A Three-Player Dynamic Majoritarian Bargaining Game[1]

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Running Title

Policy Outcomes

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ABSTRACT

We analyze an infinitely repeated divide-the-dollar bargaining game with an endogenous reversion point. In each period a new dollar is divided among three legislators according to the proposal of a randomly recognized member – if a majority prefer so – or according to previous period's allocation otherwise. Although current existence theorems for Markovian equilibria do not apply for this dynamic game, we fully characterize a Markov equilibrium. The equilibrium is such that irrespective of the discount factor or the initial division of the dollar, the proposer eventually extracts the whole dollar in all periods. We also show that proposal strategies are weakly continuous in the status quo, while the correspondence of voters' acceptance set (the set of allocations weakly preferred over the status quo) fails lower hemicontinuity.

Keywords: Endogenous Reversion Point, Legislative Bargaining, Markov Perfect Nash Equilibrium, Stage Undominated Voting strategies, Uncovered Set.

JEL Classification Numbers: C72, C73, C78, D72, D78, D79.

1. INTRODUCTION

Sequential non-cooperative models of legislative bargaining in the tradition of Romer and Rosenthal [24], Rubinstein [26], Baron and Ferejohn [4], and Banks and Duggan [1], [2], have significantly deepened our understanding of the politics of legislative decision making. Yet, this literature invariably assumes that legislative interaction ceases after a decision is reached. Consequently, the applicability of these models is limited to situations when the policy domain expires after a single decision or to policy areas where legislation cannot be modified after its initial introduction.

This is unfortunate since most legislatures have constitutional authority to legislate anew in the bulk of policy jurisdictions. In such policy domains, legislation remains in effect after its promulgation only until or unless the legislature passes a new law. Thus it appears natural to study dynamic bargaining games where (a) policy decisions can be reached in any period ,and (b) in the absence of agreement among the bargaining parties in any given period the status quo prevails.

Two reasons account for the paucity of contributions in this area despite the relevance of the above assumptions. The obvious difficulty has to do with the complexity of these games. The endogeneity of the reversion point implies that legislative decisions in the present have an impact on both the immediate payoff as well as the future stream of benefits to players. Hence, the strategic calculations involved render the characterization of equilibrium points particularly challenging. Preceding this obstacle is a more fundamental difficulty. Assuming a continuous policy space, these games belong in a class of stochastic games for which existence of Markovian equilibria is not guaranteed[2].

Previously, Markov equilibria for infinite-horizon dynamic bargaining games with endogenous status quo have only been analyzed in Baron [3]. Baron studies the same institutions as in the present analysis and assumes a one-dimensional policy space with legislators that have single-peaked stage utilities. He characterizes an equilibrium where outcomes converge to the median's ideal point from arbitrary initial decision. Thus, Baron provides a dynamic median voter theorem (Black [8]) in an environment where the core is non-empty, defined on the basis of the stage preferences of legislators.

In more than one dimensions, though, the core of the majority rule game is generically empty (Plott [22], Schofield [27], [28], Rubinstein [25]) and there are no obvious candidates for

[2] Harris, Reny, and Robson [16], provide an example of a dynamic game where subgame perfect (and hence Markov) equilbria do not exist. See also Chakrabarti [9] for a recent review of the literature on stochastic games.

the support of a steady-state distribution of the corresponding dynamic game (assuming a steady state distribution exists). Furthermore, in the absence of a core the majority preference relation induces a cycle that encompasses the entire space of alternatives (McKelvey [18], [19]), so that more interesting dynamics than those induced in one-dimension become possible. A flavor of such complex dynamics is obtained by Baron and Herron [5], who analyze a game with three players that have Euclidian preferences over a two-dimensional space of legislative outcomes. The game proves analytically intractable and Baron and Herron provide numerical calculations of Markovian equilibria for a finite period\discrete policy space version of the game.

