Capital Growth in a Global Warming Model: Will China and India Sign a Climate Treaty?

Prajit K. Dutta* and Roy Radner**

October 20, 2010

Abstract. Global warming is now recognized as a significant threat to sustainable development on an international scale. One of the key challenges in mounting a global response to it is the seeming unwillingness of the fastest growing economies such as China and India to sign a treaty that limits their emissions. The aim of this paper is to examine the differential incentives of countries on different trajectories of capital growth. A benchmark dynamic game to study global warming, introduced in Dutta & Radner (2009), is generalized to allow for exogenous capital accumulation. It is shown that the presence of capital execerbates the "tragedy of the common". Furthermore, even with high discount factors, the threat of reverting to the inefficient "tragedy" equilibrium is not sufficient to deter the emissions growth of the fastest growing economies - in contrast to standard folk theorem like results. However, foreign aid can help. If the slower growth economies - like the United States and Western Europe - are willing to make transfers to China and India then the latter can be incentivized to cut emissions. Such an outcome is Pareto improving for both slower and faster growth economies.

*Department of Economics, Columbia University, New York, NY 10025; email: pkd1@columbia.edu

**Stern School of Business, KMC 8-87, New York University, 44 W. Fourth Street, New York, NY 10012; email: rradner@stern.nyu.edu

1. Introduction

This paper addresses the following question: (when) will the fast growing economies of the East, China and India for example, agree to caps on their greenhouse gas emissions? This introductory section contextualizes the question and then provides a summary of the answers contained in this paper.

1.1. The East Versus West Debate. Global climate change has emerged as the most important environmental issue of our times and, arguably, the one with the most critical long-run import. The observed rise in temperatures and variability of climate - the hot summers in Europe and the United States, the increased frequency of storms and hurricanes including Katerina, the melting of the polar ice-caps and glaciers on Asian mountain-tops threatening to dry the rivers that water that continent, the rise in sea-levels - have all placed the problem center-stage. Since the climate change problem involves a classic "commons" - that irrespective of the source of greenhouse gas emissions it is the common stock of it that affects the global climate - it can only be solved by an international effort at reaching agreement. For such an agreement to get carried out, however, it has to align the incentives of the signatory nations so that countries will, in fact, carry out their promises. At the same time, to meaningfully contain emissions an agreement has to be signed by all the major emitting countries, both developed and developing, and they have to commit to possibly deep cuts in emissions now and in the future. In other words for an agreement to be effective it has to balance two competing forces - large enough cuts that make a difference to the climate that are yet "small enough" that countries will not cheat on their promises.

And herein lies the rub. Since emissions are tied to economic activity, countries that are growing the fastest, such as China and India, are reluctant to sign onto emission cuts that they fear will compromise their growth. They point, moreover, to the "legacy effect" - that the vast majority of existing greenhouse gas stock was accumulated in the last hundred years due to the industrialization of the West - and the per capita numbers - that per person their citizens contribute a fraction of the per capita emissions from the United States and the European Union. They argue, therefore, that they should not be asked to clean up a problem not of their making. On the other hand, leaving these countries out of a climate change treaty is simply not going to solve the problem since their growth path of emissions is high, China's total emissions are already on par with the United States and unless the emissions of the developing world are reduced they will rapidly out-strip those of today's developed economies and make it impossible to sove the climate change problem.

Put another way, finding a solution to the US/Europe versus the China/India stand-off is perhaps the most critical step in arriving at a meaningful climate change treaty. This paper is a modest attempt at analyzing that problem, critiquing a solution that has been suggested and offering an alternative that we believe is attractive.

Before getting to all that though, here are some facts on current greenhouse gas emissions related to the arguments above (details on sources and years may be found in the footnotes):

1. In the last hundred years, 63% of the cumulative emissions of greenhouse gases have come from the developed economies. Of that, the US has accounted for 25% and Western Europe for 21%. China and India, home to 40% of the world's population, have contributed, respectively, 7% and 2% of the last hundred years of cumulative emissions.[1]

[1] The data is drawn from the World Resource Institute web-site and credits two studies published in 2000 - one by Houghton & Hackler and the other by Marland et. al. For details see http://earthtrends.wri.org/features/view_feature.php?fid=31&theme=3.

2. Of 2004 emissions, the United States accounted for 22% of the total, China for 18% and the European Union for 15%. (And since then, China has surpassed the US in total emissions.) The next set of countries - each roughly at 5% - included Japan, India and Russia.[2]

3. Whilst total greenhouse gas emissions are currently lower in the developing world than in the developed economies, the rapid growth in the economies and populations of the former is expected to reverse that by 2015. According to some estimates, in the next twenty years, emissions in the developing economies will double while growing about 20% in the developed economies[3].

Given all this, the question is - what will induce China and India to sign a treaty that limits their emission growth, a treaty that they will then comply with? One possible answer is that they will perceive that the costs of climate change are so high for their economies that they have no option but to sign. These costs include the rise in sea-level along their coast-lines, the drying up of the mighty rivers that feed their agricultural plains, the possible migration into their countries from neighbors such as Bangladesh who are severely affected etc. The problem though is that these climate change induced costs still seem remote in time whereas the economic cost of abandoning a high economic growth path is immediate.

In a recent well-advertized (July 19, 2009) incident, the US Secretary of State, Hillary Clinton, was lectured to by Jairam Ramesh, India's Environment and Forestry Minister who declared "We are simply not in a position to take over legally binding emission reduction targets". As the New York Times went on to observe "Both countries (China and India) say their economic growth should not be constrained when the West never faced such restrictions during its industrialization." Indeed Secretary Clinton hastened to add that "No one wants to, in any way, stall or undermine economic growth that is necessary to lift millions more people out of poverty. The United States does not, and will not, do anything that would limit India's economic progress."[4]

In a parallel diplomatic incident (reported July 15, 2009), the US Commerce and Energy Secretaries Steven Chu and Gary Locke - themselves of Chinese ethnicity - warned the Chinese leadership on a recent visit to the country - "If China's emissions of global warming gases keep growing at the pace of the last 30 years, the country

[2]The data, corresponding to emissions in 2004, was collected in 2007 by the CDIAC (Carbon Dixide Information Analysis Center) of the US Department of Energy for the United Nations. The data considers only carbon dioxide emissions from the burning of fossil fuels. See http://en.wikipedia.org/wiki/List_of_countries_by_carbon_dioxide_emissions.

[3]These numbers are drawn from the US EPA (Environmental Protection Agency) web-site that quotes an article published in the Energy Journal. For details see http://www.epa.gov/climatechange/emissions/globalghg.html.

[4]All this and more at http://www.nytimes.com/2009/07/20/world/asia/20diplo.html?scp=5&sq=Hillary%20Clinton%

will emit more such gases in the next three decades than the United States has in its entire history" (Chu) and "Fifty years from now, we do not want the world to lay the blame for environmental catastrophe at the feet of China," (Locke).[5]

1.2. A Discussion of the Model and the Main Results. The present paper is part of an ongoing research project in which we have addressed certain elements of the global warming problem from a strategic and economic perspective. For other studies in the current project, see Dutta and Radner (2004, 2006 and 2009).

By now the basic mechanism of the greenhouse effect is well-known. The build-up of greenhouse gases - primarily CO_2 - during the course of industrialization of Western economies traps heat in a manner analogous to a greenhouse. Currently, the burning of fossil fuels accounts for most of the carbon emissions produced by humans and almost all of the burning of fossil fuels is done for the purpose of producing energy. Carbon emissions can be reduced in three different ways. Over time technology changes and typically this leads to a progressive "decarbonization"of energy production. For example this has coincided with the movement from coal to oil and natural gas. Another source of decarbonization is increased efficiency in the utilization of energy, coming from improvements in the design of electric generation and transmission systems, electric motors, combustion engines, heating and cooling systems, etc. A third source of decarbonization is a lowering of emissions through reduced utilization of energy.

The costs of climate change (CC) are subject to considerable uncertainty and debate. Roughly speaking, the costs are themselves the results of two primary effects: (1) a rise in the sea level, and (2) climate changes. The rise in the sea level, caused by melting of glacial ice, and to some extent by the thermal expansion of sea water, would damage, and even eliminate, many coastlines. Climate changes are more complex. Parts of the world, such as Sub-Saharan Africa, would probably become more arid and less productive agriculturally. Other effects would include increased energy requirements for air-conditioning, curtailed water supplies, damage to human health, increased hurricane and fire damage, costly increased immigration, etc.

The efforts to avoid CC will, of course, be costly as well. Immediate costs would be incurred if economies were forced to substitute more expensive but less carbon intensive technologies for producing energy. Cutbacks in energy use would also be costly in terms of lower levels of output of goods and services, including "amenities" such as household cooling. What is particularly significant here is the role of capital accumulation. Capital and energy are, presumably, complementary inputs in the production process. Hence, the cost imposed on a country, when energy usage is curtailed, will depend on the size of its capital stock. Constant technology, the cost

[5] All this and more at http://www.nytimes.com/2009/07/16/world/asia/16warming.html.

of energy curtailment is therefore going to be higher when capital stocks are larger - or equivalently, the long-term costs will be higher when capital grows at a faster rate. And that, of course, is part of the objection of China and India to emission cuts, that their fast growth (of capital) will imply that they have the most to lose from a climate change treaty and its attendant emission cuts.

As mentioned above, this paper is part of a project exmining climate treaties. Our approach in the project is unique in that we are the only ones to have analyzed a fully dynamic and fully game-theoretic model. By fully dynamic we mean a model in which actions in the current period have effects that persist into the future. Such intertemporal linkages are vital to the CC problem because the prominent greenhouse gas, CO_2 has a half-life of a hundred years. A game-theoretic approach is required because on the international scale of this problem there is no court that can enforce contracts and there are ndeed a few big "players". (Recall from fact 2 above that six "countries", taking the Europen Union as a single decision-making entity, produce over 70% of the current emissions.)[6] The players in our game are countries, and it is assumed that each country has the authority and political will to control its own rate of emission of greenhouse gases. In the model, each country can control its emissions essentially by controlling its level of economic activity.[7] What we look for is a treaty that countries will sign and then comply with. In game-theoretic terminology what we look for are (subgame perfect) equilibria of a dynamic game of climate change.

In our model each country emits greenhouse gases and gets a short-term benefit from doing so. The size of that benefit depends on country-specific welfare parameters and on the size of its capital stock. This capital stock grows exogenously and geometrically and hence the size of the short-term benefit itself changes over time along with the size of capital stock. The cost of CC depends on the global common - the stock of greenhouse gases that have been built up over time. We make one important simplification - that the marginal cost of CC is independent of the size of this stock. The reasoning behind this simplification is discussed at length in Dutta and Radner (2009) but it suffices to mention that our model lends itself to calibration and hence deduction of numerical magnitudes in closed form which a non-linear model would not allow.

