* because it fails to model the discontinuity at $A = \frac{2}{3}$.[11]

We further evaluated the importance of the two-thirds threshold by forming a series of dichotomous measures employing alternative thresholds at 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, and 0.90. For both the cross-sectional samples, the highest R^2 of all these alternatives was the specification using the two-thirds threshold and only the alternative using 0.70 as the threshold was close. The results of this analysis for the pooled sample were less striking but it was still the case that the best performing models were those with thresholds in the neighborhood of two-thirds. These findings highlight the utility of explicitly modeling the entry decisions of citizen candidates from ethnic groups of specific sizes rather than relying on generalized measures such as ethnic fractionalization.

Our theoretical model assumes only two social groups exist in the polity. In our empirical evaluation, we measure A as the size of the largest ethnic group relative to the sum of the two largest groups. As suggested above, this measurement strategy assumes that the two largest groups dominate the exit and entry strategies of presidential candidates. This is more likely to be true when the two largest groups make up a larger proportion of the total population. To incorporate this insight into our analysis, we reran the specifications reported in Tables 2 and 3 using weighted least squares with the percentage of the population in the two largest groups as the weighting variable. Across all four specifications, the coefficient estimates are

[11]The exception to this claim is that for the pooled 1990s sample of presidential regimes, there is essentially no difference in variation explained between the specification using A and the specification using the A greater than two-thirds variable.

qualitatively the same. For example, the coefficient for *AGT2/3*, β_2, is 1.241 (with a standard error of 0.294) for the full cross-sectional sample and is 1.472 (with a standard error of 0.457) for the cross-sectional presidential regime sample—both estimates indistinguishable from those reported in Table 2.

One potential limitation of these regression analyses is that they are essentially a difference of means test for the four exhaustive categories of elections across values of A and type of electoral system. It would be a problem for our theory if these differences were driven by values away from the $A = \frac{2}{3}$ threshold. Consequently, we evaluated for the cross-sectional samples the implication of our model that there should be a sharp discontinuity at $A = \frac{2}{3}$ under plurality systems but not under majority runoff rules (though since there is a potential downward slope for majority runoff, there could be a negative difference in the number of candidates around the two-thirds threshold).

Our procedure was to pick a small interval on either side of two-thirds and evaluate the difference in the means for the elections for the full cross-sectional sample on each side of two-thirds. We chose an interval, $[0.57, 0.77]$, that was large enough to include a sufficient number of elections to discern a difference but small enough to focus attention on cases near the two-thirds threshold. For plurality systems, the difference between the effective number of candidates for countries with $A > \frac{2}{3}$ and countries with $A < \frac{2}{3}$ was 1.20 with a t-statistic of 2.13, which is statistically significant at the 0.062 level using a one-tailed test. This indicates that the effective number of candidates in presidential elections increases by a bit over one from just below to just above the two-thirds threshold. For majority runoff systems, the difference is -1.12 with a t-value -1.52. This difference is significant at the 0.080 level using a one-tailed test. These results indicate that differences highlighted in the regression analysis are at least in part driven by the discontinuity at $A = \frac{2}{3}$ predicted by our model.[12]

[12] For the same interval, the differences were quite similar for the cross-sectional presidential regime sample (1.20 with a t-statistic of 2.13 for plurality systems and -0.675 with a t-statistic of -1.02 for majority runoff rule).

Although we think the evidence presented above is a useful test of the model and one which adds to the existing empirical literature on the correlates of the effective number of candidates or parties in elections, at least three caveats are in order in addition to the usual qualifiers for a small-n, cross-sectional analyses.

First, we have not evaluated a number of observable implications of the theory. There is of course the possibility that, by exclusively comparing $ENEP$ and A, our model makes a correct prediction on this dimension but misclassifies the case in terms of which social identity group is supporting particular candidates. Second, we have assumed that the relevant social identity to test our model is ethnicity as coded by Alesina et al (2003) and that measuring the relative size of the two largest groups characterizes the most relevant aspects of social heterogeneity. It merits investigation whether coding A according to other salient social identities—e.g. language or religion—affects the fit of the data to the model. Third, the empirical evidence that we examine here does not provide direct evidence that the social identity mechanism highlighted in the model is driving the patterns of electoral coalition formation that we observe.

6 Conclusion

An emerging consensus in the comparative politics literature concludes that there is a positive relationship between social divisions and the number of candidates or parties when electoral institutions are "permissive," but a much reduced relationship when institutions are not. This empirical description, however, lacks a theoretical mechanism describing precisely why and how varying demographic compositions matter for the candidate entry decisions that ultimately determine the equilibrium number of candidates or parties in a particular polity under a particular set of electoral rules. This paper provides a theoretical explanation by incorporating identity politics into a standard game-theoretic model of candidate entry and electoral competition.

Varying the interval from $[0.62, 0.72]$ to $[0.52, 0.82]$ yields similar results in that there is a positive difference evident for plurality rule but not for majority runoff.

