The Political Economy of Higher Public Education

Admission Tests$^1$

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October 2, 2014

$^1$We thank Bernardo Moreno and participants to seminars held in Aix-en-Provence (AMSE) and Québec (U Laval) and to the 2014 PET meeting (Seattle) for their comments. Part of this research has been done while the first author was visiting UQAM (Montréal), whose hospitality is gratefully acknowledged.

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Abstract

This paper studies the political determination of the proportion of skilled and unskilled workers in the economy when access to higher education is rationed by admission tests. Parents differ in income and in the ability of their unique child. They vote over the minimum ability level required to attend public universities, which are tuition-free and financed by proportional income taxation. University graduates become high skilled, while the other children attend vocational school and become low skilled. While preferences over the minimum ability level in universities are not single-peaked, we obtain a unique majority voting equilibrium, that can be either classical (with 50% of the population attending university) or “ends-against-the-middle”, with less than 50% attending university (and parents of low and high ability children favoring a smaller university system). A means-preserving spread of the income distribution results in a smaller public university equilibrium size, while a larger skill premium and a larger correlation between parent’s income and child’s ability result in a larger public university equilibrium size. Finally, the majority chosen size of the university is, under certain circumstances, larger than the utilitarian optimal size.

**JEL codes:** D72, I22

**Keywords:** majority voting, ends-against-the-middle, non single-peaked preferences, single-crossing
1 Introduction

In many (continental) European countries, higher education is mostly provided by public universities – see Figure 1 (source: OECD(2014)). Those universities are mostly financed through general taxation, with tuition fees quite low or even non-existent. According to Eurydice (2013), Austria, the Czech Republic, Denmark, Finland, Greece, Malta, Norway, Scotland, Slovakia, Sweden, Turkey and 15 out of 16 German Landers charged higher education students no fees or extremely low administrative fees in the academic year 2012/2013. In other countries such as Belgium, Bulgaria, Croatia, France, Iceland, Italy, Liechtenstein, Montenegro, Portugal and Spain, fees were below or very close to €1,000 per year. Moreover, youths from underprivileged backgrounds are usually eligible for generous grant programs. Rather than by fees, access to university education is rationed by some measure of ability – i.e., access is limited to students who demonstrate some minimum ability level.\footnote{The test verifying that a minimum ability is attained may take the form of a university admission test (as in Spain or the UK), or may be merged with the exam taken at the end of secondary education (as in Belgium and France, for instance).}

The objective of this paper is to better understand the political determination of this minimum ability level required to access higher education. This threshold ability level in turn determines the fraction of the population receiving a university education and becoming highly skilled. We are therefore interested in the political determination of the proportion of skilled and unskilled workers in the economy, when access to higher education is rationed by admission tests.

Figure 2 reports both the enrolment rate (full-time and part-time students in public and private institutions) among 20 to 29 year old students and the proportion of 25 to 64 years old who have obtained a tertiary degree, for the European countries listed above,
in 2010. Two features of this figure come to the fore: first, there is a lot of heterogeneity between otherwise comparable countries and, second, both measures depicted are strictly lower than one half for all countries.

Insert Figure 2 around here

Our aim in this paper is to build a simple political economy model where the minimum ability level of higher education students is determined through majority voting, to investigate the characteristics of the majority voting equilibrium, and to determine how the equilibrium is affected by factors such as the degree of income inequality, the skill premium enjoyed by higher education graduates, and the correlation between income of the parents and the academic ability of the children.

We assume that parents differ in income and in the academic ability of their unique child. Children below a minimum ability level attend a costless vocational program and become low-skilled, while those above this ability level attend university and become high-skilled. University is financed with an income tax on the whole (parents) population. The future wage earned by students is the product of their ability and of the reference wage of their skill level. The skill premium (difference in reference wage across skill levels) depends on the relative supply of each type of labour, and is thus a function of the ability threshold democratically chosen. Parents care for their child’s future wage and vote over the minimum ability level giving access to higher education.

Preferences are not single-peeked in this minimum ability level, because of the switch to vocational schooling when that level becomes larger than the child’s ability. We show that preferences are single-crossing in the sense that, for each income level, there is a threshold ability below which parents most-prefer a minimum ability level at university.

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2 The source for Figure 2 is OECD (2012), Table A1.4. and Table C1.1a., respectively.
3 This seems the most natural way to endogeneize the fraction of students attending public universities in democracies.
4 This assumption accords well with basic intuition and with state-of-the-art models of the labor market (e.g. Acemoglu, 2003; Carneiro and Lee, 2011; Acemoglu and Autor, 2011).
that leaves their child unskilled and above which they most-prefer a university just large enough to enrol their child. The intuition runs as follows. For their child to attend university (and enjoy the larger high-skilled wage), parents have to set the minimum ability threshold at most equal to their child’s ability level. This can prove very costly for parents of a low ability child (because of the tax cost of a large university system), and for rich parents (who bear a larger share of the burden of financing public universities). Such parents then most prefer their child to become low-skilled. Note that this does not entail that they oppose any positive university size, because they want to restrict the supply of low skill agents to boost their child’s wage.

We then prove the existence of a unique majority voting equilibrium (a Condorcet winning value of the minimum ability required to access university), which can be either “classical” (where parents with children in the bottom-half (resp., top half) of the ability distribution prefer a larger (resp., smaller) university size) or “ends-against-the-middle” à la Epple and Romano (1996) (where parents of both low and high ability children prefer a smaller university size, in opposition to parents of middle ability kids). While the equilibrium size of the public university is 50% of the student population in the “classical” equilibrium, it is strictly less than 50% in the other equilibrium. The ends-against-the-middle equilibrium is then in line with Figure 1, while the 50% participation rate in higher education in the classical equilibrium corresponds to targets announced during electoral campaigns by Tony Blair in 2001 for the UK, and by François Hollande in 2012 for France.

We study how the likelihood of an ends-against-the-middle equilibrium, and how its equilibrium university size, are affected by variations in income inequality, skill premium and correlation between parental income and children academic ability. We obtain that rising inequality (in the form of a means-preserving spread of the income distribution) results in a smaller public university equilibrium size, while a larger skill premium and a larger correlation between parent’s income and child’s ability result, under certain conditions, in a larger public university equilibrium size.