We start in Section 2 with quick review of the initial results from Dutta and Radner (2009), a model in which capital stock is fixed through time.[8] In that paper,

[6]Models that are fully dynamic but not strategic include Nordhaus and Boyer (2000) and Nordhaus and Zhang (1996). Models that are fully strategic but not dynamic include Barrett (2003) and Finus (2001). Also see the fuller bibliographic discussion in Section 6.

[7]One other determinant of economic activity, beyond capital and energy, is labor but that is assumed to remain fixed.

[8]Please note that the short-term benefit function is taken to be a Cobb-Douglas function here but is a more general concave function in Dutta and Radner (2009).

the basic result shows that there is a simple Markov Perfect equilibrium, termed the "Business as Usual" (BAU) equilibrium. This equilibrium exhibits a tragedy of the common in that it leads to emissions that exceed those under any Global Pareto Optimal (GPO) solution. It is further shown that there are better equilibria than the BAU including a class of equilibria whose norm behavior on emissions is sustained by the threat of reverting to the BAU. If countries are sufficiently patient GPO emissions can be sustained as an equilibrium norm as well. These results parallel the well-known results from Repeated Games using trigger strategies.[9]

In Section 3 we introduce exogenous capital accumulation into the model. Again there is a BAU equilibrium - termed a Generalized Business as Usual Equilibrium (GBAU) in this more general model. And it involves over-emission relative to the Generalized Global Pareto Optima (GGPO). The one difference though is that the size of the emissions, in both the GBAU as well as the GGPO, depends on the size of capital stock (on account of the fact that capital and energy/emissions are complementary inputs in the benefit function.) In particular, we show that the tragedy is worse under capital accumulation - in that it worsens over time as capital grows - when evaluated in terms of the differences btween GGPO and GBAU emission levels.

That makes the search for better equilibria more pressing. Our first port of call is to find analogs of the trigger strategy equilibria that we analyzed in the model without capital. And here we discover the first surprise - by way of a negative result. The fastest growing country, i.e., the one with the fastest rate of capital accumulation, will never sign a treaty that requires it to emit at GGPO levels forever. The reasoning is related to the fact - established in Section 3 - that both GGPO and GBAU emissions in each country grow at its rate of effective growth of capital.[10] What that means is that the short-term cost to the fastest growing country in adopting the GGPO emission norm rather than the GBAU emission rate also grows at that rate. In order to bear this cost there has to be, of course, a compensatory future gain from following the GGPO path. In the standard trigger strategy logic that gain arises from the lower emissions of the other countries following the GGPO path. Since the cost is growing the benefit needs to grow as well and at the same rate. However, the gain - by similar logic - grows at the rate of capital accumulation of the other countries. And, hence grows more slowly. Hence no matter what the initial conditions, at some point the gain is simply not big enough to offset the loss in own utility even though the gain persists over the infinite future.[11] Section 4 concludes by showing that the same logic

[9] Though, as noted above, the model is dynamic with intertemporal linkages rather than the static model that a repeated game studies.

[10] The effective growth rate is precisely defined in Section 4. It coincides with the actual growth rate of capital when there is constant returns to scale.

[11] The logic will, of course, be detailed in Section 4. But one quick way to see it is to take the extreme case where the other countries'capital does not grow at all. Then the future gain to the

applies to any uniform cut in GBAU emissions; sanctions that slow growing countries can muster are simply not potent enough to dissuade the fastest growing economies from their preferred emissions.

Although the sanctions route is not promising - as the Indian Minister seemed also to intimate - there is a "carrot" that works better than the "stick". And that carrot has to do with foreign aid (that is conditional on emissions). The foreign aid that we examine is made up of transfers made from the slower growing economies - like, presumably, the US and the European Union - to the fast growing economies, like China and India.[12] The aid is "budget-balanced" in that in every period the total donation equals the total received. The aid is also conditional in that aid continues just as long as the emission norm - such as the GGPO emission path - is observed but is cut off forever after in the event of a deviation. The starting intuition is that slow-growing countries might be willing to share the benefits that they get from the fast growing countries' lower trajectory of emissions. Using the analogy above, the slower growing countries benefit grows at the same rate as the fast growing countries' rate of capital accumulation. If this benefit is transferred over (in part) to a fast growing country it might be willing to suffer the loss in its own welfare due to following GGPO emissions. So a "bribe" - aka foreign aid - might work where a threat does not.

In Section 5 we prove three results. First we show that there exists a policy of (zero-sum) foreign aid transfers such that the "bribe" of conditional foreign aid - transfers made if and only if the GGPO emissions policy is followed - sustains GGPO emissions as an equilibrium outcome (at a high enough discount factor). Second, inclusive of foreign aid, both recipients as well as donors are better off than under the GBAU. Third, there is a continuum of such emission policies all of which involve uniform emission cuts to the GBAU which can be sustained as equilibria. And, again inclusive of foreign aid, both recipients as well as donors are better off than under the GBAU. These results stand in sharp contrast to the results in Section 4 which showed that the threat of sanctions is not effective.[13]

---

fastest growing country, to all other countries following the GGPO emission rather than the GBAU emission, is some finite amount. However, its short-term cost is proportional to is capital stock. As capital stock grows infinitely large, at some period, the short-term cost overwhelms.

[12] An example of such a conditional transfer - or foreign assistance - policy is the World Bank's Climate Investment Fund (CIF).

[13] A referee has suggested that the solution offered in this paper - the benefits of foreign aid in ameliorating climate change - is being realized in practice in current climate agreements, and has been implemented through the UNFCCC rules. The referre points out that the solution proposed theoretically in this paper agrees with the actual structure of the Kyoto Protocol carbon market - which is now international law since 2005, and trading in the European Union Emissions Trading System – that allows such foreign aid transfers through the structure of the UNFCCC Clean Development Mechanism, a mechanism that has already transferred over $26 billion to nations such as China and India to create similar incentives for clean development projects.

The paper concludes in Section 6 with some observations on how the model should be elaborated and generalized to make it more realistic and a brief discussion of other parts of this research project.

2. A Simple Climate Change Game

In this section we present the model and first results of the simplified "climate change game"studied in detail in Dutta and Radner (2009).

2.1. Benchmark Model. There are I countries. The *emission* of (a scalar index of) greenhouse gases during period t by country i is denoted by $a_i(t)$. [Time is discrete, with $t = 0, 1, 2, ...,$ ad inf., and the $a_i(t)$ are nonegative.] Let $A(t)$ denote the global (total) emission during period t;

$$A(t) = \sum_{i=1}^{I} a_i(t). \tag{1}$$

The total (global) stock of greenhouse gases (GHGs) at the beginning of period t is denoted by $g(t)+g_0$, where g_0 is what the "normal" steady-state stock of GHGs would be if there were negligible emissions from human sources (e.g., the level of GHGs in the year 1800). We might call $g(t)$ the *excess GHG*, but we shall usually suppress the word "excess." The law of motion for the GHG is

$$g(t+1) = A(t) + \sigma g(t), \tag{2}$$

where σ is a given parameter $(0 < \sigma < 1)$. We may interpret $(1-\sigma)$ as the fraction of the beginning-of-period stock of GHG that is dissipated from the atmosphere during the period. The "surviving" stock, $\sigma g(t)$, is augmented by the quantity of global emissions, $A(t)$, during the same period. [Note: A realistic model of GHG dynamics would be more complicated; see (Thomson, 1997) but the one above has been fairly widely used.]

Suppose that the utility of country i in period t is

$$v_i(t) = [a_i(t)]^{\beta_i} - c_i g(t). \tag{3}$$

The function $[a_i(t)]^{\beta_i}$ represents, for example, what country i's gross national product would be at different levels of its own emissions, holding the global level of GHG constant.[14] This function reflects the costs and benefits of producing and using energy from alternative sources, including fossil fuels. The parameter $c_i > 0$ represents the marginal cost to the country of increasing the global stock of GHG. Of course, it

[14]In Dutta and Radner (2009) we consider a more general form of felicity function that includes the Cobb-Douglas form $[a_i(t)]^{\beta_i}$.

is not the stock of GHG itself that is costly, but the associated climatic conditions. In a more general model, the cost would be nonlinear. The total payoff (utility) for country i s

$$v_i = \sum_{t=0}^{\infty} \delta^t v_i(t) dt. \tag{4}$$

For the sake of simplicity, we have taken the discount factor, δ, to be the same for all countries. [Note: It has implicitly been assumed here that each country's population is constant in time. The case of changing populations can be examined without much additional difficulty; we do so in Dutta and Radner (2006).]

A *strategy* for a country determines for each period the country's emission level as a function of the entire past history of the system, including the past actions of all the countries. A *stationary strategy* for country i is a function that maps the current state, g, into a current action, a_i. As usual, a *Nash Equilibrium* is a profile of strategies such that no individual country can increase its payoff by *unilaterally* changing its strategy. A *Markov Perfect Equilibrium (MPE)* is a Nash Equilibrium in which every country's strategy is stationary. A *Subgame Perfect Equilibrium (SPE)* is a profile of strategies, not necessarily stationary, that constitutes a Nash Equilibrium after every history.

2.2. The Global Pareto Optimum. Let $x = (x_i)$ be a vector of positive numbers, one for each country. A *Global Pareto Optimum (GPO)* corresponding to x is a profile of strategies that maximizes the weighted sum of country payoffs,

$$v = \sum_i x_i v_i, \tag{5}$$

which we shall call the *global welfare*. Without loss of generality, we may take the weights, x_i, to sum to I.

Theorem 1. *Let $\hat{V}(g)$ be the maximum attainable global welfare starting with an initial GHG stock equal to g; then there are a set of constant emissions $\widehat{a}_i$ determined by*

$$\hat{a}_i = \left(\frac{\beta_i x_i}{\delta w} \right)^{\frac{1}{1-\beta_i}}$$

where $w = \frac{1}{1-\delta\sigma} \sum_i x_i c_i$, that constitute the GPO emissions. Writing $\hat{A} = \sum_i \hat{a}_i$ for the total emissions, the lifetime GPO payoffs are

$$\begin{aligned} \hat{V}(g) &= u - wg, \\ u &= \frac{1}{1-\delta} \left[\sum_i x_i \hat{a}_i^{\beta_i} - \delta w \hat{A} \right], \end{aligned} \tag{6}$$

Proof. The proof uses a standard dynamic programming argument. Let $a = (a_i)$. It is sufficient to show that the value function, $\hat{V}$, given above satisfies the functional equation

$$\hat{V}(g) = \max_a \left\{ \sum_j x_j \left[\hat{a}_j^{\beta j} - c_j g \right] + \delta \hat{V} \left[\sum_j a_j + \sigma g \right] \right\}. \tag{7}$$

The first-order condition for a maximum is that, for each i,

$$x_j \beta_j \hat{a}_j^{\beta j - 1} + \delta \hat{V}' \left[\sum_j a_j + \sigma g \right] = 0.$$

But $\hat{V}' = -w$, so the optimal emission is independent of g, and is given by (7). The values of u and w are now determined by the equation

$$\hat{V}(g) = \sum_j x_j \left[\hat{a}_j^{\beta j} - c_j g \right] + \delta \hat{V} \left[\sum_j a_j + \sigma g \right],$$

which must be satisfied for all values of g.■

2.3. The Business-as-Usual Equilibrium.

The next proposition describes a Markov Perfect equilibrium, which we call the *Business-as-Usual (BAU)* equilibrium. This MPE has the unusual feature that the equilibrium emission rate of each country is constant in time, and it is the unique MPE with this property.