This innovation allows us to make a theoretical connection between social group memberships in a polity and the equilibrium number of candidates under both plurality and majority runoff electoral rules.

In our model, the explanatory variables accounting for variation in the equilibrium number of parties include, as in the existing theoretical literature, the nature of the electoral system, the cost of running as a candidate in the election, and the benefit of winning. Our model adds to these factors the existence and relative size of social identity groups in a polity. Perhaps our most striking finding is that, even in the "unpermissive" plurality system, demographics affect the number of candidates that can be supported in electoral equilibria. Specifically, we find that the existence of two-candidate equilibria in simple plurality systems depends on the size of the identity groups not being too different, and that, contrary to the prevailing Duvergerian intuition, there exist demographic configurations for which even the *effective* number of candidates in a plurality contest cannot be near two. Some of our other findings include that: (i) two-candidate equilibria do not exist under a majority runoff system; (ii) single-candidate equilibria do not exist under either plurality or majority runoff rules; and (iii) multi-candidate equilibria exist under both systems but are less likely under plurality rule than under majority runoff rule.

At one level, these theoretical results suggest a set of mechanisms—based on the premises that social identity considerations motivate behavior and that entry and exit decisions are strategic in electoral competition—to interpret the correlations that have been consistently highlighted in the existing empirical literature. However, the model also yields new insights in that it shows that these considerations can have an important impact on electoral coalitions in unexpected situations—under relatively restrictive electoral institutions—and in novel directions—greater homogeneity can be associated with more candidates under plurality rule. We present evidence broadly consistent with the model by examining the effective number of candidates in presidential elections around the world during the 1990s.

We view these results as constituting a significant step towards explaining why and how varying demographic compositions matter for the equilibrium number of candidates or parties in a polity. At the same time, the analysis is limited in scope and detail by a number of characteristics of our model. Most obviously, our model considers only two electoral institutions, and only for polities with precisely two social identity groups. Further, our model does not incorporate some factors which may encourage the formation of over-sized electoral coalitions in certain settings. For example, greater than minimum-winning coalitions may increase the credibility of electoral promises or decrease the variability of the benefits of belonging to a particular coalition. Neither of these effects is captured by our model but may be important in explaining the interaction between social cleavages and electoral coalitions in particular cases. We view these issues as important next steps for future research building on the theoretical and empirical findings presented in this paper.

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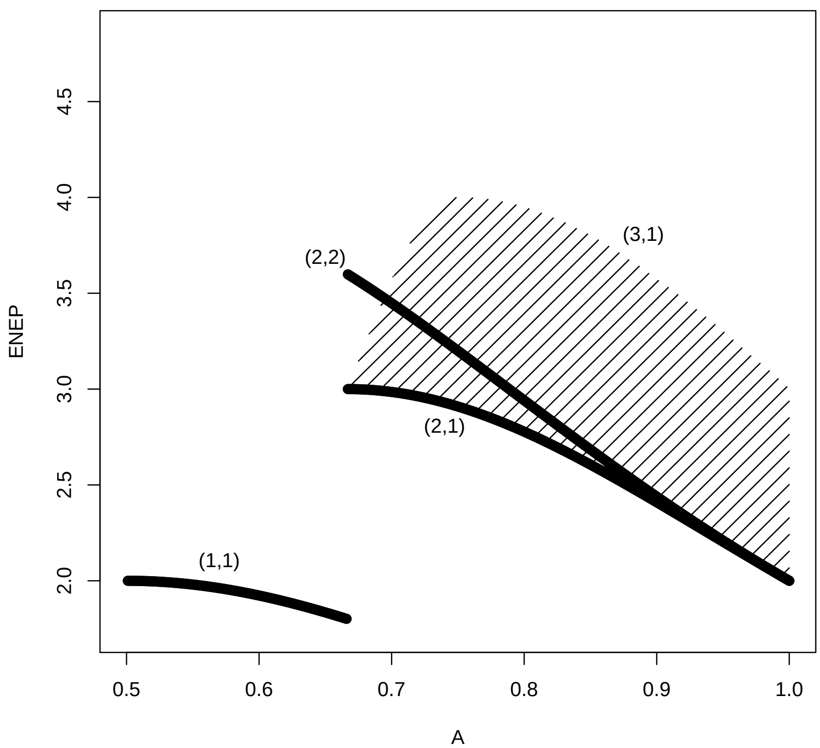

Figure 1: *Possible Effective Number of Electoral Candidates (ENEP) by Size of Largest Ethnic Group (A): Plurality Rule.*