5After completing this research, Bernardo Moreno has pointed out to our attention that individual preferences in our setting satisfy the top monotonicity requirement: see Barbera and Moreno (2011).
Our paper belongs to a relatively small but growing literature studying access conditions to higher education, together with its financing. Most contributions focus on the impact of fees and on various subsidization policies. Fernández and Rogerson (1995) assume that citizens vote over the size of a tax-financed subsidy and obtain that the political equilibrium subsidy level is not large enough to allow poor students to access higher education, resulting in redistribution from the poor to the rich. Building upon this observation, Garcia-Peñalosa and Wälde (2000) compare the efficiency and equity effects of a traditional tax-subsidy scheme, loans and a graduate tax and obtain that the latter two fare better than the former. Moreover, when education outcomes are uncertain, the graduate tax is to be preferred. Haupt (2012) extends the political economy analysis of the traditional tax-subsidy scheme to a dynamic setting and shows that high and low levels of public spending in higher education may alternate in a democracy. Del Rey and Racionero (2012, 2014) focus their attention on the political economy and the efficiency and equity properties of income-contingent loans. Borck and Wimbersky (2014) study numerically majority voting over a traditional subsidy scheme, a pure loan scheme, income contingent loans and graduate taxes by risk averse households who are heterogeneous in income.\footnote{Epple, Romano and Sieg (2006) and De Fraja and Valbonesi (2012) are less closely related to this paper.}

Two papers study admission tests either together with, or instead of, (subsidized) tuition fees. Gary-Bobo and Trannoy (2008) study the socially optimal examination-cum-fees policy. They assume that students observe only a private, noisy signal of their ability, and that universities can condition admission decisions on the results of noisy tests. Test scores are part of the optimal policy provided that test results are not public knowledge.

De Fraja (2001) is the paper most closely related to ours, with parents differing in income and in the ability of their child and facing a binary educational choice, as here. Major differences with our approach are that he assumes that universities charge fees, and that future income of children is random but determined only by their own education.
decision. An important consequence of these assumptions is that better-off children are more likely to attend university than poorer ones in a laissez-faire situation (because investing in higher education is a risky bet for parents), with this allocation then satisfying neither equality of opportunity nor production efficiency. He then studies two forms of intervention, exclusive of each other: an admission test similar to ours, or a subsidy financed by proportional income taxation. In both cases, he analyzes the majority voting equilibrium and shows that both these measures enhance equality of opportunity, but that their equity and efficiency effects are ambiguous. Preferences for the admission test are much simpler than in our model, because parents whose children do not attend university are indifferent as to its size (because they don’t finance university education through taxes, and because the future income of an unskilled agent is not affected by the proportion of university graduates), allowing for the direct application of the median voter theorem. Our paper can then be seen as a generalization of De Fraja (2001) in two dimensions: (i) the decision to attend university by an additional agent affects the other agents’ future income, and (ii) we study majority voting over the admission test level in the presence of (full) subsidy of fees. We find these two generalizations important, because (i) labor market effects are a crucial aspect of the problem, and (ii) many countries do not use fees to ration access to university, as we have shown above.

The remainder of the paper is organized as follows: after presenting the model in section 2, we describe households’ preferences over the admission ability threshold in section 3; existence of a majority voting equilibrium is studied in section 4; the outcome of the voting process is then compared to the efficient admission threshold in section 5; while section 6 contains our comparative statics analysis. Section 7 concludes.

2 The model

We model a static economy, with a continuum of individuals (parents) of mass one. Parents differ in their (exogenous) income $w$ which can take two values: $w_L$ and $w_H$ with
$w_L < w_H$. A fraction $\lambda$ has low income, so that average income is $\bar{w} = \lambda w_L + (1 - \lambda) w_H$.\footnote{This setting with two income levels is the simplest allowing us to study the impact of the income distribution.} Each parent has one child of a given (and known) ability degree $\theta$. Abilities are distributed over $[0, \bar{\theta}]$ according to the CDF $F(.)$ and density $f(.)$. While the smallest conceivable ability may tend toward zero, the smallest ability level actually observed in the economy is $\underline{\theta}$, and the density has full support over $[\underline{\theta}, \bar{\theta}]$. The median value of $\theta$ is denoted by $\theta_{med}$. With a slight abuse of language, we denote by $(\theta, i)$ with $i \in \{L, H\}$ the type of the parent. Up to the last part of section 6, we assume no correlation between parent’s income and child’s ability, so that the distribution of ability is the same whether $i = L$ or $H$.

The (binary) skill level $j$ of children is determined by education. Children who go to a vocational school ($j = V$) become low-skilled, while those who go to university ($j = u$) become high-skilled. We denote by $\theta_u$ the minimum level of ability required to be admitted to a university and to become high-skilled. After completing school, children work and obtain a wage which is the product of their idiosyncratic ability, $\theta$, and of the reference wage for their skill level, $\omega_j$.\footnote{So, even though there are only two skill levels, the actual income of workers of given skills is continuously increasing in their ability.} The skill premium $\omega_u - \omega_V$ is increasing (resp., decreasing) in the fraction of low-skilled (resp., high-skilled) agents, and is always strictly positive. To simplify the algebra (and without loss of generality for our results), we assume that the skilled reference wage $\omega_u$ is exogenous (for instance, set by the world market) while $\omega_V$ decreases with the proportion of low-skill agents. As we will see shortly, all children bright enough to be accepted at the university (i.e., with $\theta > \theta_u$) indeed attend university and become high-skilled, so that the proportion of low-skill agents is $F(\theta_u)$. To save on notation, we will often use the shortcut $\omega_V(\theta_u)$ rather than $\omega_V(F(\theta_u)))$. Note then that $\omega_V'(\theta_u) = (\partial \omega_V(F(\theta_u)) / \partial F) f(\theta_u) < 0$. Observe that, although the individual wage
\( \theta \omega_V(\theta_u) \) of low-skilled agents is decreasing in \( \theta_u \), their average wage,

\[
\frac{1}{F(\theta_u)} \int_0^{\theta_u} \theta \omega_V(\theta_u) dF(\theta),
\]

may be increasing in \( \theta_u \) by a composition effect, since their average ability increases with the threshold \( \theta_u \).\(^9\)

The cost of vocational education is normalized to zero, while universities are costly: the (constant) cost per student of university education is \( c_u \), and is financed through a proportional tax on income at rate \( t \), paid by all parents.\(^10\) The government budget constraint is then

\[
t \bar{w} = c_u (1 - F(\theta_u)).
\]

Parents care both about their own consumption (after-tax income) and about the future wage of their kid. If their child becomes highly skilled, the parent’s utility is

\[
U_u(\theta_u, w_i, \theta) = w_i (1 - t(\theta_u)) + \delta \omega_u,
\]

while it is

\[
U_V(\theta_u, w_i, \theta) = w_i (1 - t(\theta_u)) + \delta \omega_V(\theta_u),
\]

if the child remains low-skilled. The parameter \( \delta > 0 \) measures the intensity of the altruism of parents towards their child.