Like Baron and Herron [5], we analyze a three-player game. In each of an infinity of periods one of the three legislators is randomly recognized to make a proposal for the allocation of a fixed renewable resource (a dollar) among the members of the legislature. The proposed allocation is implemented if it receives a majority; otherwise, the resource is allocated as it was last period. Legislators' utility is the discounted sum of per period payoffs. Although coarse, this model combines a multidimensional policy space with the advantage of (relative) analytical tractability. Thus, our analysis serves as a first step in understanding an important family of dynamic legislative bargaining models in multi-dimensional policy spaces.

Indeed, we are able to establish existence and fully characterize a Markov equilibrium such that players condition their behavior only on the status quo. In fact, we refine the equilibrium concept and restrict players to use stage-undominated voting strategies (Baron and Kalai [6]). The equilibrium is such that, irrespective of the initial allocation of the dollar or the discount factor, policy outcomes are absorbed with probability one in a set that consists of divisions that allocate the whole dollar to the proposer.

Equilibrum dynamics can be motivated through the nature of winning coalitions that form along the equilibrium path. In the spirit of Riker [23], coalitions are minimum winning in that at most a bare majority of members receive a positive fraction of the dollar. Less equitable allocations in the current period (such that excluded minorities receive zero share of the dollar) reduce the cost of building a coalition in subsequent periods. Due to this externality, players are willing to accept proposals that exclude other members of the legislature and are even willing to accept proposals that reduce their amount compared to their allocation under the status quo. Thus, convergence to the equilibrium absorbing set of policy outcomes is fast, with a maximum expected time before absorption equal to two and a half (2.5) periods.

The equilibrium we characterize is not unique, although all additional equilibria we can find are payoff-equivalent. We show that players' equilibrium expected utility is continuous with respect to current period's decision. We also show that equilibrium expected utility fails quasi-concavity and has flat areas inducing thick indifference sets. Thus, we cannot rely on the lower-hemicontinuity of the voters' acceptance sets in arbitrary dynamic bargaining games of this type, even if the underlying stage utilities are continuous and concave.

Despite the non-continuity of the proposer's set of feasible (i.e. majority preferred) proposals, our equilibrium has the feature that (mixed) proposal strategies are weakly continuous with respect to the status quo[3]. In effect, we obtain equilibrium and continuity of proposal strategies because the pathological (flat) areas of voters' expected utility involve (weakly) suboptimal allocations. As a result these allocations are never proposed and constitute transient states in the equilibrium-induced Markov process of policy outcomes. This may not be true in more general policy spaces, making it harder to establish equilibrium if one exists.

The fact that the proposer obtains the whole dollar in each period after absorption is in contrast to the convergence to the median result of Baron [3] and the calculations of Baron and Herron [5] who find that equilibrium legislative decisions become more centrally located with higher discount factor and a longer time horizon. In the same spirit, we remark that while the uncovered set (Fishburn [15], Miller [20]) defined on the basis of stage preferences has full measure in the set of possible allocations, the equilibrium absorbing set of outcomes consists entirely of covered alternatives (Epstein [12], Penn [21]). The above suggest that the properties of the distribution of equilibrium policies in these games may depend significantly on the policy area under consideration.

Among other related contributions, Epple and Riordan [11] study subgame perfect equilibria of a three player game similar to ours where players alternate making proposals. They show that at least two subgame perfect equilibria exist for high enough discount factor. These equilibria involve two radically different allocations of the dollar, suggesting a folk theorem may apply for the set of subgame perfect equilibria in their game. Also related is the work of Ferejohn, McKelvey, and Packel [14] who consider the existence and other properties of a steady state distribution of policy outcomes in a game where proposals arise randomly from the space of alternatives and voters are impatient or myopic. Finally, Dixt, Grossman, and Gul [10] study the dynamics of compromise under efficient subgame perfect equilibria among two parties that alternate in power according to a policy-dependent stochastic rule.