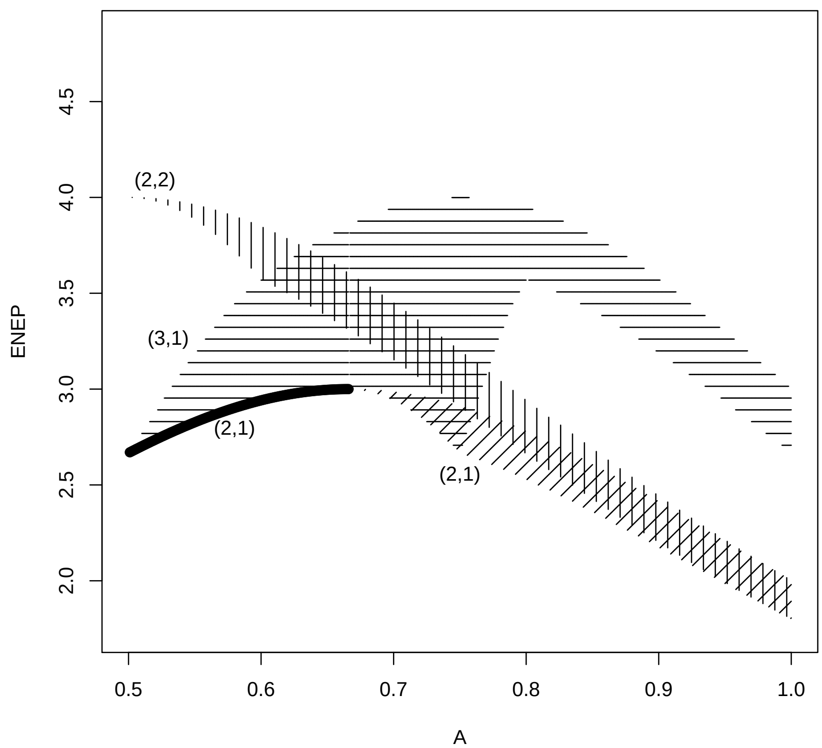

Figure 2: *Possible Effective Number of Electoral Candidates (ENEP) by Size of Largest Ethnic Group (A): Majority Runoff Rule.*

	Implied Hypothesis for Regression Coefficients
Hypothesis 1	$\beta_1 > 0$
Hypothesis 2	$\beta_1 + \beta_3 \approxeq 0$ (and $\therefore \beta_3 < 0$)
Hypothesis 3	$\beta_2 > 0$
Hypothesis 4	$\beta_2 + \beta_3 \leq 0$ (and $\therefore \beta_3 < 0$)

Table 1: *Summary of Hypotheses.*

Regressor	Full Sample	Presidential Regimes
Runoff (β_1)	2.201	2.017
	(0.299)	(0.500)
	0.000	0.001
AGT2/3 (β_2)	1.315	1.547
	(0.309)	(0.469)
	0.000	0.004
*Runoff*AGT2/3* (β_3)	-2.519	-2.923
	(0.496)	(0.726)
	0.000	0.001
Constant (β_0)	1.953	1.953
	(0.094)	(0.099)
	0.000	0.000
Standard Error of Regression	1.188	0.925
R-squared	0.166	0.387
Observations	49	23

Table 2: *Effective Number of Presidential Candidates, 1990s Cross-section.* Each cell reports OLS coefficient estimates, their robust standard errors (in parentheses), and p-values for the regression of *ENPRES* on *Runoff*, *AGT2/3*, and their interaction.

Regressor	Full Sample	Presidential Regimes
Runoff (β_1)	1.618	1.437
	(0.273)	(0.368)
	0.000	0.001
AGT2/3 (β_2)	0.982	1.352
	(0.315)	(0.371)
	0.000	0.001
*Runoff*AGT2/3* (β_3)	-1.772	-2.247
	(0.437)	(0.564)
	0.000	0.001
Constant (β_0)	2.003	2.003
	(0.100)	(0.102)
	0.000	0.000
Standard Error of Regression	1.136	0.993
R-squared	0.079	0.207
Observations	101	51

Table 3: *Effective Number of Presidential Candidates, 1990s Pooled.* Each cell reports OLS coefficient estimates, their country clustered robust standard errors (in parentheses), and p-values for the regression of *ENPRES* on *Runoff*, *AGT2/3*, and their interaction.

Appendix: Proofs of Propositions

Proof of Proposition 1. Any group A (B) citizen who fails to enter when her group does not have a candidate violates the behavioral prescription against harming the group's electoral performance, incurring a sufficiently large cost that there is always an incentive to enter. So $(0, n)$ and $(n, 0)$ are not possible for any n. ■

Proof of Proposition 2. In any $(1, n)$ setting, candidate A wins regardless of her policy, which is unconstrained by B candidates' choices. Given this and since $n > 1$, B candidates will only have an incentive to stay in if they win a share of group leadership. As such, in any $(1, n)$ equilibrium, there must be an n-way tie among group B candidates. An A citizen at the ideal point of the A "incumbent" could enter and tie the election without affecting the winning policy (since $n \geq 2$ and $A > B$, $A/2 > B/n$) earning utility $\gamma/2 - c > 0$, so there is an incentive to enter and $(1, n)$ is not possible for $n > 1$. ■