Theorem 2. *(BAU) Let g be the initial stock of GHG. For each country i, let $\overline{a}_i$ be determined by*

$$\overline{a}_i = \left(\frac{\beta_i}{\delta w_i} \right)^{\frac{1}{1-\beta_i}}$$

where $w_i = \frac{c_i}{1-\delta\sigma}$, and let its strategy be to use a constant emission equal to $\overline{a}_i$ in each period; then this strategy profile is a MPE, and, writing $\overline{A} = \sum_j \overline{a}_j$ for the aggregate emissions, country i's corresponding payoff is

$$\begin{aligned} \overline{V}_i(g) &= u_i - w_i g, \\ u_i &= \frac{1}{1-\delta} \left[\overline{a}_i^{\beta_i} - \delta w_i \overline{A} \right]. \end{aligned} \tag{8}$$

Proof. The proof uses an argument similar to that of Theorem 1. If the emissions of all countries other than i are constant, say a_j for country j, then country i faces a standard dynamic programming problem. It is sufficient to show that the value function $\overline{V}_i$ satisfies the functional equation,

$$\overline{V}_i = \max_{a_i} \left\{ a_i^{\beta_i} - c_i g + \delta \overline{V}_i \left(\sum_j a_j + \sigma g \right) \right\}.$$

The argument now proceeds as in the proof of Theorem 1.■

If the cost of the stock of GHG were nonlinear, then one would expect the GPO and BAU emissions to vary with the stock, and in fact one would expect higher stocks to lead to lower emissions. In the next section we will see that, once we introduce capital stock, emissions will no longer be constant in time.

2.4. Comparison of the GPOs and the BAU. The preceding results enable us to compare the emissions in the GPOs with those in the BAU equilibrium:

$$\text{GPO} \; : \qquad \beta_i \hat{a}_i^{\beta_i - 1} = \frac{\delta \sum_j x_j c_j}{x_i(1-\delta\sigma)}, \tag{9}$$

$$\text{BAU} \; : \qquad \beta_i \overline{a}_i^{\beta_i - 1} = \frac{\delta c_i}{1-\delta\sigma}.$$

From

$$x_i c_i < \sum_j x_j c_j,$$

it follows that

$$\frac{\delta c_i}{1-\delta\sigma} < \frac{\delta \sum_j x_j c_j}{x_i(1-\delta\sigma)}.$$

Since $a_i^{\beta_i}$ is concave, it follows that

$$\overline{a}_i > \hat{a}_i. \tag{10}$$

Note that this inequality holds for all vectors of strictly positive weights (x_i).[15] It follows from these results that there is an open set of strictly positive weights (x_i) such that the corresponding GPO is strictly Pareto superior to the BAU. We are therefore led to search for (non-Markovian) Nash equilibria of the dynamic game that sustain a GPO, or at least are superior to the BAU.

[15] We conjecture that this inequality would hold in a variety of models. It certainly does in the concave model of Dutta and Radner (2009). Indeed, one can show in a quite general model that a GPO cannot be a BAU, or even that, starting from a GPO, each country will want to increase its emissions unilaterally by a small amount.

2.5. BAU Sanctions. In Dutta and Radner (2009) we further characterize equilibria in this game that are sustained by the threat of reverting to the BAU equilibrium. We report here, without proof, two of the main results. First, for all discount factors, the third-best solution qualitatively mirrors the BAU and GPO solutions; there is a constant emission level $\widetilde{a}_i$ that country i emits, independently of the stock of GHGs. Second, if discount factors are high enough, then, in fact, the GPO emission levels are themselves the third-best solution.

Let $x = (x_i)$ be a vector of positive numbers, one for each country. A *Third-Best Optimum (TBO)* corresponding to x is a profile of "norm" strategies that maximizes the weighted sum of country payoffs,

$$v = \sum_i x_i v_i, \tag{11}$$

subject to BAU reversion, i.e., subject to the constraint - detailed below - that should any country i not follow the norm, all countries would switch to BAU emissions vector $\overline{a} = (\overline{a}_1, \overline{a}_2, .. \overline{a}_I)$ forever from the following period on. As before, and without loss of generality, we may take the weights, x_i, to sum to I. The following result characterizes the TBO:

Proposition 1 *There exists a vector $\tilde{a}$ of constant emission levels $\tilde{a}_i$ such that on the equilibrium path country i's TBO strategy is to use a constant emission equal to $\tilde{a}_i$* in *all periods, where $\tilde{a}_i$ satisfies the incentive constraint:*

$$\text{for every } i, \quad \widetilde{a}_i^{\beta_i} - \delta w_i(\widetilde{a}_i + \delta \sum_{j \neq i} \widetilde{a}_j) \geq \overline{a}_i^{\beta_i} - \delta w_i \left[\overline{a}_i + \delta \sum_{j \neq i} \overline{a}_j \right].$$

It is immediate that the BAU emission policy is sustainable by the threat of BAU reversion - of course! - since the inequality is trivially satisfied when $\widetilde{a}_i = \overline{a}_i$. What is also not very difficult to show is that the GPO emission policy also becomes sustainable at a high enough δ. Formally, we have:

Proposition 2 *a) The welfare that is achievable under the threat of BAU emissions is at least as high as $\overline{u}$.*

b) Suppose that the GPO solution under equal country weighting, ($x_i = x_j$ for all i,j) Pareto-dominates the BAU solution for all high $\delta \geq \delta'$ Then, there is a cut-off discount factor $\widetilde{\delta} \in (\delta', 1$)such that, above it, the GPO emission policy is sustainable as an equibrium norm.

3. A Generalized Model with Exogenous Capital Accumulation

We now generalize the model of Section 2.1 to include the possibility that the capital stock in each country changes exogenously over time. For simplicity, we assume that each capital stock evolves geometrically, although models with other stock dynamics would also be tractable.

3.1. The Model. Let $K_i(t)$ denote the size of capital stock of country i at the beginning of period t, and let $K(t)$ be the vector with coordinates $K_i(t)$. The *state of the system at the beginning of period* t is now the pair $[g(t), K(t)]$.

Corresponding to Eq. 3 of Section 2.1, the utility of country i in period t is

$$v_i(t) = [a_i(t)]^{\beta_i} [K_i(t)]^{\gamma_i} - c_i g(t) \tag{12}$$

where the coefficients β_i and γ_i are positive fractions. [Of course one special situation is the CRS case $\beta_i + \gamma_i = 1$.] It is convenient, though not necessary, to think of the utility function, $[a_i(t)]^{\beta_i} [K_i(t)]^{\gamma_i}$, as the nation's GDP function. Note that emissions are an "input" into the GDP "production" function because there is a one to one link between emissions and energy usage in the economy - and energy usage is an actual input into the production function. In the Cobb-Douglas form assumed here, emissions/energy and capital stock are complementary inputs in that the marginal product of one input increases in the level of the other input. Again, the total payoff (utility) for country i is given by the sum of discounted one-period utilities, as in Eq. 4 of Section 2.

A Markov strategy for country i is a function that maps the current state, (g, K) into a current action, a_i. As in (2) of Section 2.1, the level of greenhouse gas evolves according to the linear difference equation

$$g(t+1) = A(t) + \sigma g(t), \tag{13}$$

where $A(t) = \sum_{i=1}^{I} a_i(t)$. We assume that the capital stock in country i evolves according to the geometric growth equation

$$K_i(t+1) = \theta_i K_i(t), \tag{14}$$

where the parameter θ_i satisfies $\theta_i > 1$. Thus the capital stock in country i becomes unboundedly large. To preserve boundedness of solutions, we shall require that discounted growth is bounded, i.e., that $\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}} < 1$ for all countries i. Note that in the CRS case - $\gamma_i = 1 - \beta_i$ - this condition reduces to the more familiar one that $\delta\theta_i < 1$, $\forall i$.

3.2. Generalized Business-as-Usual Equilibrium. Reversing the order followed in Sections 2.2 and 2.3, we first derive the analog of the Markov Perfect Equilibrium that was called there "business-as usual" (BAU); hereinafter, Generalized "business-as usual" equilibrium (GBAU).

Theorem 3. *(GBAU Equilibrium). Let g be the initial stock of greenhouse gas, and let K be the vector of initial capital stocks. For each i, let country i use the Markovian strategy $\overline{a}_i = \overline{\alpha}_i(K_i)$ determined by*

$$\beta_i \overline{a}_i^{\beta_i - 1} K_i^{\gamma_i} = \delta w_i, \tag{15}$$

where $w_i = \frac{c_i}{1-\delta\sigma}$. *Then this strategy profile is a MPE, and country* i*'s corresponding payoff is*

$$\overline{V}_i(g,K) = \overline{u}_i(K) - w_i g, \tag{16}$$

where the function $\overline{u}_i(K)$ *is separable in being the sum of two functions,* $\overline{u}_i(K) = \overline{u}_i^i(K_i) + \sum_{j\neq i}^I \overline{u}_i^j(K_j)$, *and, furthermore,* $\overline{u}_i^i(K_i) = \Phi_i^i K_i^{\frac{\gamma_i}{1-\beta_i}}$ *and* $\overline{u}_i^j(K_j) = \Phi_i^j K_j^{\frac{\gamma_j}{1-\beta_j}}$ *both of which are continuous in their arguments and solve the functional equations*

$$\begin{aligned} \overline{u}_i^i(K_i) &= \overline{a}_i^{\beta_i} K_i^{\gamma_i} + \delta[\overline{u}_i^i(K_i') - w_i \overline{a}_i(K_i)], \qquad (17)\\ \overline{u}_i^j(K_j) &= -\delta w_i \overline{a}_j(K_j) + \delta \overline{u}_i^j(K_j'), \qquad (18)\\ K_i' &\equiv \theta_i K_i. \end{aligned}$$