We now proceed to a detailed presentation of the legislative setup. We outline equilibrium

[3]Continuity is with respect to the topology of weak convergence.

analysis in section 3 and state the main result in section 4. We conclude in section 5.

2. LEGISLATIVE SETUP & EQUILIBRIUM NOTION

The problem involves a set $N = \{1, 2, 3\}$ of three legislators that choose a legislative outcome $\mathbf{x}^t$ for each $t = 1, 2, ...$ Possible decisions within each period are divisions of a dollar among the three legislators, *i.e.* $\mathbf{x}^t$ is a triple $\mathbf{x}^t = \left(x_1^t, x_2^t, x_3^t\right)$ with $x_i^t \geq 0$ for $i = 1, 2, 3$ and $\sum_{i=1}^{3} x_i^t = 1$. Thus, the legislative outcome $\mathbf{x}^t$ is an element of the unit simplex in $\Re^3$ which we denote by Δ. Legislative interaction is as follows: at the beginning of each period legislator $i = 1, 2, 3$ is recognized with probability $\frac{1}{3}$ to make a proposal $\mathbf{z} \in \Delta$. Having observed the proposal legislators vote *yes* or *no*. If a majority of two or more vote *yes* then $\mathbf{x}^t = \mathbf{z}$; otherwise $\mathbf{x}^t = \mathbf{x}^{t-1}$ [4]. Thus, previous period's decision, $\mathbf{x}^{t-1}$, serves as the *status quo* or *reversion point* in the current period, t.

Legislators derive vNM stage utility $u_i : \Delta \longrightarrow \Re$, $i \in N$, from the implemented proposal $\mathbf{x}^t$. In particular, we assume that legislators are risk-neutral and care only about the share of the dollar they receive, i.e. $u_i\left(\mathbf{x}^t\right) = x_i^t$. Legislators discount the future with a common discount factor $\delta \in [0, 1)$, so that the utility of player i from a sequence of outcomes $\left\{\mathbf{x}^t\right\}_{t=1}^{+\infty}$ is given by:

$$V_i\left(\left\{\mathbf{x}^t\right\}_{t=1}^{+\infty}\right) = \sum_{t=1}^{+\infty} \delta^{t-1} u_i\left(\mathbf{x}^t\right) \tag{1}$$

In general, strategies in this game are functions that map *histories*[5] to the space of proposals Δ and voting decisions $\{yes, no\}$. In what follows, though, we restrict analysis to cases when players condition their behavior only on a summary of the history of the game that accounts for *payoff-relevant* effects of past behavior (Maskin and Tirole [17]). Specifically, define the *state* in period t as previous period's decision $\mathbf{x}^{t-1}$. We denote the state by $\mathbf{s} \in S$ so we have $\mathbf{s} = \mathbf{x}^{t-1}$ and $S = \Delta$. We restrict players to Markov strategies such that they condition their proposals and voting decisions only on the state $\mathbf{s}$, even if that state arises from different histories.

Denote the set of probability measures over Δ by $\wp(\Delta)$. In general, a *mixed Markov proposal strategy* for legislator i is a function $\mu_i : S \longrightarrow \wp(\Delta)$. Without delving into measurability issues, it is sufficient for the purposes of our analysis to assume that for every state $\mathbf{s}$, μ_i has finite support. Thus, we shall use the notation $\mu_i[\mathbf{z} \mid \mathbf{s}]$ to represent the probability that legislator i makes the proposal $\mathbf{z}$ when recognized conditional on the state being $\mathbf{s}$. A *voting strategy* is

[4] We assume $\mathbf{x}^0$ exogenously given.

[5] A history is a vector that records all proposals as well as all voting decisions that precede an action (voting or proposing).

an *acceptance set* $A_i(\mathbf{s}) \equiv \{\mathbf{z} \in X \mid i \text{ votes } yes \text{ if state is } \mathbf{s}\}$ for legislator i over proposals $\mathbf{z}$. A (mixed) *Markov strategy* for legislator i is a pair of proposal and voting strategies which we denote by $\sigma_i(\mathbf{s}) = (\mu_i[\cdot \mid \mathbf{s}],\ A_i(\mathbf{s}))$.