Proof of Proposition 3. First suppose $A \in (\frac{1}{2}, \frac{2}{3})$. For $(1, 1)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (ii): Along with the fact that there can be no identity reason for a solo group candidate to exit, $\gamma > c$ ($g(B) > c$) implies the A (B) candidate won't drop out. (iii) There is no identity incentive for A entry. Suppose the A candidate's policy is at the median A voter. Then a potential A entrant must always lose, either to incumbent A (paying entry cost with no policy or winning benefit), or to the B candidate (suffering lexicographic identity losses), since $B > \frac{A}{2}$. So an A incumbent at the A voter median can deter entry by A citizens. (iv) There is no identity incentive for B entry. Suppose the B candidate is at the median B voter. Then a potential B entrant can do no more than tie for group B support, without affecting policy, which will not be worth the cost of entry so long as $c > \frac{g(B)}{2}$. Thus (i)-(iv) can be simultaneously satisfied and so $(1, 1)$ is possible. Now suppose $A \in (\frac{2}{3}, 1)$. Now an A citizen who shared the incumbent A's policy would be able to enter and win vote share $\frac{A}{2}$, tying for first since now $\frac{A}{2} > B$. Since $\frac{\gamma}{2} > c$ there will be an incentive to enter so $(1, 1)$ is not possible. ■

Proof of Proposition 4. In a $(2, 1)$ setting, if the two A candidates are not tied, then the trailing candidate, who pays entry costs but does not influence policy or receive winning benefits, will drop out because there will never be identity costs for doing so. So if $(2, 1)$ is possible, it must involve a tie between the two A candidates.

Suppose that $A \in (\frac{1}{2}, \frac{2}{3})$. Each A candidate wins vote share $\frac{A}{2} < B$, so B wins the race and either A candidate would wish to exit for identity reasons to ensure victory for the other A candidate. So $(2, 1)$ is not possible. Now suppose $A \in (\frac{2}{3}, 1)$. For $(2, 1)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (ii): Because there are clearly no identity reasons for exit, as exit would improve neither group vote share nor the probability of group victory, and because the A candidates tie with vote share $\frac{A}{2} > B$, $\frac{\gamma}{2} > c$ ($g(B) > c$) implies the A (B) candidates wouldn't drop out. (iii) There are clearly no identity incentives for A entry. Because they must tie, the two A candidates must be symmetrically spaced around the median A voter; they cannot be at the same position, since an arbitrarily close potential A entrant could get at least arbitrarily close to half of the A vote (because the distribution of ideal points is continuous), and win since $\frac{A}{2}$ defeats the B candidate. In Proposition 2 of Osborne and Slivinski, two incumbents spaced around a median voter can deter entry by a citizen who cares about winning and policy in the same way that ours do; the competition by group A candidates for group A voters takes on the same form here, except that additional constraints are imposed (for example, winning group A does not imply victory, as one must also defeat group B candidates in order to win). As such their deterrence result implies that all potential A entrants can be deterred here as well. (iv) There are clearly no identity incentives for B entry. Suppose the B candidate is positioned at the median B voter. Then a potential B entrant can do no more than tie for group B support, without affecting policy or identity, which will not be worth the cost of entry so long as $c > \frac{g(B)}{2}$. Note that (i)-(iv) can be simultaneously satisfied; so $(2, 1)$ is possible. ■

Proof of Proposition 5. In a $(2, 2)$ setting, if the two A candidates are not tied, then the trailing candidate, who pays entry costs but does not win or influence policy, would pay no identity costs for exiting, and will wish to. So any equilibrium must involve a tie between the two A candidates. Further, because either A candidate could ensure the victory of the other by dropping out, A identity concerns imply that all B candidates lose for sure in equilibrium. Given this and since there are multiple B candidates, B candidates can be motivated only by group leadership payoffs, as their policies have no effect on the A candidates. As any B candidate not tied for the lead would then wish to drop out, both group B candidates must win vote share $\frac{B}{2}$.

For $(2, 2)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (iv) There can be no identity motivation for B entry as all A candidates beat all B candidates. Since B entrants cannot affect policy outcomes, entry incentives are limited to B group leadership. The only way for tied B candidates to deter entry is with candidates symmetrically spaced about the median. (ii) There will be identity incentives to drop out if this increases the probability of a B candidate victory. For $(2, 2)$ this is true if $B > \frac{A}{2}$, that is $A < \frac{2}{3}$. If there is no such identity incentive then candidates will wish to stay in so long as $\frac{g(B)}{2} > c$. (i) The A candidates tie for the win for sure, so there are no identity reasons for exit. Further, $\frac{\gamma}{2} > c$ ensures that neither A candidate will wish to exit. (iii) By the same logic as in part (iii) of the proof of Proposition 4, it is possible to deter entry by further A citizens. Note that (i)-(iv) can be simultaneously satisfied, and equilibria are therefore possible, when $A > \frac{2}{3}$ for $(2, 2)$. ■