Proof: That the value associated with the strategies given by Eq. 15 is continuous and separable of the form given in Eq. 17 is established by way of a bootstrapping argument and the Bellman equation. Presuming that the value function is of that form, we write the Bellman equation as:

$$\overline{u}_i^i(K_i) + \sum_{j\neq i}^I \overline{u}_i^j(K_j) = \max_{a_i}\left[a_i^{\beta_i} K_i^{\gamma_i} - \delta w_i a_i\right] + \delta \overline{u}_i^i(K_i') + \delta \sum_{j\neq i}^I \left[-w_i \overline{a}_j(K_j) + \overline{u}_i^j(K_j')\right] \tag{19}$$

It is seen that the Bellman equation preserves both properties, continuity and separability. Substituting the maximizing emission values

$$a_i = \left[\frac{\beta_i K_i^{\gamma_i}}{\delta w_i}\right]^{\frac{1}{1-\beta_i}}, \; a_j = \left[\frac{\beta_j K_j^{\gamma_j}}{\delta w_j}\right]^{\frac{1}{1-\beta_j}}$$

and recognizing the separable nature of the equation we get that the above reduces to

$$\overline{u}_i^i(K_i) = f_i(\beta_i) K_i^{\frac{\gamma_i}{1-\beta_i}} + \delta \overline{u}_i^i(K_i')$$

where $f_i(\beta_i) = \left[\frac{\beta_i}{\delta w_i}\right]^{\frac{\beta_i}{1-\beta_i}} [1-\beta_i]$ and

$$\overline{u}_i^j(K_j) = \delta\left[-g_j(\beta_j) K_j^{\frac{\gamma_j}{1-\beta_j}} + \overline{u}_i^j(K_j')\right]; j \neq i$$

where $g_j(\beta_j) = w_i \left[\frac{\beta_j}{\delta w_j}\right]^{\frac{1}{1-\beta_j}}$. Writing

$$\Phi_i^i = \frac{f_i(\beta_i)}{1-\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}}}, \; \Phi_i^j = \frac{-\delta g_j(\beta_j)}{1-\delta\theta_j^{\frac{\gamma_j}{1-\beta_j}}}$$

it further follows that

$$\overline{u}_i^i(K_i) = \Phi_i^i K_i^{\frac{\gamma_i}{1-\beta_i}},$$
$$\overline{u}_i^j(K_j) = \Phi_i^j K_j^{\frac{\gamma_j}{1-\beta_j}},$$

Standard arguments then show that the space of continuous, power functions is a complete metric space. The Bellman equation is a contraction and hence it has a fixed point, i.e., the value function. Finally, the characterization of the GBAU emissions follows immediately from the maximization above. The theorem is proved.■

Remarks:

1. In the GBAU-equilibrium strategy of country i, the current emission depends only on the country's own current capital stock. Own value u_i^i is also affected only by own capital stock K_i.

2. For *any* profile of stationary strategies with property that a country's current action depends only on its own current capital stock, the value function of country i has the separable form given by Eq. 17, with w_i given above.

3. It should be clear that if the growth rates of capital stock are not equal, i.e., if $\theta_i \neq \theta_j$ then the country with the highest growth rate will eventually come to dominate in terms of utility. This happens both because its own utility $\overline{u}_i^i$ grows at the fastest rate and also because the disutility it imposes on others through its own emissions - $\overline{u}_j^i$, $j \neq i$ - grows at the fastest rate as well.

3.3. Generalized Global Pareto Optima. We define a global Pareto Optimum as in Section 2.2. The following theorem, which corresponds to Theorem 1, characterizes the generalized global Pareto optimum (GGPO) for a given set of welfare weights, (x_j). The proof is omitted, since the method is similar to that used in the previous theorem.

Theorem 4. *(GGPO) Given strictly positive welfare weights (x_i), let $\hat{V}(g,K)$ be the maximum attainable global welfare starting with an initial GHG stock equal to g and capital stocks K; then, after writing $w = \sum_i x_i w_i$,*

$$\hat{V}(g,K) = \widehat{u}(K) - wg, \tag{20}$$

where

$$\widehat{u}(K) = \sum_i x_i \widehat{u}_i(K_i) \tag{21}$$

and the $\widehat{u}_i$ are the solution of the functional equation

$$x_i \widehat{u}_i(K_i) = x_i \widehat{a}_i^{\beta_i} K_i^{\gamma_i} + \delta[x_i u_i(K_i') - w\widehat{\alpha}_i(K_i)].$$

Country i's GGPO emission $\widehat{\alpha}_i(K_i)$ is the stationary strategy determined by

$$x_i \beta_i \widehat{a}_i^{\beta_i - 1} K_i^{\gamma_i} = \delta w. \tag{22}$$

3.4. Comparison of BAU and GPO Emission Rates. Comparing the GBAU and GGPO strategies, we have:

$$GBAU \quad : \qquad \beta_i \overline{a}_i^{\beta_i - 1} K_i^{\gamma_i} = \delta w_i, \tag{23}$$

$$GGPO \quad : \qquad \beta_i \widehat{a}_i^{\beta_i - 1} K_i^{\gamma_i} = \delta \left(\frac{1}{x_i} \right) \sum_j x_j w_j. \tag{24}$$

Therefore, since $\left(\frac{1}{x_i}\right) \sum_j x_j w_j > w_i$, *for all* K, i, *and vectors* (x_i),

$$\overline{\alpha}_i(K_i) > \widehat{\alpha}_i(K_i), \tag{25}$$

i.e., the BAU emission rates will exceed the GPO emission rates.

Indeed, for future usage, it will be useful to note the exact relationship between the two emission levels:

$$\frac{\widehat{\alpha}_i(K_i)}{\overline{\alpha}_i(K_i)} = \left[\frac{w_i x_i}{w} \right]^{\frac{1}{1-\beta_i}}$$

Note in particular that the ratio of emission levels is actually *independent* of the size of capital stock even though each emission is a function of that variable. Put another way, the GGPO emission level $\widehat{\alpha}_i(K_i)$ is a constant fraction of the GBAU emission level $\overline{\alpha}_i(K_i)$ and the size of the fraction is independent of the capital stock. Put yet another way, the GGPO is achieved by a simple across the board cut in emissions from the GBAU level. All cuts of this form we will call **uniform cuts**.

Definition A uniform cut in emissions is achieved by a (capital-dependent) emission policy $\widetilde{a}_i(.)$ where, for all capital stock K_i, emissions are a constant fraction, say λ_i, of the GBAU emission level, i.e.,

$$\widetilde{a}_i(K_i) = \lambda_i \overline{\alpha}_i(K_i)$$

The reader will notice that the Kyoto agreement attempted to bring about just such a uniform cut in emissions. In the next two sections we shall investigate the sustainability of such uniform emission cuts.

3.5. Effects of Capital Stock on Emission Levels. As we saw in the previous subsection, there is a tragedy of the common with capital stocks, exactly as there was without. The question though is: does the presence of capital exacerbate the tragedy, possibly because capital and energy are complementary inputs in the production function? As we shall now see, the answer is that the tragedy does indeed get worse when we consider absolute levels of emissions but not when we consider percentages (or ratios of emission levels). Note that the GBAU as well as the GGPO emission level for country i only depends on its own capital stock.

Theorem 5. *(Capital Effect on Tragedy of the Common) i) Absolute Levels - Consider the absolute difference in emission levels* $\overline{a}_i - \widehat{a}_i$. *That increases at the rate* $K_i^{\frac{\gamma_i}{1-\beta_i}}$.

ii) Percentages - Consider the ratio difference in emission levels $\frac{\overline{a}_i}{\widehat{a}_i}$. *That is independent of the size of capital stock.*

The proof follows immediately from the characterizations provided in the previous subsections. Indeed, the second part was explicitly derived in the immediate prequel.■

4. Uniform Emission Cuts Under BAU Sanctions

In this subsection, we characterize the emission policies that are sustainable under the threat of BAU reversion in the model with capital. The answers are largely negative. We start by asking whether the GGPO policy can be so sustained. The answer, it will turn out, is in general *no*. The GGPO is an example of a broader class of emission policies that involve uniform cuts from GBAU emission levels. So the next question that we then ask is whether *any* uniform cut emissions policy is sustainable as part of an equilibrium norm. And the answer, again, is *no*.

The reason why the GGPO cannot be sustained as a SPE, by threatening to revert to the GBAU emission, is critically linked to the growth of capital. To understand the intuition, suppose for a moment that there are two countries and suppose, furthermore, that production is subject to CRS, i.e., that $\frac{\gamma_i}{1-\beta_i} = 1$. Finally, without loss of generality, let us suppose that the growth rate of capital is higher in country 1 i.e., that $\theta_1 > \theta_2$.

From the discussion in the previous section it follows that emissions in each country grows at rate K_i. Of course, under the GGPO emissions are lower; they are a (fixed) fraction of emissions under the GBAU. Imagine that an agrement is proposed in which the two countries are to cut their GBAU emissions to the fractions that the GGPO represents. The clear "loss" for country i in doing so is the loss every period t in own utility - $[a_i(t)]^{\beta_i}\,[K_i(t)]^{1-\beta_i} - \delta w_i a_i(t)$ - where by loss we mean the difference between own utility under the GBAU and that under the GGPO.[16] By definition, this loss is proportional to K_i since the emissions $a_i(t)$ are proportional to K_i. The "gain" though for country i is that the other country is also going to reduce its emissions and hence the damage inflicted by the other country - $-\delta w_i a_j(t)$ $-$ is lower if the GGPO agrement is adhered to. How much lower though and does it offset the

[16] That this is the utility consequence to country i from emission $a_i(t)$ is easily seen by noting that that level of emission causes first, an immediate "GDP" payoff $[a_i(t)]^{\beta_i}\,[K_i(t)]^{1-\beta_i}$ (where we have used the CRS simplification). However, next period there is $\sigma c_i a_i(t)$ of GHG damage, the period after that $\sigma^2 c_i a_i(t)$, two periods after that $\sigma^3 c_i a_i(t)$, all of which discounted back at rate δ yields a present discounted cost of $\delta w_i a_i(t)$.

loss in own utility? Well, the gain is - by similar logic as above - of order K_j. So country 1, in our two country example, gives up own utility which loss grows at rate θ_1 in return for a gain in damages as imposed by country 2's emissions. Yet that gain only grows at rate θ_2. It is clear that no matter what the initial conditions are, at some point the gain is simply not going to be big enough to offset the loss in own utility. Put another way, at that point in the future, the agreement will break down. Knowing that - or given the constraints of subgame perfection - such an agrement will never get written in period 0. In the proof of the theorem below, it will be seen that the logic generalizes when there are many countries and when there is not CRS in the production function.