The timing of the model is as follows. Parents first vote over the threshold \( \theta_u \). They then decide individually whether to enrol their unique child at university. Solving the second stage is straightforward: since the skill premium is always positive, i.e. \( \omega_u >

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\(^9\)All results in this paper can be generalized to a setting with uncertainty (as to the probability of actually graduating or the future wage amount) as long as the expected wage of students increases with \( \theta \) (for instance because of a lower dropout rate) and is larger when attending university rather than the vocational school, whatever \( \theta \).

\(^{10}\)Since all children get some form of education in our model, adding a cost for vocational education would not change our results provided we interpret \( c_u \) as the difference between the per student university and vocational school costs. Also, the assumption of proportional taxation is made for simplicity only, with all our results continuing to hold provided that taxes paid increase with income.
\( \omega_V(\theta_u) \forall \theta_u \), we have that \( U_u(\theta_u, w_i, \theta) > U_V(\theta_u, w_i, \theta) \) for all \( \theta \): parents pay the same tax in both cases (since the university has the same size), but their child has a larger income when highly skilled than when unskilled. All parents whose child ability \( \theta \) reaches the threshold \( \theta_u \) thus enrol their child at university.

We next look at parents’ preferences over the threshold level \( \theta_u \) before aggregating these preferences through majority voting.

### 3 Individual preferences over \( \theta_u \)

We proceed in two steps. We first look at individuals’ preferences over \( \theta_u \) as a function of the (exogenous) type of education received by the child (university or vocational), and we then look at overall preferences over \( \theta_u \) when the education type is determined by whether the child’s ability reaches the threshold \( \theta_u \) or not.

We start with the preferences over \( \theta_u \) of parents whose children attend a university:

\[
\frac{\partial U_u(\theta_u, w_i, \theta)}{\partial \theta_u} = -w_i t'(\theta_u) > 0
\]

since

\[
t'(\theta_u) = -\frac{c_u f(\theta_u)}{\bar{w}} < 0. \tag{1}
\]

Conditional on their child going to university, parents always most-prefer a smaller university size (i.e., larger \( \theta_u \)) since it decreases their tax bill without affecting the exogenous reference wage received by high-skilled agents.\(^{11}\)

Alternatively, the most-preferred value of \( \theta_u \) of a parent whose child remains low-skilled, which we denote by \( \theta_u^*(w_i, \theta) \), satisfies the following FOC:

\[
\delta \theta \frac{\partial \omega_V(F(\theta_u^*)))}{\partial F} = -c_u \frac{w_i}{\bar{w}}. \tag{2}
\]

This individually optimal size trades off the smaller vocational wage associated to a smaller university (the left-hand side of (2)) with the smaller tax bill (the right-hand side of (2)).

\(^{11}\) Adding university peer effects would reinforce the attractiveness of a smaller (and more elitist) university.
The necessary and sufficient condition for the SOC

$$\delta \theta \frac{\partial^2 \omega_V(F(\theta_u))}{\partial F^2} f(\theta_u) < 0$$

to be satisfied is that $\partial^2 \omega_V(F(\theta_u))/\partial F^2 < 0$. We assume from now on that the SOC holds and that $\underline{\theta} < \theta^*_u(w_i, \theta) < \bar{\theta}$ for all $(\theta, i)$.

The following lemma performs the comparative statics analysis of $\theta^*_u$.

**Lemma 1** $\theta^*_u(w_i, \theta)$ decreases with $\theta$ and $\delta$ and $\theta^*_u(w_L, \theta) < \theta^*_u(w_H, \theta)$.

**Proof.** Applying the implicit function theorem on the FOC (2), and assuming an interior solution, we obtain that

$$\frac{\partial \theta^*_u(w_i, \theta)}{\partial w_i} = -e'(\theta_u) > 0,$$  \hspace{1cm} (3)
$$\frac{\partial \theta^*_u(w_i, \theta)}{\partial \delta} = \delta \frac{\partial \omega_V(F(\theta_u))}{\partial F} < 0,$$
$$\frac{\partial \theta^*_u(w_i, \theta)}{\partial \theta} = \delta \frac{\partial \omega_V(F(\theta_u))}{\partial F} < 0.$$  

Richer parents pay more taxes and are thus in favor of a smaller university when their child does not attend university. Also, parents of brighter low-skilled kids put more weight on the reference vocational wage (because their kid’s wage increases with $\theta$) and thus favor a larger university to restrict the supply of low-skilled agents and so boost this reference wage. A similar phenomenon occurs for all parents when the degree of altruism is increased.

The following lemma will prove useful.

**Lemma 2** There exists a unique value of $\theta$ for each income level $i \in \{L, H\}$, denoted by $\hat{\theta}_i$, such that $\theta^*_u(w_i, \theta) > \theta$ for all $\theta < \hat{\theta}_i$ and $\theta^*_u(w_i, \theta) < \theta$ for all $\theta > \hat{\theta}_i$.

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\footnote{Observe that we keep $\bar{w}$ constant in (3) since our objective is simply to compare $\theta^*_u(w_L, \theta)$ and $\theta^*_u(w_H, \theta)$.}
Proof. Observe that \( \lim_{\theta \to 0} \theta_u(w_i, \theta) = \tilde{\theta} > 0, \theta_u(w_i, \tilde{\theta}) < \tilde{\theta} \) and \( \partial \theta_u(w_i, \theta)/\partial \theta < 0. \)

We now study the preferences over \( \theta_u \) when the child’s educational track is endogenous. This means that a parent anticipates that his child will be low-skilled if \( \theta_u > \theta \) and will attend university and become high-skilled if and only if \( \theta_u < \theta \). His utility over \( \theta_u \) is then given by

\[
U(\theta_u, w_i, \theta) = U_u(\theta_u, w_i, \theta) \text{ if } \theta_u < \theta, \quad U_V(\theta_u, w_i, \theta) \text{ if } \theta_u > \theta.
\]

Observe that there is a discontinuous decrease in utility for all agents at \( \theta_u = \theta \), since the skill premium is positive and so \( U_u(\theta_u, w_i, \theta) > U_V(\theta_u, w_i, \theta) \forall \theta, \theta_u \). It is straightforward to see that preferences are single-peaked in \( \theta_u \) if \( \theta_u(\theta_u, \theta) < \theta \) (i.e., if \( \theta > \hat{\theta}_i \)) (see Figure 3) but are not if \( \theta_u(\theta_u, \theta) > \theta \) (i.e., if \( \theta < \hat{\theta}_i \)) (see Figure 4).