Proof of Proposition 6. $(3, 1)$ configurations potentially involve: (1) An A candidate wins outright; (2) Two or more A candidates tie for the win; (3) An A candidate and the B candidate tie for the win; (4) Two or more A candidates and the B candidate tie for the win; and (5) the B candidate wins outright. But in (4), at least one of the A candidates would wish to drop out for identity reasons, to increase the probability of an A candidate winning. And in (1) or (3), there are two A candidates who lose outright, and who therefore do not share the winning A's policy. If the losing A's share the same policy, either of these will wish to drop out, whereas if they do not share the same policy, at least one of them (and possibly both) must not be the centrist candidate and will wish to drop out because they experience no winning, policy, or identity benefit from entry. If (5), the B candidate wins outright, and the losing A candidates do not even influence policy, so to stay in they must win some group leadership benefit and not wish to exit for identity reasons. Because of the former all three A candidates must receive vote share $\frac{A}{3}$. There are three ways the A candidates could tie: (I) all have the same policy; (II) exactly two candidates share the same policy; (III) all have different policies. Case I cannot form the basis for an equilibrium, because there must exist a potential A entrant who can win a vote share that is at least arbitrarily close to $\frac{A}{2}$ by continuity of F, which would give benefit $g(A) > c$ and therefore an incentive to enter. For both cases II and III, the largest vote share for the new A vote-winner after one candidate exits is $\frac{2A}{3}$ (for case II, when one of the coincident A candidates exits; for case III, when either of the A candidates who are at the "extremes" exits). If the vote share for the top A candidate exceeds B, then the A candidate wins; as such, there can be no identity incentive for exit only when $\frac{2A}{3} < B$, or $A < \frac{3}{5}$. Now consider a potential B entrant at the ideal point of the B incumbent candidate. Tying for first in an election is always worthwhile ($\frac{\gamma}{2} > c$), so such entry entrant can only be deterred if it would cause the now-tied B group candidates to place below the A candidates, i.e. if $\frac{B}{2} < \frac{A}{3}$ or $A > \frac{3}{5}$. As this is incompatible with the condition above, no equilibrium corresponds to cases II or III, and therefore to (5). This leaves (2), which is not feasible for all A: specifically, two A candidates can tie for the win only if $A \in (\frac{2}{3}, 1)$, whereas three A candidates can tie for the win only if $A \in (\frac{3}{4}, 1)$ because the B candidate must win vote share $1 - A$. The B candidate will wish to remain in the race for identity reasons (or simply because $g(B) > c$), and as B citizens have no identity or policy incentive to enter, they can be deterred from entry, for example if the B incumbent is at the median B voter and if $c > \frac{g(B)}{2}$. There is no identity incentive for A entry, and Proposition 3 of Osborne and Slivinski is sufficient to demonstrate that further A citizens can be deterred from entering for policy or winning reasons for either the two-way or three-way tie cases, since entrants in our models not motivated by identity must meet their conditions (as well as further constraints that are not necessary to consider). Finally, there is no identity incentive for A exit, and Proposition 3 of Osborne and Slivinski demonstrates that the further necessary and sufficient conditions can also be met. So $(3, 1)$ is possible for any $A \in (\frac{2}{3}, 1)$ and the possible equilibrium vote shares are as described. ■

Proof of Proposition 7. Same logic as in the corresponding plurality case. ■

Proof of Proposition 8. For $n > 1$, the proof is almost identical to that in the corresponding plurality case (Proposition 2). For $n = 1$, consider a potential group A entrant who shares the incumbent A candidate's ideal point. If she enters, she wins vote share $\frac{A}{2}$ in the first round. There are then three cases for the first round depending on A: (1) the A candidates tie for first; (2) the A candidates tie for second; and (3) all candidates tie for first. In (1), the two A candidates both advance to a runoff, which is also tied; each wins with probability $\frac{1}{2}$. In (2), each of the A candidates advances to (and then certainly wins) a runoff against B with probability $\frac{1}{2}$. In (3), with probability $\frac{1}{3}$, the two A candidates both advance to the runoff, which each wins with equal probability; in addition, each of the A candidates advances to (and then certainly wins) a runoff against B with probability $\frac{1}{3}$. In all three cases, the entrant wins with probability $\frac{1}{2}$. Because A candidate(s) always win(s) the election and maximum possible vote share regardless of entry, there are no identity costs or benefits to entry; and because here the A candidates share the same policy position, there are no policy costs or benefits to entry. The entry condition is then just $\frac{\gamma}{2} - c > 0$, which is true by assumption. So $(1, 1)$ is not possible. ■

Proof of Proposition 9. Take $A \in (\frac{1}{2}, \frac{2}{3})$. For $(2, 1)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (iii): The B candidate wins vote share $(1 - A) \in (\frac{1}{3}, \frac{1}{2})$ in the first round, earning no worse than second place, so the B candidate always makes it to the runoff against one ultimately victorious A opponent. Thus, being the top first-round A candidate is tantamount to election, and the strategic problem facing A candidates in the first round of the runoff system in a divided society is exactly the same as the one they would face in a *plurality* system in which the A group comprised the entire electorate. As such, Proposition 2 of Osborne and Slivinski, along with the observation that there are no identity reasons for A exit or entry, demonstrate that (i) and (iii) can can both be satisfied. (ii) The B candidate will clearly not wish to exit because $g(B) > c$ as well as for identity reasons. (iv) Group B entrants could be motivated either by group leadership concerns (which can be deterred by a B incumbent at the median B voter if $c > \frac{g(B)}{2}$) or by identity concerns. A solo B candidate in a runoff always loses; identity-motivated entry can occur here if and only if it leads *both* the B candidates to at least tie the top A candidate in the first round. The most efficient (and always feasible) allocation of B votes is to divide them equally between the B candidates, so deterrence of this case is necessary and sufficient for condition (iv). Since A candidates must tie in $(2, 1)$, the deterrence condition is $\frac{B}{2} < \frac{A}{2}$, which holds since $A > B$.