Recall from the last section that the difference in greenhouse gas growth rates are proportional to $K_i^{\frac{\gamma_i}{1-\beta_i}}$ which is effectively proportional to $\theta_i^{\frac{\gamma_i}{1-\beta_i}}$. Based on that, let us call $\theta_i^{\frac{\gamma_i}{1-\beta_i}}$ the effective growth rate of capital stock. We shall say that there is a **unique maximal effective growth** rate if, without loss of generality

$$\theta_1^{\frac{\gamma_1}{1-\beta_1}} > \theta_i^{\frac{\gamma_i}{1-\beta_i}},\ \forall i \neq 1$$

In the sequel we shall prove two results. The first says that the GGPO emission level cannot be sustained by the threat of reverting to the GBAU emission. Then we go on to show the more general result that no emission policy that involves a uniform reduction from the GBAU is sustainable. (This is a more general result since - as we have seen in the previous section - the GGPO does in fact involve a uniform reduction in emissions from the GBAU.) Indeed that is the result we prove.

Theorem 6. *Suppose that there is a unique maximal effective growth rate. Then, no matter what the discount factor, and no matter what the initial levels of capital stock are, the GGPO cannot be sustained as part of a SPE by the threat of reverting to the GBAU. In particular for country* 1*, with the maximal effective growth rate, there will be a date, say* T_1*, such that it will deviate from the GGPO agrement in every period after* T_1*.*

Theorem 7. *Suppose that there is a unique maximal growth rate. Then, no matter what the discount factor, and no matter what the initial levels of capital stock are, no emission policy involving uniform cuts from the GBAU can be sustained as part of a SPE by the threat of reverting to the GBAU. In particular for country* 1*, with the maximal effective growth rate, there will be a date, say* T_1*, such that it will deviate from the GGPO agrement in every period after* T_1*.*

Proof of Theorem 7: Recall the GBAU emission policy $\overline{\alpha}_i(K_i) = \left(\frac{\beta_i}{\delta w_i}\right)^{\frac{1}{1-\beta_i}} K_i^{\frac{\gamma i}{1-\beta_i}}$. By extension, for an emission policy that involves a uniform cut in the GBAU emissions we have $\overrightarrow{a}_i(K_i) = \lambda_i\overline{\alpha}_i(K_i)$, where λ_i is any fraction. Consider the life-time payoff to any such emission policy for country i. By the decomposition given by Eq. 19, which we repeat here in slightly modified form for easy access, we have

$$\overrightarrow{u}_i^i(K_i) = \lambda_i \overline{a}_i^{\beta_i} K_i^{\gamma_i} - \delta w_i \lambda_i \overline{a}_i + \delta \overrightarrow{u}_i^i(K_i')$$

and

$$\sum_{j \neq i}^{I} \overrightarrow{u}_i^j(K_j) = \delta \sum_{j \neq i}^{I} \left[-w_i \lambda_j \overline{a}_j(K_j) + \overrightarrow{u}_i^j(K_j') \right]$$

The first equation above yields by simple substitution

$$\overrightarrow{u}_i^i(K_i) = \left[\frac{\beta_i}{\delta w_i} \right]^{\frac{\beta_i}{1-\beta_i}} \left[\lambda_i^{\beta_i} - \beta_i \lambda_i \right] K_i^{\frac{\gamma i}{1-\beta_i}} + \delta \overrightarrow{u}_i^i(K_i')$$

Note that the immediate own-payoff - the first term in the expression above - is maximized at the GBAU emission, i.e., is maximized when $\lambda_i = 1$. Substituting a conjectured solution $\overrightarrow{u}_i^i(K_i) = \overrightarrow{\Phi}_i^i K_i^{\frac{\gamma i}{1-\beta_i}}$ and using the fact that $K_i' = \theta_i K_i$ we can see right away that

$$\overrightarrow{u}_i^i(K_i) = \frac{\left[\frac{\beta_i}{\delta w_i} \right]^{\frac{\beta_i}{1-\beta_i}} \left[\lambda_i^{\beta_i} - \beta_i \lambda_i \right]}{1 - \delta \theta_i^{\frac{\gamma_i}{1-\beta_i}}} K_i^{\frac{\gamma i}{1-\beta_i}}$$

By similar logic

$$\overrightarrow{u}_i^j(K_j) = \frac{-\delta w_i \lambda_j}{1 - \delta \theta_j^{\frac{\gamma j}{1-\beta_j}}} \left(\frac{\beta_j}{\delta w_j} \right)^{\frac{1}{1-\beta_j}} K_j^{\frac{\gamma_j}{1-\beta_j}}$$

Again, it is clear that the greatest damage is inflicted in the GBAU case, i.e., when $\lambda_j = 1$.

Armed with the lifetime payoffs, we now turn to the sustainability of any uniform cut policy. We shall show that such a policy cannot be sustained by simply showing that for country 1, the one with the highest effective growth rate of capital, the lifetime payoff under GBAU must eventually exceed the lifetime payoff from the uniform cut policy. Say it exceeds by time $T+1$. In particular therefore, at time T, country 1 has no further incentive to continue with the cuts since - by construction - next period onwards the payoffs are higher by switching to the GBAU policy. That switch can be affected by deviating in the current period when own payoffs are in any case going to be higher from the deviation.[17] Using the results above, the difference between GBAU and uniform emissions payoffs is given by

$$A_1 K_1^{\frac{\gamma_1}{1-\beta_1}} - \delta \sum_{j \neq 1}^{I} B_{1j} K_j^{\frac{\gamma_j}{1-\beta_j}}$$

where

$$A_1 = \frac{\left[\frac{\beta_1}{\delta w_1}\right]^{\frac{\beta_1}{1-\beta_1}}}{1 - \delta \theta_1^{\frac{\gamma_1}{1-\beta_1}}} \left[1 - \lambda_1^{\beta_1} - \beta_1(1-\lambda_1)\right] > 0$$

and

$$B_{1j} = \frac{-\delta w_i}{1 - \delta \theta_j^{\frac{\gamma_j}{1-\beta_j}}} \left(\frac{\beta_j}{\delta w_j}\right)(\lambda_j - 1) > 0$$

Since $K_1^{\frac{\gamma_1}{1-\beta_1}}$ is arbitrarily bigger than $K_j^{\frac{\gamma_j}{1-\beta_j}}$ by some time period, say $T+1$, it follows that the expression must be strictly ositive from that period onwards. The theorem is proved.■

5. Foreign Aid

The main point of the previous section is that capital growth makes it difficult to sustain equilibria better than the GBAU. It certainly makes it impossible to sustain the most natural ones that involve a uniform cut in BAU emissions. The reason is straightforward enough as we saw above. Countries where capital accumulation is fastest have an ever increasing "potential loss" from agreeing to emission cuts - they would prefer the BAU emissions and lose by reducing emissions to, say, GPO levels. The fact that capital is complementary to emissions means that larger and larger amounts of capital amplify this loss in own-welfare. The only way then that such a country would agree to emissions reductions is if it is "made good" on this loss. One way the loss can be made good is by the threat of other countries raising their own emissions in the event that the fast-growing country does not cut its own emissions. That is the way in which a reversion to GBAU levels works. However, as we saw in the previous section, the threat is not strong enough since it is, in turn, tied to the rate of capital expansion in those countries. And if country 1 is the fastest growing country then the threat of being affected by the slower growth of country 2's GBAU emissions is simply not enough of a threat.

[17] As with all Nash equilibrium logic, other countries - whom country 1 is best responding to at time T - will be presumed to be carrying on with the cuts in that period. Hence the T period payoff consequence from the others' actions is identical for country 1 whether it deviates or not.

In this section we show that foreign aid, however, works. The starting point is that another way in which a fast growing country can be "made good" on the loss from not pursuing BAU emissions is that other countries might be willing to share the benefits that they get from this country's lower trajectory of emissions. Using the analogy above, country 2 - the slower growing one - benefits from country 1's reduced emissions and it will be seen that this benefit grows at the same rate as country 1's capital growth (since 1's emissions are indeed linked to its rate of capital expansion). Now if this benefit is transferred over in part to country 1 it might be willing to suffer the loss in its own welfare due to following GGPO emissions. So a "bribe" - aka foreign aid - might work where a threat does not.

Even if such a bribe works to induce the fast-growing country to limit its emissions, one may wonder whether the bribe will be given. Put differently, would the foreign aid donor, inclusive of aid, be better off relative to the GBAU? Put yet differently, can foreign aid be Pareto improving?

In this section we prove three results. First we show that there exists a policy of (zero-sum) foreign aid transfers such that the "bribe" of conditional foreign aid - transfers made if and only if the GGPO emissions policy is followed - sustains GGPO emissions as an equilibrium outcome (at a high enough discount factor). Second, inclusive of foreign aid, both recipients as well as donors are better off than under the GBAU. Third, there is a continuum of such emission policies all of which involve uniform emission cuts to the GBAU which can be sustained as equilibria. And, again inclusive of foreign aid, both recipients as well as donors are better off than under the GBAU. These results stand in sharp contrast to the results in the previous section which showed that the threat of sanctions is not effective.

5.1. Foreign Aid: A Definition. First a defiinition regarding foreign aid. We shall consider a "clearing-house" mechanism of providing foreign aid rather than bilateral aid between countries. (Though we believe that the results and the intuition can be carried forward to the bilateral case as well.) Imagine that there is an international aid agency, much like the World Bank, which makes a transfer Υ_i to country i. We will adopt the usual convention that $\Upsilon_i > 0$ implies that country i is a recipient of aid whilst $\Upsilon_i < 0$ implies that it is a donor.

Definition A *feasible foreign aid policy* (related to climate change) is a sequence of time and emission-dependent aid levels $\{\Upsilon_{it}\}$ with the requirement that in every period the transfers aggregate to zero, i.e.,

$$\sum_i \Upsilon_{it} = 0, \forall t$$

Furthermore, ii) the transfers are made to country i in period t only if the appropriate emissions are recorded for country i in that period.[18]

5.2. Sustainability of the GGPO Emission Policy under Foreign Aid. In this subsection we show that there is a feasible foreign aid policy such that the equally wighted GGPO emission level can be sustained as part of a SPE by sufficiently patient countries.[19] In the next subsection we will then show that indeed there is a continuum of such emission reduction policies that are also sustainable - though possibly at different discount factors.

Definition The **Aid Induced GGPO strategy** that we consider is the following:

Norm - Start at period 0, given capital stock K_{i0}, by following GGPO emission level $\widehat{\alpha}_i(K_{i0})$ and transferring Υ_{i0} upon observing it. Follow thereafter in every period t with GGPO emission level $\widehat{\alpha}_i(K_{it})$ and transferring Υ_{it} provided these emissions have been followed in the past.