For a given set of voting strategies, we define the *win set* of $\mathbf{x} \in \Delta$ as the set of proposals that beat $\mathbf{x}$ by majority rule:

$$W(\mathbf{x}) = \left\{\mathbf{y} \in \Delta \mid \sum_{i=1}^{3} I_{A_i(\mathbf{x})}(\mathbf{y}) \geq 2\right\} \tag{2}$$

Then, for a triple of Markov strategies $\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$, we can write the transition probability to decision $\mathbf{x}$ when the state is $\mathbf{s}$, $Q[\mathbf{x} \mid \mathbf{s}]$, as follows:

$$\begin{aligned} Q[\mathbf{x} \mid \mathbf{s}] \equiv\ & I_{W(\mathbf{s})}(\mathbf{x}) \sum_{i=1}^{3} \frac{1}{3} \mu_i[\mathbf{x} \mid \mathbf{s}] \\ & + I_{\{\mathbf{s}\}}(\mathbf{x}) \sum_{i=1}^{3} \frac{1}{3} \sum_{\mu_i[\mathbf{y}|\mathbf{s}]>0} I_{\Delta/W(\mathbf{s})}(\mathbf{y})\, \mu_i[\mathbf{y} \mid \mathbf{s}] \end{aligned} \tag{3}$$

The first part of (3) reflects the probability of transition to allocations that are proposed by one of the three players and obtain a majority, while the second part is the probability that legislative decision is the same as in the last period (i.e. the *reversion point* or *status quo* $\mathbf{s}$) because the proposal fails majority passage. Note that since we have assumed that μ_i is a discrete measure, so is $Q[\mathbf{x} \mid \mathbf{s}]$.

Thus, we can recursively define the *continuation value*, $v_i(\mathbf{s})$, of legislator i when the state is $\mathbf{s}$ as:

$$v_i(\mathbf{s}) = \sum_{\mathbf{x} \ni Q[\mathbf{x}|\mathbf{s}]>0} [u_i(\mathbf{x}) + \delta v_i(\mathbf{x})]\, Q[\mathbf{x} \mid \mathbf{s}] \tag{4}$$

On the basis of (4) write the expected utility of legislator i, $U_i(\mathbf{x}^t)$, solely as a function of the current decision $\mathbf{x}^t$:

$$U_i(\mathbf{x}^t) = u_i(\mathbf{x}^t) + \delta v_i(\mathbf{x}^t) \tag{5}$$

where it is understood that $v_i(\mathbf{x}^t)$ – hence $U_i(\mathbf{x}^t)$ – are defined for given Markov strategies σ.

Given that legislators are otherwise identical, we focus on Markov proposal and voting strategies that are symmetric with respect to permutations of the state vector. To be precise, let the one-to-one and onto function $l : N \longrightarrow N$ denote a permutation of $N = \{1, 2, 3\}$ and let $\widehat{l}(\mathbf{x}) \in \Delta$ denote the permutation of the vector $\mathbf{x} \in \Delta$ induced by l. Then, a *symmetric Markov strategy profile* σ is such that for all $i \in N$, $\mathbf{s} \in S$, and any permutation l:

$$\mathbf{x} \in A_i(\mathbf{s}) \Leftrightarrow \widehat{l}(\mathbf{x}) \in A_{l(i)}\left(\widehat{l}(\mathbf{s})\right) \tag{6}$$

and

$$\mu_i \left[\mathbf{z} \mid \mathbf{s}\right] = \mu_{l(i)} \left[\widehat{l}\left(\mathbf{z}\right) \mid \widehat{l}\left(\mathbf{s}\right)\right] \tag{7}$$