Now take $A \in (\frac{2}{3}, 1)$. Clearly the B candidate cannot tie or beat both of the A candidates. And, any A candidate trailing B does not make the runoff, so would wish to drop out to save entry costs. So either (1) both the A's beat the B in the first round or (2) one of the A's beats the B while the other ties. Two A candidates in a runoff must tie in the runoff, or the trailing candidate would drop out; so the A's either have the same policy or are symmetrically arranged around the overall median voter. An A always wins the election. For $(2, 1)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) There are clearly no identity reasons for A exit. For (1), $\frac{\gamma}{2} > c$ implies that there will be no incentive for exit; for (2), this will still be true so long as $\frac{\gamma}{4} > c$. (ii) The incumbent B candidate will clearly not wish to exit because $g(B) > c$ and for identity reasons. (iv) If the B incumbent is at the median B voter, all potential B entrants can be deterred so long as $c > \frac{g(B)}{2}$, as there are no identity motivations for entry in a situation where the sole B candidate was no better than tied for second to begin with. (iii) Consider (1). Suppose that the A incumbents have positions symmetric about the overall median voter. There are no identity incentives for entry since an A candidate always wins the election. For entry at a position that is either to the left or to the right of both A incumbents, there can be no policy incentive since entrants drain votes only from the incumbent whose policy the entrant prefers. An A incumbent would still make the runoff for sure, and would beat any such entrant who also made the runoff because of distance from the median voter. By familiar logic, it is also possible to deter entry between the two A incumbents; if the incumbents are sufficiently close together, such entrants would fail to make the runoff or change the composition of the candidates who do make the runoff. The remaining possibility is of a potential entrant at the policy of one of the incumbent candidates. An entrant at the policy of candidate A_j will receive vote share $\frac{A_j}{2}$; clearly the entry incentive will be at least as great at x_1 as at x_2 if $A_1 \geq A_2$ (which we assume without loss of generality). If $A_1 < 2A_2$, such an entrant would finish no better than a two-way tie for second in the first round: this best case scenario leads to a runoff place with probability $\frac{1}{2}$, and conditional on that a victory with probability $\frac{1}{2}$, for a best-case victory probability of $\frac{1}{4}$, and no impact on the probability distribution of policy outcomes, so that there will be an incentive to enter if and only if $\frac{\gamma}{4} > c$. As such, in this case, entry can be deterred if $\frac{\gamma}{2} > c > \frac{\gamma}{4}$. (If the entrant does worse than a two-way tie for second place, the expected winning and policy benefits of entry will both be at least weakly worse, so deterrence will be possibly for a weakly wider range of conditions.) If $A_1 = 2A_2$, then the entrant would be tied for first place among the A candidates, with an expected winning benefit $\frac{\gamma}{3}$ and an improved distribution of policy outcomes for the entrant. The deterrence condition here is $\frac{\gamma}{2} > c > \frac{\gamma}{3} + \frac{\delta}{6}$ which is clearly possible if the policy separation δ between the A incumbents is not too large. Finally, if $A_1 > 2A_2$, then the entrant would be tied for first, and win the runoff with probability $\frac{1}{2}$, so that there would always be an incentive for entry since $\frac{\gamma}{2} > c$. So in this case, entry cannot be deterred at all. As such, for (1), A entrants can be deterred as long as $A_1 \leq 2A_2$. (Therefore to determine the vote shares that are possible in equilibrium, considering A incumbents with identical positions is not necessary since $A_1 = A_2$ is already included here.) Now consider (2), with first-round vote shares $A_1 > A_2 = B$. Entry at the extremes of A_1 and A_2 and at x_1 and x_2 involve the same considerations and thus deterrence conditions as above. Entry in between A_1 and A_2 is not the same because now an infinitesimal measure of support garnered between A_1 and A_2 could potentially change the vote share orderings. Now suppose that all A_2's support comes from her "extreme" flank away from A_1, that A_1 gets from her "extreme" flank support less than A_2, that A_1's "centrist" support is at least three times closer to x_1 than to x_2, and that x_1 and x_2 are sufficiently close. Then there is no incentive for entry in between the incumbents (no chance to win since policies sufficiently close; and entrants cannot achieve policy improvements since the relevant voters are out of reach). So the conditions are the same in (2) as in (1). Thus, (i)-(iv) can be simultaneously satisfied, so $(2, 1)$ is possible under the conditions described. ■