The following proposition studies which parents most-prefer a university size compatible with their child becoming high-skilled.

**Proposition 1** For each parent’s income level \( w_i \), there exists a unique value of \( \theta \), denoted by \( \tilde{\theta}_i \), such that all agents with \( \theta < \tilde{\theta}_i \) most-prefer putting their kids in a vocational school with \( \theta_u = \theta_u(w_i, \theta) > \tilde{\theta}_i \), while all \( \theta > \tilde{\theta}_i \) most-prefer enrolling their kid at the smallest university that accepts him: \( \theta_u = \theta \). Moreover, we have that \( \tilde{\theta}_i < \hat{\theta}_i \).

**Proof.** See Appendix A. ■

The parent of a higher ability child benefits relatively more from going to university, for two reasons: (i) the child benefits more from the skill premium \( \omega_u - \omega_V \) and (ii) it is socially (and individually) less costly for the university to enrol his child (because the implied size of the university, and thus its tax cost, is lower). This explains why there exists a unique threshold value of \( \theta \) below (resp., above) which parents most-prefer a university size consistent with their child becoming low-skilled (resp., high-skilled).
The next lemma performs the comparative statics analysis of this threshold individual $\tilde{\theta}_i$.

**Lemma 3** $\tilde{\theta}$ is decreasing in $\delta$ with $\tilde{\theta}_L < \tilde{\theta}_H$.

**Proof.** (a) $\tilde{\theta}_i$ is such that $U_u^*(w_i, \theta) - U_v^*(w_i, \theta) = 0$. Applying the implicit function theorem, we obtain that

$$\frac{\partial \tilde{\theta}_i}{\partial w_i} = -\frac{t(\theta_u^*(w_i, \tilde{\theta}_i)) - t(\tilde{\theta}_i)}{-w_i t'(\tilde{\theta}_i) + \delta [\omega_u - \omega_V(\theta_u^*(w_i, \tilde{\theta}_i))]} > 0,$$

since $\tilde{\theta}_i < \hat{\theta}$ (Proposition 1) and by Lemma 2,

$$\frac{\partial \tilde{\theta}_i}{\partial \delta} = -\frac{\tilde{\theta}_i \left[\omega_u - \omega_V(\theta_u^*(w_i, \tilde{\theta}_i))\right]}{-w_i t'(\tilde{\theta}_i) + \delta [\omega_u - \omega_V(\theta_u^*(w_i, \tilde{\theta}_i))]} < 0.$$

Recall that individual $\tilde{\theta}_i$ is indifferent between the smallest university accepting his child ($\theta_u = \tilde{\theta}_i$) and a smaller (at $\theta_u^*(w_i, \tilde{\theta}_i) > \tilde{\theta}_i$) university system that would exclude his child. A richer individual pays a larger fraction of the cost of the university system, and thus has to be the parent of a more gifted child to be indifferent between the two options. Alternatively, a more altruistic parent puts more weight on the skill premium enjoyed by university graduates and thus has to be the father of a less gifted child to be indifferent between the two options.

We now move to the determination of the majority voting equilibrium threshold ability.

## 4 Majority voting equilibrium

We first introduce this straightforward definition.

**Definition 1** Let $\theta_u^{MV}$ be the median most-preferred value of $\theta_u$ in the population.

The following lemma compares $\theta_u^{MV}$ with $\theta_{med}$.

**Lemma 4** (a) $\theta_u^{MV}$ is unique. (b) If $\theta_u^*(w_H, \theta) < \theta_{med}$ then $\theta_u^{MV} = \theta_{med}$. (c) otherwise $\theta_u^{MV} > \theta_{med}$.
Proof. (a) Follows from the facts that $\theta_u^*(w_i, \theta)$ and $\theta$ are continuous and strictly monotone in $\theta$ for $i \in \{L, H\}$ and that $f(\theta)$ has full support. (b) If $\theta_u^*(w_H, \theta) < \theta_{med}$ then $\tilde{\theta}_L < \tilde{\theta}_H < \theta_{med}$ (since $\partial \theta_u^*(w_i, \theta)/\partial \theta < 0$ and $\partial \tilde{\theta}_i/\partial w_i > 0$) so that all $\theta > \theta_{med}$ most-prefer $\theta_u = \theta > \theta_{med}$, while all $\theta < \theta_{med}$ most-prefer either $\theta_u = \theta < \theta_{med}$ or $\theta < \theta_u^*(w_i, \theta) < \theta_{med}$ (since $\partial \theta_u^*(w_i, \theta)/\partial \theta < 0$). Hence $\theta_u^{MV} = \theta_{med}$. (c) If $\theta_u^*(w_H, \theta) > \theta_{med}$ then it is clear that more than one half of the polity (made of rich parents with low ability children, and of parents of higher-than-$\theta_{med}$ ability children) most-prefer a higher-than $\theta_{med}$ value of $\theta_u$. It is then straightforward that $\theta_u^{MV} > \theta_{med}$. ■

Lemma 4 is straightforward when one looks at Figures 5 (Lemma 4(b)) and 6 (Lemma 4(c)).

Insert Figures 5 and 6 around here

Proposition 2 proves the existence of a majority voting equilibrium and shows that it can be of two types. It makes use of the following assumption.

Assumption 1 $\max \left[ \theta_u^*(w_H, \tilde{\theta}_H), \theta_u^*(w_L, \tilde{\theta}_L) \right] \leq \theta_u^{MV}$.

Assumption 1 is essentially technical and guarantees the existence of a Condorcet winner when voting over $\theta_u$.\(^{13}\)

Proposition 2 (a) If $\theta_u^*(w_H, \theta) < \theta_{med}$, then $\theta_u^{MV} = \theta_{med}$ is the unique Condorcet winning value of $\theta_u$. (b) If $\theta_u^*(w_H, \theta) > \theta_{med}$ and if Assumption 1 is satisfied, then $\theta_u^{MV} > \theta_{med}$ is the unique Condorcet winning value of $\theta_u$.