We can now define the equilibrium solution concept as follows:

Definition 1 *A Symmetric Markov Perfect Nash Equilibrium in Stage-Undominated Voting strategies (MPNESUV) is a symmetric Markov strategy profile* $\sigma^* = \{(\mu_i^* [\cdot \mid \mathbf{s}],\ A_i^* (\mathbf{s}))\}_{i=1}^3$*, such that for* $i = 1, 2, 3$ *and all* $\mathbf{s} \in S$*:*

$$\mathbf{y} \in A_i^* (\mathbf{s}) \Longleftrightarrow U_i (\mathbf{y}) \geq U_i (\mathbf{s}) \tag{8}$$

$$\mu_i^* [\mathbf{z} \mid \mathbf{s}] > 0 \Longrightarrow \mathbf{z} \in \arg\max \{U_i (\mathbf{x}) \mid \mathbf{x} \in W (\mathbf{s})\} \tag{9}$$

Equilibrium condition (8) requires that legislators vote *yes* if and only if their expected utility from the status quo is not larger than their expected utility from the proposal. Such stage-undominated (Baron and Kalai [6]) voting strategies rule out uninteresting equilibria where voting decisions constitute best responses solely due to the fact that legislators vote unanimously. Equilibrium condition (9) requires that proposers optimize and play "no-delay" strategies, *i.e.* their proposals are always approved by a majority[6].

3. A PREVIEW OF RESULTS

Even in this considerably simplified setup, characterization of a symmetric MPNESUV constitutes a challenging problem due to the cardinality of the state space that makes it difficult to ascertain the validity of equilibrium conditions 8 and 9. The solution we present arises from an informative guess about the nature of the equilibrium-induced Markov process on policy outcomes defined in (3).

Suppose that proposers build coalitions by allocating a positive amount to at most two legislators. With such proposals, we certainly have $s_i = 0$ for some i for all periods but the first. Further suppose that legislator i, with $s_i = 0$ does not object to new (optimal) divisions of the dollar $\mathbf{z}$ with $z_i = 0$, so that if $j \neq i$ is recognized in period $t + 1$, a coalition of i and j vote *yes* on a proposal that allocates the whole dollar to j. But then both legislators i and $h \neq j$ receive zero, so that any of the three legislators can successfully form a coalition to extract the whole dollar in all subsequent periods.

[6] Since voting strategies guarantee $\mathbf{s} \in W (\mathbf{s})$ the restriction to "no-delay" proposal strategies in (9) is consistent with equilibrium.

If this conjectured path of play is part of an equilibrium, then it is possible to solve this game backwards from the period when absorption to the set of outcomes that give zero to two legislators takes place, to arbitrary initial allocation of the dollar. It is by means of this strategy that we demonstrate the advertised result. In this section we offer a brief description of some basic steps that can prove enlightening both as to the nature of the solution and the process via which it is derived.

Additional notation will be necessary before we can proceed. First, partition the space of policy outcomes into subsets $\Delta_\theta \subset \Delta$, where $0 \leq \theta < 3$ indicates the number of legislators receiving zero share of the dollar:

$$\Delta_\theta = \left\{ \mathbf{x} \in \Delta \mid \sum_{i=1}^{n} I_{\{0\}}(x_i) = \theta \right\} \tag{10}$$

In the following three subsections we will describe equilibrium proposals for the cases θ is equal to 0, 1, and 2, respectively. We will illustrate how continuation values can be derived on the basis of these proposal strategies. We note that in the remainder of this section we assume (and only prove in the following section) that these proposals achieve majority passage and constitute optima for the proposers.