Proof of Proposition 10. An A candidate must win the election for sure; otherwise either A candidate would wish to drop out for identity payoff reasons. As such, the B candidates must tie each other; otherwise, trailing B candidates would wish to drop out, because group leadership payoffs provide the only incentive for entry. Also, an A candidate with no chance of winning would drop out, so both A candidates must make the runoff with some probability. And at least one A must be in the runoff every time as an A candidate must win for sure: so either (1) both A's beat the B's, or (2) one A beats the B's while the other A ties the B's. If both A candidates advance to the runoff, they must tie in the runoff (or one would wish to exit). For $(2, 2)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (iv) There can be no identity motivation for B entry as all A candidates beat or tie all B candidates in the first round. Since B entrants cannot affect A policy choices, entry incentives are limited to B group leadership. The only way for tied B candidates to deter such entry is with candidates symmetrically spaced about the median. (ii) Identity motivations for exit can exist only if a B candidate's exit creates positive probability that two B candidates will simultaneously qualify for the runoff (if only one B candidate is in the runoff, she would lose for certain). For $(2, 2)$ this cannot exist because exit leaves only one B candidate. As such, B candidates will not exit so long as $\frac{g(B)}{2} > c$. (i) There can be no identity incentive for A exit. If the two A candidates both defeat the B's outright, $\frac{\gamma}{2} > c$ implies there will be no incentive for exit; if one of the A candidates ties the B's, it is necessary and sufficient that $\frac{\gamma}{6} > c$, because this candidate makes the runoff with probability $\frac{1}{3}$ and conditional on that wins half the time. So it is possible for both A candidates to wish to stay in for either of cases (1) and (2). (iii) The argument in part (iii) of the proof of Proposition 9 for $A \in (\frac{2}{3}, 1)$ holds here, replacing B in that proof with $\frac{B}{2}$ here. Note that (i)-(iv) can be simultaneously satisfied, so $(2, 2)$ is possible under the conditions described. ■

Proof of Proposition 11. For $A \in (\frac{1}{2}, \frac{2}{3})$, the argument in Proposition 9 for $(2, 1)$, $A \in (\frac{1}{2}, \frac{2}{3})$, holds here except that the relevant reference in Osborne and Slivinski ("OS") is Proposition 3, and the deterrence condition for B entry is instead $\frac{B}{2} < \frac{A}{3}$, or $A > \frac{3}{5}$, for the three-way tie specified in OS Proposition 3, and $\frac{B}{2} < xA$, or $A > \frac{1}{2x+1}$ for the two-way tie (two A candidates get xA, $x \in (\frac{1}{3}, \frac{1}{2})$, while the third trails with $(1-2x)A$). Note $\frac{1}{2x+1} \in (\frac{1}{2}, \frac{3}{5})$ so that for the two-way tie A can take on any value between $\frac{1}{2}$ and $\frac{2}{3}$, as long as $x \in (max(\frac{1}{3}, \frac{1}{2A} - \frac{1}{2}), \frac{1}{2})$.

Now take $A \in (\frac{2}{3}, 1)$. Taking A candidate vote shares $A_1 \geq A_2 \geq A_3$, there are 20 different relative orderings (including potential indifference) of these vote shares along with that of the B candidate. The six with A_1 and A_2 unambiguously as the top two cannot be in equilibrium; if the last-placed A candidate exited, it would not affect who made the runoff, and therefore not affect policy, nor does the trailing A get identity or winning gains from staying in. The six with A_1 and B unambiguously as the top two also cannot be in equilibrium. A_2 and A_3 do not get winning or identity benefits from running, since an A candidate ultimately wins regardless, so only policy reasons could keep them from exiting. Only a candidate in the middle of three dispersed candidates could have such an incentive; extreme or coincident candidates can only draw support away from their most favored alternative. But clearly A_2 and A_3 cannot both be the central of three dispersed candidates, so at least one must wish to exit.

We consider the eight remaining orderings in turn. In each instance, the incumbent B candidate will not wish to withdraw because of identity reasons (and $g(B) > c$), and potential B entrants can be deterred, since there is no identity motive for entry (both B candidates cannot make the runoff since for $A > \frac{2}{3}$, $\frac{B}{2} < \frac{A}{3}$), if the B incumbent is at the median B voter, so long as $\frac{g(B)}{2} < c$. As such we consider only A candidate incentives below.