Punishment - In the event that there has been a unilateral deviation in period t - country i did not emit at GGPO levels or withheld promised foreign aid - switch for all countries j to the GBAU emissions $\overline{a}_j(K_{jt+1)}$ from period $t+1$ onwards with no foreign aid from period t onwards.[20]

Recall that $\theta_i^{\frac{\gamma_i}{1-\beta_i}}$ is the effective growth rate of capital stock in country i (θ_i being the actual growth rate, γ_i the coefficient for capital in the production function and β_i the emissions coefficient).[21] Recall too that, without loss of generality, we have adopted the convention that this growth rate is highest in country $1, i.e.,$

$$\theta_1^{\frac{\gamma_1}{1-\beta_1}} \geqslant \theta_i^{\frac{\gamma_i}{1-\beta_i}},\ \forall i$$

Note that - unlike in the previous section - the above is a weak inequality, i.e., that country 1 need not have the uniquely maximum effective growth rate. Recall too that for the problem to be bounded we have imposed the restriction that $\delta\theta_1^{\frac{\gamma_1}{1-\beta_1}} < 1$. Call any such discount factor *feasible*.

[18]The closest institutional mechanism to climate change related foreign aid is the aid that is disbursed by the World Bank via its Climate Investment Fund (CIF). In the CIF though, there is no requirement that the transfers should aggregate to zero. Clearly having that as an additional requirement only makes our task of showing the beneficial effects of aid more difficult. Equally clearly, some kind of budget-balance, but possibly over a long horizon, will be required of any such policy. We choose to work with the most stringent budget balance policy.

[19]By the equally weighted GGPO emission level what we mean is that we consider the GGPO where each country is given equal weight. In terms of the notation of Section 3, the weight $x_i = \frac{1}{I}$, for all i.

[20]As always, given Nash equilibrium logic, one can ignore multiple simultaneous deviations.

[21]In the CRS case the effective and actual growth rates of capital coincide.

Theorem 8. *There is a cut-off discount factor* $\widehat{\delta} < \theta_1^{\frac{\gamma_1}{\beta_1 - 1}}$ *and a feasible foreign aid policy with the property that the Aid Induced GGPO strategy defined above is a SPE for all feasible discount factors above* $\widehat{\delta}$. *Furthermore, for every country* i, *donor as well as recipient, life-time payoffs inclusive of foreign aid strictly Pareto dominates the GBAU lifetime payoffs.*

Proof: Evidently the proposed aid induced GGPO strategy is an equilibrium if no country i has a profitable deviation against it at any time τ, i.e., if for all i and all τ it is the case that

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau} \left(\widehat{\alpha}_{it}^{\beta_i} K_{it}^{\gamma_i} - \delta w_i \widehat{\alpha}_{it} - \delta w_i \sum_{j \neq i} \widehat{\alpha}_{jt} + \Upsilon_{it} \right) \geqslant \max_{a_i} [a_i^{\beta_i} K_{i\tau}^{\gamma_i} - \delta w_i a_i] - \delta w_i \sum_{j \neq i} \widehat{\alpha}_{j\tau} + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau-1} \left(\overline{a}_{it}^{\beta_i} K_{it}^{\gamma_i} - \delta w_i \overline{a}_{it} - \delta w_i \sum_{j \neq i} \overline{a}_{jt} \right)$$

It is immediate that the best deviation, the solution to the maximimization above, is attained at the GBAU emission associated with capital stock $K_{i\tau}$, what we have denoted $\overline{a}_{i\tau}$. Substituting that - and doing a bit of re-arranging - we can show that the above is equivalent to the holding of the following *Individual Incentive Constraints* (IIC):

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau} \left(\widehat{\alpha}_{it}^{\beta_i} K_{it}^{\gamma_i} - \delta w_i \widehat{\alpha}_{it} - \delta w_i \sum_{j \neq i} \widehat{\alpha}_{jt} + \Upsilon_{it} \right) \geqslant \sum_{t=\tau}^{\infty} \delta^{t-\tau} \left(\overline{a}_{it}^{\beta_i} K_{it}^{\gamma_i} - \delta w_i \overline{a}_{it} - \delta w_i \sum_{j \neq i} \overline{a}_{jt} \right) + \delta w_i \sum_{j \neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}), \ for\ all\ i, \tau \tag{26}$$

The proof will rely on the following "Aggregation Lemma" which essentially says that we can replace the I incentive constraints, one for each country, with a single incentive constraint that sums up - across countries - both sides of the I individual constraints. The proof that this single incentive constraint is all that is required to be checked, is in the Appendix. The intuition for it is that the simplifying force of foreign aid is just this - if there is sufficient slack in the incentives of some countries then they can transfer some of that slack via foreign aid to those countries whose incentives are not being met. Is there enough slack to make those transfers, i.e., to make up the shortfall? Yes, if the total slack is more than the total shortfall.

Aggregation Lemma *The IIC above, as given by Eq. 26, hold if and only if the* *following Aggregate Incentive Constraints (AIC) hold*

$$\sum_{i=1}^{I}\sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}\right) \geqslant \sum_{i=1}^{I}\sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}\right) + \delta\sum_{i=1}^{I}w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau}),\ for\ all\ \tau \tag{27}$$

Proof: In the Appendix. ■

Continuing with the proof of the theorem, we shall now show that the AIC holds, i.e., that Eq. 27 holds (at every τ). To conserve on notation - and because the cases are qualitatively identical - we shall focus in the immediate sequel on the case $\tau = 0$, i.e., we will show that

$$\sum_{i=1}^{I}\sum_{t=0}^{\infty}\delta^{t}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}\right) \geqslant \sum_{i=1}^{I}\left[\begin{array}{c}\sum_{t=0}^{\infty}\delta^{t}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}\right)\\ +\delta w_i\sum_{j\neq i}(\overline{a}_{j0}-\widehat{\alpha}_{j0})\end{array}\right] \tag{28}$$

Interchanging the order of summation in Eq. 28 we get that the requirement is

$$\sum_{t=0}^{\infty}\sum_{i=1}^{I}\delta^{t}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}\right) \geqslant \begin{array}{c}\sum_{t=0}^{\infty}\sum_{i=1}^{I}\delta^{t}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}\right)\\ +\delta\sum_{i=1}^{I}w_i\sum_{j\neq i}(\overline{a}_{j0}-\widehat{\alpha}_{j0})\end{array} \tag{29}$$

Clearly Eq. 29 can be re-written as

$$\sum_{t=0}^{\infty}\delta^{t}\sum_{i=1}^{I}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta\widehat{\alpha}_{it}\sum_{j}w_j\right) \geqslant \sum_{t=0}^{\infty}\delta^{t}\sum_{i=1}^{I}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta\overline{a}_{it}\sum_{j}w_j\right)+\delta\sum_{i=1}^{I}w_i\sum_{j\neq i}(\overline{a}_{j0}-\widehat{\alpha}_{j0}) \tag{30}$$

which is, of course, equivalent to

$$\sum_{i=1}^{I}\sum_{t=0}^{\infty}\delta^{t}\frac{1}{I}\left[\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta\widehat{\alpha}_{it}\sum_{j}w_j\right)-\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta\overline{a}_{it}\sum_{j}w_j\right)\right] \geqslant \delta\frac{1}{I}\sum_{i=1}^{I}w_i\sum_{j\neq i}(\overline{a}_{j0}-\widehat{\alpha}_{j0}) \tag{31}$$

Term by term, for every i, the LHS of Eq. 31 is precisely the difference between the GGPO lifetime payoffs (when using the GGPO welfare function with equal weights for all countries) and the lifetime payoffs under the same welfare function but under GBAU emissions. From the construction of the GGPO each term, for each country, is strictly positive. We will now show that more is true. That in fact the LHS blows up to infinity as $\delta \uparrow \theta_1^{\frac{\gamma_1}{\beta_1-1}}$. To see this, note that from the characterization of the GBAU and GGPO emission levels in Section 3 it follows that the difference in country 1's payoffs is

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{I} \left[\left(\widehat{\alpha}_{1t}^{\beta_1} K_{1t}^{\gamma_1} - \delta \widehat{\alpha}_{1t} \sum_j w_j \right) - \left(\overline{a}_{1t}^{\beta_1} K_{1t}^{\gamma_1} - \delta \overline{a}_{1t} \sum_j w_j \right) \right] = \frac{(\widehat{\Phi}_1 - \overline{\Phi}_1) K_{10}^{\frac{\gamma_1}{1-\beta_1}}}{1 - \delta \theta_1^{\frac{\gamma_1}{1-\beta_1}}}$$

where $\widehat{\Phi}_1 - \overline{\Phi}_1 = \frac{1}{I} \left(\left[\frac{\beta_i}{\delta \sum_j w_j} \right]^{\frac{\beta_i}{1-\beta_i}} (1 - \beta_i) - \left[\frac{\beta_i}{\delta w_i} \right]^{\frac{\beta_i}{1-\beta_i}} (1 - \beta_i \frac{\sum_j w_j}{w_i} \right)$ which by construction is positive. Re-writing the incentive constraint we have the requirement that

$$\sum_{i=1}^{I} \frac{(\widehat{\Phi}_i - \overline{\Phi}_i) K_{i0}^{\frac{\gamma_i}{1-\beta_i}}}{1 - \delta \theta_i^{\frac{\gamma_i}{1-\beta_i}}} \geqslant \delta w \sum_{i=1}^{I} (\overline{a}_{i0} - \widehat{\alpha}_{i0})$$

where $w = \frac{1}{I} \sum_{i=1}^{I} w_i$. Note that $(\overline{a}_{i0} - \widehat{\alpha}_{i0})$ is also proportional to $K_{i0}^{\frac{\gamma_i}{1-\beta_i}}$, say is equal to $\lambda_i K_{i0}^{\frac{\gamma_i}{1-\beta_i}}$. Hence we need to show that

$$\sum_{i=1}^{I} \left[\frac{(\widehat{\Phi}_i - \overline{\Phi}_i)}{1 - \delta \theta_i^{\frac{\gamma_i}{1-\beta_i}}} - \lambda_i \right] K_{i0}^{\frac{\gamma_i}{1-\beta_i}} \geqslant 0$$

The coefficients on the LHS stay bounded away from zero even as $\delta \uparrow \theta_1^{\frac{\gamma_1}{\beta_1 - 1}}$. Naturally it follows that the LHS of the above inequality blows up and hence is strictly positive above a feasible cut-off discount factor.

To simplify notation we had taken the starting point of the deviation to be $\tau = 0$. What if the deviation happens at $\tau > 0$? It is easily seen the arguments repeat with no change other than notation. Given the observation that $K_{it}^{\frac{\gamma_i}{1-\beta_i}}$ is equal to $\theta_i^{\frac{\gamma_i t}{1-\beta_i}} K_{i0}$ means that the positive terms - such as the incentive slack for country 1 - only get disproportionately larger than th negative terms. Hence the inequality holds at every time period if it holds at period 0.