Proof. See Appendix C ■

The type of majority voting equilibrium as well as the chosen size of the university system depend crucially on the preferences of a rich parent with the lowest ability child.

\(^{13}\)We refer the reader to Appendix B for a description of the equilibrium existence issues faced when Assumption 1 is not satisfied.
If such a parent (who most-prefers not to enrol his child at university) prefers a relatively large university system, then the decisive voters are the (poor and rich) parents of a child with median skill and we obtain a “classical equilibrium” where half of students are enrolled in the university and where the top half of the ability distribution favors a smaller university while the bottom half favors a larger one (see Figure 5). Among those who favor a larger university, parents of low ability children do this in order to boost the vocational wage of their child while parents of children with larger abilities would like their child to become highly skilled. Observe that a university system enrolling one half of the student population is precisely the target of current French policy (and of UK policy under Prime Minister Blair).

If the rich parent of the lowest ability child prefers a relatively small university system (and if Assumption 1 is satisfied), the majority voting equilibrium is of the “ends-against-the-middle” type (see Figure 6), with four decisive voters, and where parents with either low and high (strictly larger than median) ability children prefer a smaller university size (to decrease their tax bill in both cases), while parents of children with medium abilities prefer a larger university system (to enable access to university for the higher ability children in this group, and to further boost the vocational wage for the lower ability group). The equilibrium proportion of children attending university is strictly less than one half, which corresponds to the empirical data reported in the Introduction.

We next briefly compare the majority chosen value of $\theta_u$ with its socially optimal level, before performing its comparative statics analysis.

5 Comparison with the utilitarian optimum

A utilitarian social planner would choose the value of $\theta_u$ which maximizes the sum of individual utilities. Assuming to simplify, in this section, that there is a single parental income level $w$, the optimal value of $\theta_u$ would maximize

$$\delta \int_{\theta_u}^{\hat{\theta}} \theta \omega_V(\theta_u)dF(\theta) + \delta \int_{\theta_u}^{\hat{\theta}} \theta \omega_d dF(\theta) - c_u(1 - F(\theta_u)),$$
with the following FOC, where $\theta_u^W$ denotes the utilitarian optimum,

$$c_u = \delta \left[ \theta_u^W (\omega_u - \omega_V(\theta_u^W)) - \frac{\partial \omega_V(F(\theta_u^W))}{\partial F} \frac{\theta_u^W}{2} \int_2^{\theta_u^W} \theta dF(\theta) \right]. \quad (4)$$

The LHS of (4) denotes the marginal social benefit of increasing $\theta_u$, while the RHS represents its marginal cost. The latter can be decomposed into two effects: the first term represents the loss of the skill premium by the marginal agent $\theta_u$ who loses access to university, while the second term measures the decrease in vocational wage of all low-skilled agents when the university size is decreased.

We now compare this utilitarian level $\theta_u^W$ with the one chosen under majority voting. We concentrate on the ends-against-the-middle equilibrium. Denoting by $\theta_{dec}$ the decisive voter who most-prefers $\theta_u^{MV} > \theta_{dec}$ (i.e., whose child becomes low-skilled, see Figure 6), the FOC for $\theta_u^{MV}$ is

$$c_u = -\delta \theta_{dec} \frac{\partial \omega_V(F(\theta_u^{MV}))}{\partial F} = 0. \quad (5)$$

The decisive voter $\theta_{dec}$ does not consider the first marginal cost of increasing $\theta_u$, namely the loss of the skill premium by the marginal student $\theta_u^{MV}$. Moreover, the decisive voter considers only the impact of $\theta_u$ on the vocational wage of his child, rather than on all low-skilled agents. We then obtain the following proposition, where the two effects reinforce each other.

**Proposition 3** In an ends-against-the-middle equilibrium, if

$$\theta_{dec} < \int_2^{\theta_u^W} \theta dF(\theta),$$

then the majority chosen university size is too large compared to its utilitarian level—i.e., $\theta_u^{MV} < \theta_u^W$. 

14
6 Comparative statics analysis of the majority chosen ability threshold level

With the classical equilibrium, we have $\theta^\text{MV}_u = \theta^\text{med}$, so that the result that 50% of students go to university is not affected by changes in parameters of the model. We then investigate the circumstances under which an ends-against-the-middle equilibrium emerges, as well as how the majority voting size of the university sector is affected in that equilibrium.

**Proposition 4** Factors that favor an ends-against-the-middle equilibrium are (i) a larger income inequality (in the form of a means-preserving spread of income levels), (ii) a poorer society (i.e., a larger proportion of low income agents, driving the average income down), (iii) a more expensive university (larger $c_u$), (iv) a smaller degree of altruism of parents (i.e., a lower value of $\delta$), (v) a lower minimum ability level of children (keeping $\theta^\text{med}$ constant), (vi) a vocational wage less sensitive to supply (i.e., a smaller absolute value of $\omega'_V(\theta_u)$).

**Proof.** An ends-against-the-middle equilibrium arises when $\theta^*_u(w_H, \theta) > \theta^\text{med}$. We then look at all factors that increase $\theta^*_u(w_H, \theta)$, (with $\theta^\text{med}$ constant) which is determined by the following FOC (where we have made use of (2) and (1))

$$\frac{\partial \omega_V(F(\theta_u))}{\partial F} + \frac{w_H}{w} c_u = 0.$$

Repeated application of the implicit function theorem gives results (i) to (vi). □

An ends-against-the-middle equilibrium is more likely, other things equal, when the rich parent of the child with the lowest ability level most-prefers a public university enrolling less than one half of the students. Three factors make the university system more expensive for this individual: a larger income level (since tax financing of universities is proportional to income), a smaller average income (i.e., tax base) and a larger cost per student $c_u$. Three factors decrease the benefit of a large university for this individual: a less altruistic society, a lower minimum level of ability and a smaller sensitivity of the
vocational wage to the number of low-skilled workers (the latter two factors decrease the incentive to restrict the supply of low skills in order to increase the child’s vocational wage).