i. Recurrent Allocations: $\mathbf{s} \in \Delta_2$

According to the conjectured equilibrium, Δ_2 is an irreducible absorbing set. In particular, let generic elements of Δ_2 be denoted by $\mathbf{e}^i = \left(e_1^i, e_2^i, e_3^i\right)$, with $e_i^i = 1,\ e_j^i = 0,\ j \neq i$ and assume $\mu_i^* \left[\mathbf{e}^i \mid \mathbf{s}\right] = 1, i = 1, 2, 3$, and $\mathbf{s} \in \Delta_2$, *i.e.* proposers always obtain the whole dollar when any one of the three legislators received the whole dollar in the previous period. Since players receive the whole dollar when recognized and zero otherwise, their expected payoff in each period is $\frac{1}{3}$, so that the continuation value of player i is:

$$v_i(\mathbf{s}) = \overline{v} = \frac{1}{3(1-\delta)},\ \mathbf{s} \in \Delta_2, i = 1, 2, 3. \tag{11}$$

ii. Transient Allocations: $\mathbf{s} \in \Delta_1$

Moving backwards, consider states $\mathbf{s}$ for or prior to the period of transition in Δ_2, starting with the case $\mathbf{s} \in \Delta_1$ first, *i.e.* cases when a single player received zero in the previous period. As a consequence of the focus on symmetric MPNESUV, it is sufficient to characterize equilibrium Markov strategies for $\mathbf{s} = (s_1, s_2, s_3)$ with $s_1 \geq s_2 \geq s_3$ – *i.e.* $s_3 = 0$ in this case. Since $s_3 = 0$, player 1's and 2's proposal strategy takes the form $\mu_i^* \left[\mathbf{e}^i \mid (s_1, s_2, 0)\right] = 1, i = 1, 2$. In other words, players 1 and 2 obtain the consent of player 3 in order to pass a proposal that allocates them the whole dollar.

With regard to the proposal strategy of player 3, a natural candidate for equilibrium is for this player to form a coalition with player 2 who appears less 'expensive' compared to player 1 since $s_1 \geq s_2$. It turns out that this intuition is not consistent with equilibrium for some values of $\mathbf{s} = (s_1, s_2, 0)$, as we show shortly. In particular consider the case, in accordance with this intuition, that player 3 indeed plays a pure proposal strategy and allocates a positive amount to player 2 only. By invoking symmetry we deduce that player 2 is indifferent between $\mathbf{s} = (s_1, s_2, 0)$ and a proposal $(0, s_2, s_1)$ so that according to the above conjecture:

$$\mu_3^* [(0, s_2, s_1) \mid (s_1, s_2, 0)] = 1. \tag{12}$$

On the basis of the above we can write the continuation values of players as follows:

$$v_1 (\mathbf{s}) = \frac{1}{3} [1 + \delta \overline{v}] + \frac{1}{3} [0 + \delta \overline{v}] + \frac{1}{3} [0 + \delta v_1 (0, s_2, s_1)] \tag{13a}$$

$$v_2 (\mathbf{s}) = \frac{1}{3} [0 + \delta \overline{v}] + \frac{1}{3} [1 + \delta \overline{v}] + \frac{1}{3} [s_2 + \delta v_2 (0, s_2, s_1)] \tag{13b}$$

$$v_3 (\mathbf{s}) = \frac{1}{3} [0 + \delta \overline{v}] + \frac{1}{3} [0 + \delta \overline{v}] + \frac{1}{3} [s_1 + \delta v_3 (0, s_2, s_1)] \tag{13c}$$

Symmetry implies that $v_1 (0, s_2, s_1) = v_3 (\mathbf{s})$, $v_2 (0, s_2, s_1) = v_2 (\mathbf{s})$, and $v_3 (0, s_2, s_1) = v_1 (\mathbf{s})$ so that after substitution (13b) can be solved for $v_2 (\mathbf{s})$ and equations (13a) and (13c) can be solved for $v_3 (\mathbf{s})$ and $v_1 (\mathbf{s})$ to obtain:

$$v_1 (\mathbf{s}) = \frac{1}{3 (1 - \delta)} - \frac{\delta s_2}{(9 - \delta^2)} \tag{14a}$$

$$v_2 (\mathbf{s}) = \frac{1}{3 (1 - \delta)} + \frac{s_2}{(3 - \delta)} \tag{14b}$$

$$v_3 (\mathbf{s}) = \frac{1}{3 (1 - \delta)} - \frac{3 s_2}{(9 - \delta^2)} \tag{14c}$$