Evidently, there is no profitable deviation in which a country withholds aid after lowering its emissions. Since in that case it gets the GBAU emissions from the next period onwards and loses out on aid as well. If there is going to be a deviation it might as well be on emissions as well. Which we have shown to be unprofitable. Finally, the GBAU punishment regime, once started, does get carried out, i.e., the punishment is credible.

We have so far shown that the Aid Induced GGPO strategy is a subgame perfect equilibrium. To see that - inclusive of aid - it implies a Pareto improvement vis-a-vis the GBAU one need only look at Eq. 26. The theorem is proved.■

5.3. Sustainability of Other Emission Reduction Policies. In this subsection we examine the sustainability of other emission reduction policies. In particular, we will consider any policy that involves uniform reductions from the GGPO but is at least as high an emission level as the equally weighted GGPO. Modifying the definition given above we reproduce it here for easy access:

Definition A uniform cut in emissions is achieved by a (capital-dependent) emission policy $\widetilde{a}_i(.)$ where, for all capital stock K_i, emissions are a convex combination, with weight say λ_i, of the GBAU and equally weighted GGPO emission level, i.e.,

$$\widetilde{a}_i(K_i;\lambda_i) = \lambda_i \overline{\alpha}_i(K_i) + (1-\lambda_i)\widehat{a}_i(K_i)$$

We shall consider - as in the previous subsection - an aid induced emissions policy with the obvious difference that the Norm emission policy will be given by $\widetilde{a}_i(K_i;\lambda_i)$ rather than the GGPO emissions. The punishment - as above - will be the witholding of aid coupled with a reversal to the GBAU emissions.

Theorem 9. *There is a cut-off discount factor $\delta(\lambda) < \theta_1^{\frac{\gamma_1}{\beta_1 - 1}}$ and a feasible foreign aid policy with the property that the Aid Induced emission reduction strategy $\widetilde{a}_i(.;\lambda_i)$ is a SPE for all feasible discount factors above $\delta(\lambda)$. Furthermore, for every country i, donor as well as recipient, life-time payoffs inclusive of foreign aid strictly Pareto dominates the GBAU lifetime payoffs.*

Proof: The proof is identical to that for the proof of the immediately preceding theorem, with the obvious changes of notation. Note first that Eq. 26 is the IIC with the norm emission policy being $\widetilde{a}_i(.)$ rather than the GGPO emission policy considered above. The Aggregation Lemma applies without any change because it clearly made no use of the specific emission policy. Hence, after making the same substitutions and interchanges as we made in the previous proof we get

$$\sum_{i=1}^{I}\sum_{t=0}^{\infty}\delta^t \frac{1}{I}\left[\left(\widetilde{a}_{it}^{\beta_i}K_{it}^{\gamma_i} - \delta\widetilde{a}_{it}\sum_j w_j\right) - \left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i} - \delta\overline{a}_{it}\sum_j w_j\right)\right] \geqslant \delta\frac{1}{I}\sum_{i=1}^{I} w_i \sum_{j\neq i}(\overline{a}_{j0} - \widetilde{\alpha}_{j0}) \tag{32}$$

The function $a_{it}^{\beta_i}K_{it}^{\gamma_i} - \delta a_{it}\sum_j w_j$ is strictly concave and at every K_{it} it reaches a maximum at $\widehat{a}_{it}$. Hence it follows that the bracketed terms are all strictly positive. More is true since $\widetilde{a}_i(K_i;\lambda_i)$ is proportional to the effective capital - $K_{i0}^{\frac{\gamma_i}{1-\beta_i}}$ since the component emissions - the GGPO and GBAU emissions - are. Using that fact and aggregating payoffs in the same way that we did above we get that the AIC above holds iff

$$\sum_{i=1}^{I}\left[\frac{(\widetilde{\Phi}_i - \overline{\Phi}_i)}{1-\delta\theta_i^{\frac{\gamma_i}{1-\beta_i}}} - \lambda_i\right] K_{i0}^{\frac{\gamma_i}{1-\beta_i}} \geqslant 0$$

where $\widetilde{\Phi}_i - \overline{\Phi}_i > 0$. By identical logic to that above, the terms above on the LHS - especially that involving country 1's payoffs - blows up as $\delta \uparrow \theta_1^{\frac{\gamma_1}{\beta_1 - 1}}$. Naturally it follows that the LHS of the above inequality blows up and hence is strictly positive above a feasible cut-off discount factor $\delta(\lambda)$. Finally, identical logic as in the GGPO case shows that if the above AIC holds at time period 0 then it also holds at every other time period. That the aid induced emission reduction involves a Pareto-improvement over the GBAU follows immediately from Eq. 32. The theorem is proved.■

Remark - Given that the GGPO is the maximum value of the function $a_{it}^{\beta_i} K_{it}^{\gamma_i} - \delta a_{it} \sum_j w_j$ it follows that the LHS of Eq. 32 achieves its maximum at the GGPO emission norm rather than at the emission norm $\widetilde{a}$. This might suggest that the GGPO is easiest to sustain as a norm. However, that might not be true because the payoffs on the RHS of Eq. 32 are also proportional to the size of the emission cuts and hence are highest at the GGPO emission. If that effect is stronger the cut-off discount factor $\delta(\lambda)$ might be lower for $\lambda > 0$, i.e., when we sustain a norm that has a higher level of emission than the GGPO.

6. Discussion and Conclusion

To the best of our knowledge we are the first to investigate, within a fully formed model, the possibility of getting China and India to sign a climate treaty. As has been widely reported in the press, these fast growing economies are reluctant to sign onto emission caps fearing that it will compromise growth. They have also claimed that they do not have the resources to make the technological switches that are required and have pointed to the fact that the problem is not of their making. In response, Western economies have discussed various "punishment" options that range from the possibility of trade-related sanctions[22] to escalating targets on emission cuts if the first targets are not met.[23] In this paper we investigate the effectiveness of retaliatory emissions - if reductions are not made, then all countries are free to increase their emissions to BAU levels. We show that such a sanction is ineffective - the fast growing countries always have an incentive to cheat; their loss from reductions grows as quickly as their own rate of accumulation while the loss from the sanctions grow at the slower rate of others' accumulation.

We then examine a mechanism similar to the World Bank's CIF - contingent

[22]The United States House of Representatives passed a bill in June, 2009 that would place tariffs on countries that do not adjust their carbon emissions. See http://www.nytimes.com/2009/06/29/us/politics/29climate.html.

[23]Subsequent to Kyoto, at the Hague November 2000 meeting, the most popular proposal (which came from the Dutch Environment Minister Jan Pronk) was that countries would face an escalating series of target reductions in the future if they failed to comply in the current stage. A watered-down version of this proposal was adopted in Bonn in March, 2001

foreign aid. And we show that this is effective at getting fast (and slow) growing economies to curtail the growth of emissions. Furthermore, even inclusive of the aid given, the outcome is Pareto-superior to the BAU equilibrium.

A key simplification of our model is (1) "power functions" - the one-period payoff for each country is a Cobb-Douglas function whilst capital grows geometrically, and (2) "cost linearity" - the (incremental) damage cost is linear in the current stock of greenhouse gas. These properties of the model allows us to get closed-form solutions for the Business as Usual and Pareto optimal solutions, characterize the equilibrium subject to BAU reversion, and investigate aid-contingent equilibria. It also facilitates the possible calibration of the model, the numerical calculation of various trajectories, and sensitivity analyses. The disadvantage is that it results in a number of cases in unrealistic "unbounded" strategies, that is, strategies in which the emission rates grow infinitely large along with capital stock. In particular, a country's cost of damage due to climate change, and/or the amount of foreign aid it has promised could become unrealistically large. This aspect of the results needs to be taken "with a grain of salt." In a more realistic model, one would expect that these strategies would display a more gradual adaptation to capital growth, and capital growth might even be bounded in the long run. Our conjecture is that the analysis of the affine model yields reasonable approximations to equilibrium and optimal trajectories in the medium term. However, precise tests of this conjecture will have to await future research.

In Dutta and Radner (2006) we generalized our bench-mark model to allow for population change and in Dutta and Radner (2004) we allowed for simple technological change and presented some theoretical and numerical results on the GPO and BAU solutions. In Dutta and Radner (2007a) we incorporate technical change in a more meaningful way. Eventually, we hope to develop and analyze a "complete" model that incorporates all of the above features.

The literature on (symmetric) dynamic commons games is exceedingly rich and goes back over twenty-five years. The earliest model was that of Levhari and Mirman (1980) who studied a particular functional representation of the neo-classical growth model with the novel twist that the capital stock could be "expropriated" by multiple players. Subsequently several authors (Sundaram 1989, Sobel 1990, Benhabib and Radner 1992, Rustichini 1992, Dutta and Sundaram 1992 and 1993, Sorger 1998) studied this model in great generality and established several interesting properties relating to existence of equilibria, welfare consequences, and dynamic paths. Another variant of that model has been studied by Tornell and Velasco (1992) and, subsequently, Long and Sorger (2006).[24]

[24] Some of these papers allow asymmetry; however, none of them analyzes the effect of asymmetries. One significant exception is the recent paper of Long and Sorger that explicitly considers asymmetry in appropriation costs within the Tornell and Velasco model.

More recently in a series of papers by Dockner and his co-authors, the growth model has been directly applied to environmental problems including the problem of global warming. The paper closest to the current one is Dockner et al. (1996). It studies a model of global warming that has some broad similarities to the one we have studied here. In particular, the transition equation is identical in the two models. What is different is that they impose linearity in the emissions payoff function (whereas we have assumed it to be Cobb-Douglas and hence strictly concave) while their cost to g is strictly convex (as opposed to ours which is linear).

A large volume of literature exists that directly focuses on the economics of climate change. A central question there is to determine the level of emissions that is globally optimal. An excellent example of this is Nordhaus and Boyer (2000).[25] Several of those papers, including the Nordhaus and Boyer paper, analyze only the "competitive" model, not taking strategic considerations fully into account.[26] A smaller volume of literature emphasizes the need for treaties to be self-enforcing, presenting a strategic analysis of the problem. (See Barrett 2003 and Finus 2001). Where we depart from that literature is in the dynamic modelling; we allow greenhouse gases to accumulate and stay in the environment for a (possibly long) period of time. By contrast the Barrett and Finus studies restrict themselves to purely repeated games, which implies that the state variable, gas stock, remains constant over time.