Observe that a small variation in the skilled reference wage $\omega_u$ does not impact the type of majority voting equilibrium: although it affects the set of agents who most prefer to enrol their child at university (i.e., $\tilde{\theta}_i$), it does not affect the most-preferred threshold $\theta_u$ of those who prefer their child to remain low skill. Also, introducing some correlation between $w$ and $\theta$ (while keeping the marginal distributions of skills and of income unchanged) does not affect the nature of the voting equilibrium (classical or ends-against-the-middle), because the introduction of correlation does not impact the government’s budget constraint (since all children whose ability is above the chosen threshold go to university, whatever the income level of their parent). We will show in Proposition 6 the impact of the correlation between income and ability on the size of the university in an ends-against-the-middle equilibrium.

We now look at the comparative statics analysis of $\theta_{u}^{MV}$ in an ends-against-the-middle equilibrium. Observe that this requires understanding not only the impact of (changing) parameters on $\theta_{u}^{*}(w_i, \theta)$, but also on $\tilde{\theta}_i$.

**Proposition 5** Factors that decrease the majority chosen university size (i.e., increase $\theta_{u}^{MV}$) in an ends-against-the-middle equilibrium are (i) a higher income inequality (in the form of a means-preserving spread of income levels), provided that income inequality is large enough to start with, (ii) a lower degree of altruism of parents (i.e., a lower value of $\delta$) and (iii) a lower skill premium.

**Proof.** See Appendix D

A higher income inequality decreases the most-preferred university size of rich parents of low ability children (because their tax bill increases) but increases that of the poor parents (for the symmetrical reason). If income inequality is large enough so that all poor parents most-prefer a larger university size than the one chosen under majority voting in an ends-against-the-middle equilibrium, the latter effect has no impact on the median
most-preferred size, while the former effect decreases this median most-preferred size. A lower degree of altruism has the straightforward impact of decreasing the equilibrium size of the (costly) university, while a larger skill premium does not affect the most-preferred size of university for parents sending their kid to vocational school, but induces parents to prefer sending their child to university for lower ability levels, thereby (weakly) increasing the equilibrium fraction of skilled agents.

Note that the impact of both a higher income inequality and a lower degree of altruism is consistent across Propositions 4 and 5, since they both favor a move from the classical ($\theta_u^{MV} = \theta_{med}$) to the ends-against-the-middle ($\theta_u^{MV} > \theta_{med}$) equilibrium, while increasing $\theta_u^{MV}$ in the latter case.

We now introduce positive correlation between $w$ and $\theta$ using the concept of “median-preserving spread” introduced by Allison and Foster (2004). Assume that we have

$$F_i(\theta) = \lambda F_{L_i}(\theta) + (1 - \lambda) F_{H_i}(\theta),$$

where $F_i(\theta)$ denotes the distribution function of $\theta$ among parents with income $w_i$, and $f_i(\theta)$ the corresponding density function. We assume that these density functions satisfy

$$f_L(\theta) \geq f_H(\theta) \text{ for all } \theta < \theta_{med},$$

$$f_L(\theta) \leq f_H(\theta) \text{ for all } \theta > \theta_{med}.$$

The case with no correlation between income and ability corresponds to $f_L = f_H$ for all $\theta$. We increase the correlation between $w$ and $\theta$ by having $f_L(\theta) - f_H(\theta)$ increase for all $\theta < \theta_{med}$ and decrease for all $\theta > \theta_{med}$, while keeping $F(\theta)$ unchanged. In words, for any value of $\theta < \theta_{med}$, the fraction of children having a $w_L$ parent increases, while the opposite occurs for children with $\theta > \theta_{med}$. Note that $\theta_{med}$ is not affected, since the marginal distribution $F(\theta)$ is not affected by assumption.

We then obtain the following proposition.

**Proposition 6** In an end against-the-middle equilibrium with $\theta_{med} > \tilde{\theta}_H$, increasing the correlation between $w$ and $\theta$ in the way just defined increases the majority-chosen university size (i.e., lowers $\theta_u^{MV}$).
Proof. Recall from Proposition 4 that the correlation between income and ability affects neither the type of equilibrium nor individual preferences over $\theta_u$, so that $\tilde{\theta}_H$ is not affected either. All agents with $\theta > \tilde{\theta}_H$ prefer $\theta_u = \theta$, whatever their income level, so that the increase in correlation does not affect their preferences. Among agents with $\theta < \tilde{\theta}_H$, low income parents most-prefer a lower value of $\theta_u$ than high income agents of the same $\theta$ (with $\theta_u^*(w_L, \theta) < \theta_u^*(w_H, \theta)$ for $\theta < \tilde{\theta}_L$ and $\theta < \theta_u^*(w_H, \theta)$ for $\tilde{\theta}_L < \theta < \tilde{\theta}_H$), and the fraction of low income parents among these agents increases provided that $\theta_{med} > \tilde{\theta}_H$, hence the result.

The preferences of parents with higher-than-$\tilde{\theta}_H$ ability children are not affected by their income, since they most-prefer the smallest university that enrols their children. The preferences over $\theta_u$ of parents with $\theta < \tilde{\theta}_H$ are affected by their income level, with low-income parents preferring a larger university system (i.e., a lower $\theta_u$) than rich parents (whether they prefer their child to remain unskilled or not) because of their lower tax cost of universities. Increasing the correlation between income and ability, by increasing the share of poor parents among those with lower-than-median abilities, then results in a larger majority chosen size of the university system. Observe that Proposition 6 is driven not by the increase in the fraction of rich parents with smart kids, but rather by the increase, among children with lower abilities, of the fraction of those with poor parents. Also, since children with $\theta > \theta_u^{MV} > \theta_{med}$ end up attending university, an increase in the correlation between income and ability (in the way defined above) results in a larger fraction of high-skilled children coming from rich families (even though more people get access to universities).

7 Conclusion

In this paper, we have built a simple model to assess the political support for higher education in a setting where university admission is conditioned on a minimum ability level, and is fully subsidized. This in turn means that no household is credit constrained and that all students who reach the minimum ability level do become high skilled. We have
also assumed away any uncertainty regarding the ability level of one’s child. Finally, we have assumed away any private alternatives to public education. Lifting those assumptions would doubtless improve the model.