Three remaining orderings involve: (1) three A candidates tie for first place ($A > \frac{3}{4}$); (2) all four candidates tie for first place ($A = \frac{3}{4}$); and (3) three A candidates tie for second place ($A < \frac{3}{4}$). In their Proposition 8, OS describe alternative policy configurations leading to this vote share; to demonstrate existence here, it is sufficient to focus on a runoff equilibrium in which all three A candidates have different positions but win equal vote shares, and in which the two extreme candidates are symmetric about the (overall) median voter. Consider A exit incentives, noting there is no identity incentive for exit since an A always has to win. For (1), the analysis is identical to OS, and demonstrates that the entrants don't exit for $\frac{\gamma}{6} > c$. For (3), the A candidates compete for only one runoff spot. Each gets it with probability $\frac{1}{3}$, and after getting it, beats B in the runoff. So there will be no exit incentive here so long as $\frac{\gamma}{3} > c$. For (2), each of the A candidates competes in three of six possible runoff pairings; the central (non-central) candidate(s) win all of them (win one, tie one, and lose one), with no exit incentive so long as $\frac{\gamma}{2} > c$ ($\frac{\gamma}{4} > c$). Now consider incentives of potential A entrants. There are no identity-related motives for entry. OS show in their setting that entrants whose objective is to finish first or second among the A candidates can be successfully deterred. This is sufficient to show entry deterrence is possible here for (1), (2), and (3).

Two further orderings are (4) $A_1 > A_2 = A_3 > B$ and (5) $A_1 > A_2 = A_3 = B$. Consider x_1 and x_2 symmetric about the overall median voter, with $x_1 < x_3 < x_2$. Further suppose that the distribution of A_1 voters has support $[y, x_1]$, $y < x_1$; the distribution of A_2 voters has support $[x_2, z]$, $z > x_2$; and all A_3 voters are contained within $(\frac{3x_3+x_1}{4}, \frac{3x_3+x_2}{4})$. Here A_1 is always in the runoff. When $A_1 > 2A_2$, there is clearly no means of deterring entry (as an entrant at x_1 can win with probability $\frac{1}{2}$ and ensure her ideal policy), so we restrict our attention to the complementary cases. In (4), A_1 faces either of the other A candidates; A_1 ties A_2 but loses to A_3 in runoff matchups, so that A_1 and A_2 (A_3) win with probability $\frac{1}{4}$ ($\frac{1}{2}$). In (5), A_1 faces any of the other three candidates, beating B, tying A_2, and losing to A_3, so A_1 (A_2) $\{A_3\}$

win with probabilities $\frac{1}{2}$ ($\frac{1}{6}$), and $\{\frac{1}{3}\}$. For (4,5), one can write conditions for non-exit for all three A candidates in terms of these probabilities and the probabilities of victory that would hold if the candidates individually dropped out (which are clearly determined by the preference distribution described); γ; c; and the policy distances between candidates, which can clearly be satisfied simultaneously when γ is large enough relative to c and potential policy costs of entry. (Note also that the conditions for B can also be simultaneously satisfied.) For A entry, for the given preference distribution, no entrant can win or obtain identity benefit, and entrants who are able to win positive vote shares take them from their most favored candidate and therefore obtain no policy benefit. This establishes that A entry can be deterred for general (4) and (5) if and only if $A_1 < 2A_2$.

The final three orderings are (6,7) $B \geq A_1 = A_2 > A_3$ and (8) $A_1 > A_2 = B > A_3$. For (8), A_3 must clearly prefer the policy of A_1; if A_3 instead preferred the policy of A_2, she could not harm but might help A_2's prospects by dropping out of the race, and so would not wish to pay the costs of entry. Similarly label as A_1 the candidate whose policy A_3 prefers in (6,7) (without loss of generality). Consider x_1 and x_2 symmetric about the overall median voter, with $x_1 < x_3 < \frac{x_1+x_2}{2} < x_2$. Note first that for (8), as above, entry cannot be deterred if $A_1 > 2A_2$, so we take $A_1 < 2A_2$ (automatically true for (6,7)). Consider then a preference distribution for which vote share less than A_2 lies left of x_1; vote share less than A_2 lies right of x_1 but left of $\frac{3x_1+x_3}{4}$; vote share $y < A_2 - A_3$ lies right of x_2 while $A_2 - y$ lies left of x_2 but right of $\frac{x_3+3x_2}{4}$; and vote share A_3 lies on $[\frac{x_1+x_2}{2}, z]$ for some $z < \frac{3x_3+x_2}{4}$. In (6,7), A_1 and A_2 each win with probability $\frac{1}{2}$ while in (8) A_1 (A_2) wins with probability $\frac{3}{4}$ ($\frac{1}{4}$). For (6,7,8), one can write conditions for non-exit for all three A candidates in terms of these probabilities and the probabilities of victory that would hold if the candidates individually dropped out (which are clearly determined by the preference distribution described); γ; c; and the policy distances between candidates, which can clearly be satisfied simultaneously when c is small enough relative to the policy distances (so that A_3 will wish to enter to influence policy) and when γ is large enough relative to c and potential policy costs (for A_1 and A_2) of entry. (Note also that the conditions for B can also be simultaneously satisfied.) For A entry, for the given preference distribution, no entrant can make the runoff or obtain identity benefit, and entrants who are able to win positive vote shares take them from their most favored candidate and therefore obtain no policy benefit. This establishes that A entry can be deterred for general (6), (7), and (8) if and only if $A_1 < 2A_2$. ■