What this paper does not address is a set of complementary issues regarding the economics of climate change and many of them have been addressed by other papers in this volume. These issues include whether taxes or quotas are the best instruments to achieve abatement (Karp and Zhang, this issue), whether lower level "polycentric" bodies can substitute for treaty formation at national level (Ostrom, this issue), whether the BAU solution can be Pareto-improved across generations by appropriate mitigation investment by existing generations (Rezai, Foley and Taylor, this issue), and whether climate effects are mitigated if agents have preferences that value the long-run future (Asheim, Mitra and Tungodden; Figuieres and Tidball as well as Chichilnisky, this issue). Indeed this article is part of a Special Issue of Economic Theory on the topic of the Global Environment, which includes also the following articles: “Unspoken Ethical Issues in the Climate Affair Insights From a Theoretical Analysis of Negotiation Mandates” by Lecocq and Hourcade, “Intergenerational equity, efficiency, and constructability”, by Lauwers, “Carbon Leakages: A General Equilibrium View” by Burniaux and Martins, and “Detrimental Externalities, Pollution Rights, and the “Coase Theorem”” by Chipman and Tian.

[25] But also see Chichilnisky (2006).

[26] To be fair, Nordhaus and Boyer (2000) and Nordhaus and Zhang 1996 do consider strategic models but restrict themselves to open-loop strategies.

7. APPENDIX

Aggregation Lemma *The IIC above, as given by Eq. 26 in Section 5, hold if and only if the following Aggregate Incentive Constraints (AIC) holds*

$$\sum_{i=1}^{I}\sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}\right) \geqslant \sum_{i=1}^{I}\sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}\right) + \delta\sum_{i=1}^{I}w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau}),\ for\ all\ \tau \tag{33}$$

Proof: We are required to show that Eq. 33 above implies the existence of a feasible foreign aid policy $(\Upsilon_t)_{t\geqslant 0}$ such that the Individual Incentive Constraints (IIC) hold for every country, i.e., that

$$\sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}+\Upsilon_{it}\right) \geqslant \sum_{t=\tau}^{\infty}\delta^{t-\tau}\left(\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}\right) + \delta w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau}),\ for\ all\ i,\tau. \tag{34}$$

To simplify the notation let $\widehat{\alpha}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\widehat{\alpha}_{it}-\delta w_i\sum_{j\neq i}\widehat{\alpha}_{jt}$ be denoted $\widehat{u}_{it}$ and likewise let $\overline{a}_{it}^{\beta_i}K_{it}^{\gamma_i}-\delta w_i\overline{a}_{it}-\delta w_i\sum_{j\neq i}\overline{a}_{jt}$ be denoted $\overline{u}_{it}$. Fix any time-period τ and separate the group of countries into two exclusive groups where Group 1 is defined as all countries such that

$$\sum_{t=\tau}^{\infty}\delta^{t-\tau}(\widehat{u}_{it}-\overline{u}_{it})-\delta w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau})\geqslant 0$$

and Group 2 is made up of countries for which the inequality is reversed. For Group 2, where the IIC does not hold in the absence of foreign aid, define the life-time foreign aid receipts $\Gamma_{i\tau}$ by

$$\Gamma_{i\tau}=\left|\sum_{t=\tau}^{\infty}\delta^{t-\tau}(\widehat{u}_{it}-\overline{u}_{it})-\delta w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau})\right|$$

Let the parameter μ_τ be defined by the following equality which ensures that the total of foreign aid grants is equal to the total of foreign aid receipts:

$$\sum_{i\in Group\ 2}\Gamma_{i\tau}=\mu_\tau\sum_{i\in Group\ 1}\left[\sum_{t=\tau}^{\infty}\delta^{t-\tau}(\widehat{u}_{it}-\overline{u}_{it})-\delta w_i\sum_{j\neq i}(\overline{a}_{j\tau}-\widehat{\alpha}_{j\tau})\right] \tag{35}$$

Note that since the AIC holds at time τ, the parameter $\mu_\tau \leqslant 1$. For Group 1, countries where the IIC does hold in the absence of foreign aid, define the life-time of foreign aid donations $\Gamma_{i\tau}$ by[27]

$$\Gamma_{i\tau} = -\mu_\tau \left[\sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) \right]$$

An implication of Eq. 35 is that the total life-time foreign aid $\sum_i \Gamma_{i\tau} = \sum_{i\in Group\ 1} \Gamma_{i\tau} + \sum_{i\in Group\ 2} \Gamma_{i\tau} = 0$. Finally, the lifetime aid amounts are decomposed into period by period aid amounts through the following decomposition. For every i and for every τ

$$\Upsilon_{i\tau} = \Gamma_{i\tau} - \delta\Gamma_{i\tau+1}$$

It immediately follows that $\sum_i \Upsilon_{i\tau} = 0$ given that $\sum_i \Gamma_{i\tau} = 0$ and $\sum_i \Gamma_{i\tau+1} = 0$. So the foreign aid that is proposed aggregates to zero in every period as required. To see that Eq. 33 holds, note that for Group 1, the countries that starting at period τ are a net donor of foreign aid

$$\begin{aligned} & \sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) + \sum_{t=\tau}^{\infty} \delta^{t-\tau}\Upsilon_{it} \\ = & \sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) + \Gamma_{i\tau} \\ = & (1-\mu_\tau) \left[\sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) \right] \geqslant 0 \end{aligned}$$

since $1 - \mu_\tau \geqslant 0$. For Group 2, the countries that starting at period τ are a net recipient of foreign aid,

$$\begin{aligned} & \sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) + \sum_{t=\tau}^{\infty} \delta^{t-\tau}\Upsilon_{it} \\ = & \sum_{t=\tau}^{\infty} \delta^{t-\tau}(\widehat{u}_{it} - \overline{u}_{it}) - \delta w_i \sum_{j\neq i} (\overline{a}_{j\tau} - \widehat{\alpha}_{j\tau}) + \Gamma_{i\tau} = 0 \end{aligned}$$

Clearly the argument repeats at every time period τ. In other words the IIC holds (for all countries and all time-periods). Put differently, the lemma is proved.■

[27] Recall the convention is that donations are negative while receipts are postive numbers.

8. References

Asheim, G., T. Mitra, B. Tungodden, 2010, Sustainable Recursive Social Welfare Functions, Economic Theory, this issue.

Barrett, S., 2003. Environment and Statecraft: The Strategy of Environmental Treaty-Making. Oxford: Oxford University Press.

Benhabib, J., Radner, R., 1992. Joint exploitation of a productive asset. Economic Theory 2, 155-190.

Burniaux, J-M., and Martins, J., 2010, Carbon Leakages: A General Equilibrium View, Economic Theory, this issue.

Chichilnisky, C., 2006. Global Property Rights: The Kyoto Protocol and the Knowledge Revolution, Discussion Paper, Institut du Development Durable et Relations Internationales (IDDRI), Ecole Polytechnique, Paris.

Chichilnisky, C., 2010. Sustainable Markets with Short Sales, Economic Theory, this issue.

Chipman, J., and Tian, G., 2010, Detrimental Externalities, Pollution Rights, and the "Coase Theorem", Economic Theory, this issue.

Cline, W., 1992. The Economics of Global Warming. Washington, DC: Institute for International Economics.

Dockner, E., Long, N, Sorger, G., 1996. Analysis of Nash equilibria in a class of capital accumulation games. Journal of Economic Dynamics and Control 20, 1209-1235.

Dockner, E., Nishimura, K., 1999. Transboundary pollution in a dynamic game model. Japanese Economic Review 50, 443-456.

Dutta, P., Radner, R., 2004. Self-Enforcing climate change treaties, Proceedings of the National Academy of Sciences, USA 101-14, 5174-5179.

Dutta, P., Radner, R., 2006. Population growth and technological change in a global warming model. Economic Theory 29, 251-270.

Dutta, P., Radner, R., 2007a. Choosing cleaner technologies: Global warming and technological change, (in preparation).

Dutta, P., Radner, R., 2009. A strategic model of global warming: Theory and some numbers, Journal of Economic Behavior and Organization.

Dutta, P., Sundaram, R, 1992. Markovian equilibrium in a class of stochastic games: Existence theorems for discounted and undiscounted models. Economic Theory 2, 197-214.

Dutta, P., Sundaram, R., 1993. How different can strategic models be? Journal of Economic Theory 60, 42-61.

Figuieres, C., M. Tidball, 2010, Sustainable Exploitation Of A Natural Resource: A Satisfying Use Of Chichilnisky's Criterion, Economic Theory, this issue.

Finus, M, 2001. Game Theory and International Environmental Cooperation.

Cheltenham: Edward Elgar.

Inter-Governmental Panel on Climate Change, 2007. Climate Change, The Synthesis Report. Geneva, Switzerland: IPCC. Available at http://www.ipcc.ch/ipccreports/assessments-reports.htm. Accessed, December 5, 2007.

Karp, L., Zhang, Q, 2010, Taxes Versus Quantities for a Stock Pollutant with Endogenous Abatement Costs and Asymmetric Information, Economic Theory, this issue.

Lauwers, L., 2010, Intergenerational Equity, Efficiency, and Constructability, Economic Theory, this issue.

Lecocq, F., and Hourcade, J-C., 2010, Unspoken Ethical Issues in the Climate Affair Insights From a Theoretical Analysis of Negotiation Mandates, Economic Theory, this issue.

Levhari, D., Mirman, L., 1980. The great fish war: An example using a dynamic Cournot-Nash solution. Bell Journal of Economics 11, 322-334.

Long, N., Sorger, G., 2006. Insecure property rights and growth: The role of appropriation costs, wealth effects and heterogenity. Economic Theory 28, 513-529.

Nordhaus, W., Yang, Z., 1996. A regional dynamic general equilibrium model of alternative climate-change strategies. American Economic Review 86, 741-765.

Nordhaus, W., Boyer, J., 2000. Warming the World: Economic Models of Global Warming, Cambridge, MA: MIT Press.

Ostrom, E., 2010, Nested externalities and polycentric institutions: must we wait for global solutions to climate change before taking actions at other scales?, Economic Theory, this issue.

Rezai, S., D. Foley, L. Taylor, 2010, Global Warming and Economic Externalities, Economic Theory, this issue.

Rustichini, A., 1992. Second-best equilibria for games of joint exploitation of a productive asset. Economic Theory 2, 191-196.

Sorger, G., 1998. Markov-Perfect Nash equilibria in a class of resource games. Economic Theory 11, 79-100.

Stern, N., 2006. Review on the Economics of Climate Change. London, UK; HM Treasury, http://www.sternreview.org.uk. Accessed, December 5, 2007.

Sundaram, R., 1989. Perfect equilibrium in a class of symmetric dynamic games. Journal of Economic Theory 47, 153-177.

Thomson, David J., "Dependence of Global Temperatures on Atmospheric CO_2 and Solar Irradiance," Proc. Nat. Acad. Sci. USA, 94 (1997), 8370-77.

Tornell, A., and Velasco, A., 1992. The tragedy of the commons and economic growth: Why does capital flow from poor to rich countries? Journal of Political Economy 100, 1208-1231.