Appendix A: Proof of Proposition 1

We denote by
\[ U_u^*(w_i, \theta) = U_u(\theta, w_i, \theta) \]
the highest utility level a parent of type (\(\theta, i\)) can attain by sending his child to university (i.e., when setting \(\theta_u = \theta\)), and by
\[ U_V^*(w_i, \theta) = U_V(\theta_u^*(w_i, \theta), w_i, \theta) \]
the highest utility level attained when his child attends vocational school (i.e., when setting \(\theta_u = \theta_u^*(w_i, \theta)\)). We have
\[ U_u^*(w_i, \theta) - U_V^*(w_i, \theta) = w_i [t(\theta_u^*(w_i, \theta)) - t(\theta)] + \delta \theta [\omega_u - \omega_V(\theta_u^*(w_i, \theta))] . \]  
(6)

Using the envelope theorem, we obtain
\[ \frac{\partial}{\partial \theta} (U_u^*(w_i, \theta) - U_V^*(w_i, \theta)) = -w_i t'(\theta) + \delta [\omega_u - \omega_V(\theta_u^*(w_i, \theta))] > 0. \]

It is easy to see that \( \lim_{\theta \to 0} U_u^*(w_i, \theta) < \lim_{\theta \to 0} U_V^*(w_i, \theta) \) since \( \lim_{\theta \to 0} \theta_u^*(w_i, \theta) = \tilde{\theta} \) so that \( t(\tilde{\theta}) < \lim_{\theta \to 0} t(\theta) \), while \( U_u^*(w_i, \tilde{\theta}) < U_V^*(w_i, \tilde{\theta}) \) since \( \theta_u^*(w_i, \tilde{\theta}) > 0 \) and \( \omega_V(\theta_u^*(w_i, \tilde{\theta})) < \omega_u \). Hence the existence and unicity of \( \tilde{\theta}_i \). Moreover, \( U_u^*(w_i, \tilde{\theta}_i) > U_V^*(w_i, \tilde{\theta}_i) \), implying that \( \tilde{\theta}_i < \hat{\theta}_i \).

Appendix B: Assumption 1

To convey the intuition for why Assumption 1 is necessary to guarantee the existence of a majority voting equilibrium when \( \theta_u^*(w_H, \theta) > \theta_{med} \), assume that there is only one income level, \( w \). Figure 7a shows a situation under which Assumption 1 is not satisfied. In that case, the individual \( \hat{\theta} \) is indifferent between \( \theta_u = \hat{\theta} < \theta_u^{MV} \) and \( \theta_u^*(w, \hat{\theta}) > \theta_u^{MV} > \hat{\theta} \). Figure 7b reports the utility function \( U(\theta_u, w, \hat{\theta}) \) of individual \( \hat{\theta} \). It is clear that, unlike in the proof of Proposition 2 (b), \( \theta_u^{MV} \) is not preferred to all \( \theta < \theta_u^{MV} \) by individual \( \hat{\theta} \), since this individual attains a higher utility level with \( \theta_u = \hat{\theta} - \varepsilon \) with \( \varepsilon > 0 \) low enough.
This opens up the possibility of a Condorcet cycle and of the inexistence of a Condorcet winning value of \( \theta_u \).

Insert Figures 7a and 7b here

Observe that Assumption 1 is not satisfied when

\[
F(\theta^*_u(w, \hat{\theta})) - F(\hat{\theta}) > 1/2,
\]

which implies that \( \theta_{med} \in ]\hat{\theta}, \theta^*_u(w, \hat{\theta})[ \) —i.e., that a large fraction of the population is concentrated around the median ability level.

**Appendix C: Proof of Proposition 2**

(a) Assume that \( \theta^*_u(w_H, \bar{\theta}) < \theta_{med} \), so that we claim that \( \theta^M = \theta_{med} \) (see Lemma 4 (a)) is preferred by a majority of parents to any other value of \( \theta_u \). It is easy to see that all agents with \( \theta \geq \theta_{med} \) prefer \( \theta_{med} \) to any value of \( \theta_u < \theta_{med} \) (since \( U_u(\theta_u, w, \theta) > U_V(\theta_u, w, \theta) \) when \( \theta_u < \theta \), and since \( U_u(\theta_u, w_i, \theta) \) increases with \( \theta_u \)). Since they form a majority, \( \theta_{med} \) cannot be beaten by any \( \theta_u < \theta_{med} \). We now look at agents with \( \theta < \theta_{med} \). They all have \( \theta^*_u(w_i, \theta) < \theta_{med} \), since \( \theta^*_u(w_H, \bar{\theta}) < \theta_{med} \) together with \( \partial \theta^*_u(w_i, \theta)/\partial w_i > 0 \) and \( \partial \theta^*_u(w_i, \theta)/\partial \theta < 0 \). They then all prefer \( \theta_{med} \) to any \( \theta_u > \theta_{med} \), and since they form a majority \( \theta_{med} \) cannot be beaten by any \( \theta_u > \theta_{med} \) and constitute the unique Condorcet winner.

(b) Assume that \( \theta^*_u(w_H, \bar{\theta}) > \theta_{med} \), so that we claim that \( \theta^M > \theta_{med} \) (see Lemma 4 (b)) is preferred by a majority of parents to any other value of \( \theta_u \). Since \( \partial \theta^*_u(w_i, \theta)/\partial \theta < 0 \), its inverse is unique over its range \( [\theta^*_u(w_i, \tilde{\theta}_i), \theta^*_u(w_i, \bar{\theta})] \). We then denote by \( \theta^*_i(\theta_u) \) the unique type \( \theta \) of a parent of income \( w_i \) who would most-prefer \( \theta_u \in [\theta^*_u(w_i, \tilde{\theta}_i), \theta^*_u(w_i, \bar{\theta})] \) (and send his child to vocational school) and we define \( \theta^*_i(\theta_u) = \theta \) for \( \theta_u > \theta^*_u(w_i, \bar{\theta}) \). It is clear that \( \theta^*_i(\theta_u) \) decreases with \( \theta_u \) on \( [\theta^*_u(w_i, \tilde{\theta}_i), \theta^*_u(w_i, \bar{\theta})] \). We then define by

\[
V(\theta_u) = 1 - F(\theta_u) + \lambda F(\theta^*_L(\theta_u)) + (1 - \lambda) F(\theta^*_H(\theta_u))
\] (7)
the proportion of parents who most-prefer a larger value of the tracking university threshold than $\theta_u$. Note that this set of parents is constituted both of parents of low $\theta$ kids who would not be enrolled at university with this $\theta_u$, and of parents of large $\theta$ kids who would go to university with this $\theta_u$. Observe that $V(\theta_{med}) > 1/2$ and that $\partial V(\theta_u)/\partial \theta_u < 0$. Assumption 1 guarantees that $V(\theta_u^{MV}) = 1/2$, so that $\theta_u^*(\theta_u^{MV})$ is well defined with $\theta \leq \theta_u^*(\theta_u^{MV})$ for $i \in \{L, H\}$. Moreover, we have that $\theta_{med} < \theta_u^{MV} < \theta_u^*(w_H, \theta)$.

We now prove that $\theta_u^{MV}$ is a Condorcet winner. As in part (a) above, $\theta_u^{MV}$ is preferred to any lower value of $\theta_u > \theta_u^{MV}$ by the individuals who most-prefer a value of $\theta_u$ lower than $\theta_u^{MV}$.

In this case, this group is made of agents with $\theta_u^* (\theta_u^{MV}) \leq \theta \leq \theta_u^{MV}$ and, by definition of $\theta_u^{MV}$, constitutes one half of the electorate. As in part (a) above, agents with $\theta > \theta_u^{MV}$ prefer $\theta_u^{MV}$ to any smaller value of $\theta_u$. We then have to prove that the remaining group, made of parents of low $\theta$ children who favor a larger than $\theta_u^{MV}$ value of $\theta_u$ also prefer $\theta_u^{MV}$ to any lower value of $\theta_u$. This group is formed of all parents with $\theta < \theta_u^* (\theta_u^{MV})$. For this group, we then have that $\theta < \theta_u^{MV} \leq \theta_u^*(w_i, \theta)$. Since (see Figure 2) $\partial U_u(\theta_u, w_i, \theta)/\partial \theta_u > 0$ and $\partial U_V(\theta_u, w_i, \theta)/\partial \theta_u > 0$ for $\theta_u < \theta_u^*(w_i, \theta)$, together with $U_u(\theta_u, w_i, \theta) > U_V(\theta_u, w_i, \theta)$ for $\theta_u < \theta$, a necessary and sufficient condition for $\theta_u^{MV}$ to be preferred to any lower value of $\theta_u$ is thus $U_u^*(w_i, \theta) < U_V(\theta_u^{MV}, w_i, \theta)$ for all $\theta < \theta_u^* (\theta_u^{MV})$. It is easy to see (from the proof of Lemma 1) that $\partial (U_u^*(w_i, \theta) - U_V(\theta_u^{MV}, w_i, \theta))/\partial \theta > 0$ so that, since $U_u^*(w_i, \theta_u^* (\theta_u^{MV})) < U_V(\theta_u^{MV}, w_i, \theta_u^* (\theta_u^{MV}))$, all agents with $\theta < \theta_u^* (\theta_u^{MV})$ strictly prefer $\theta_u^{MV}$ to any lower value of $\theta_u$. By definition of $\theta_u^{MV}$, we then have that (at least) one half of the population share this preference, so that $\theta_u^{MV}$ cannot be defeated at the majority voting and is the unique Condorcet winner.

**Appendix D: Proof of Proposition 5**

(i) The proof of Lemma 1 has shown that $\theta_u^*(w, \theta)$ increases with $w$ when $\bar{w}$ is kept constant. Lemma 3 has established that $\tilde{\theta}_i$ increases with $w_i$. If $\theta_u^*(w_L, \theta) < \theta_u^{MV}$ (i.e., if the income inequality is large enough that $\theta_u^*(w_L, \theta) < \theta_u^{MV} < \theta_u^*(w_H, \theta)$) then a means-preserving spread of the income distribution increases $\theta_u^{MV}$ since it increases the fraction of rich parents who favor $\theta_u^*(w_H, \theta) > \theta_u^{MV}$, while keeping unchanged the fraction of poor
agents who prefer $\theta^*_u(w_L, \theta) < \theta^{MV}_u$ (since we have that all poor parents have $\theta^*_u(w_L, \theta) < \theta^{MV}_u$ to start with).

(ii) Lemma 1 has shown that $\theta^*_u(w, \theta)$ decreases with $\delta$ while Lemma 3 has established that $\tilde{\theta}_i$ decreases with $\delta$. Putting the two together, we obtain that $\theta^{MV}_u$ decreases with $\delta$.

(iii) The easiest way to model an increase in the skill premium ($\omega_u - \omega_V(.)$) is to increase $\omega_u$. This does not affect $\theta^*_u(w_i, \theta)$ but decreases $\tilde{\theta}_i$ since

$$\frac{\partial}{\partial \omega_u} \left( U^*_u(w_i, \theta) - U^*_V(w_i, \theta) \right) = \delta \theta > 0.$$ 

We then have that $\theta^{MV}_u$ weakly decreases with $\omega_u$.

References


http://dx.doi.org/10.1787/eag-2012-en

Figure 1 - Public universities' share of university students
Figure 2: Higher education in European countries in 2010

Education enrolment rates among 20-29
Proportion tertiary education
Figure 3: Single-Peaked Preferences

Utility of \((w, \theta)\)

\[ U_u(\theta_u, w, \theta) \]
\[ U(\theta_u, w, \theta) \]
\[ U_i(\theta_u, w, \theta) \]

\[ \theta_u(w, \theta) \]
\[ \theta \]

Child goes to university  
Child goes to vocational school

Figure 4: Non Single-Peaked Preferences

Utility of \((w, \theta)\)

\[ U_u(\theta_u, w, \theta) \]
\[ U(\theta_u, w, \theta) \]
\[ U_i(\theta_u, w, \theta) \]

\[ \theta \]
\[ \theta_u(w, \theta) \]

university  
vocational school
Figure 5: Classical Equilibrium

Most-preferred $\theta_u$ of $(w, \theta)$

Figure 6: Ends-against-the-middle equilibrium

Most-preferred $\theta_u$ of $(w, \theta)$
Figure 7 a

Most-preferred $\theta_u$ of $(w, \bar{\theta})$

$\theta_u^M$, $\theta_u^M$

$\theta_u^*(w, \bar{\theta})$

$\theta_u = \theta$

Figure 7 b

Utility of $(w, \bar{\theta})$

$U_u(\theta_u, w, \bar{\theta})$

$U(\theta_u, w, \bar{\theta})$

$U_r(\theta_u, w, \bar{\theta})$

$\hat{\theta}$ $\theta_u^{MV}$ $\theta_u^*(w, \bar{\theta})$

University Vocational school