We will now show that the proposal strategy in equation (12) is not optimal for player 3 for all values of s_1, s_2. To see this calculate the amount legislator 1 *demands*[7] from player 3 in order to vote *yes* on a proposal that excludes legislator 2, assuming the game is subsequently played according to (12). Denote this amount by d_1; we have:

$$d_1 + \delta v_1 (d_1, 0, 1 - d_1) = s_1 + \delta v_1 (\mathbf{s}) \tag{15}$$

[7] A precise definition of *demands* appears in the following section.

which, substituting for $s_1 = 1 - s_2$, $v_1(d_1, 0, 1-d_1) = \frac{1}{3(1-\delta)} + \frac{d_1}{(3-\delta)}$, and $v_1(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{\delta s_2}{(9-\delta^2)}$ from (14b) and (14a) respectively, we can solve for d_1 to get $d_1 = \frac{(9-\delta^2)-9s_2}{3(3+\delta)}$. This is smaller than the amount demanded by legislator 2, s_2, when:

$$s_2 > \frac{(9-\delta^2)}{3(6+\delta)} \tag{16}$$

As a consequence, when the difference between s_1 and s_2 is small, *i.e.* when equation (16) holds, legislator 3 has an incentive to mix between coalescing with legislator 1 and legislator 2.

Player 3's proposal strategy can[8] take the following form: propose $(q, 0, 1-q)$ with probability $\mu_3^*[(q,0,1-q) \mid \mathbf{s}]$ and $(0, q, 1-q)$ with probability $\mu_3^*[(0,q,1-q) \mid \mathbf{s}] = 1 - \mu_3^*[(q,0,1-q) \mid \mathbf{s}]$, where $q = \frac{(9-\delta^2)}{3(6+\delta)}$. Using the shorter notation $\mu_3^* = \mu_3^*[(0,q,1-q) \mid \mathbf{s}]$, we can write the continuation values for $\mathbf{s}$ such that equation (16) holds as follows:

$$v_1(\mathbf{s}) = \frac{1}{3}(1+\delta\overline{v}) + \frac{1}{3}\delta\overline{v} + \frac{1}{3}\left((1-\mu_3^*)(q + \delta v_1(q,0,1-q)) + \mu_3^*\delta v_1(0,q,1-q)\right) \tag{17a}$$

$$v_2(\mathbf{s}) = \frac{1}{3}\delta\overline{v} + \frac{1}{3}(1+\delta\overline{v}) + \frac{1}{3}\left((1-\mu_3^*)\delta v_2(q,0,1-q) + \mu_3^*(q + \delta v_2(0,q,1-q))\right) \tag{17b}$$

$$v_3(\mathbf{s}) = \frac{1}{3}\delta\overline{v} + \frac{1}{3}\delta\overline{v} + \frac{1}{3}(1 - q + \delta v_3(0,q,1-q)) \tag{17c}$$

Note that for all $\mathbf{s} \in \Delta_1$ with $s_2 \leq \frac{(9-\delta^2)}{3(6+\delta)}$, the continuation values of players are given in equations (13a) to (13c). Hence, since $q = \frac{(9-\delta^2)}{3(6+\delta)}$, we can use equations (13a), (13b), and (13c) to substitute for $v_1(0,q,1-q) = v_2(q,0,1-q) = \frac{1}{3(1-\delta)} - \frac{1}{(6+\delta)}$, $v_2(0,q,1-q) = v_1(q,0,1-q) = \frac{1}{3(1-\delta)} + \frac{(3+\delta)}{3(6+\delta)}$, and $v_3(0,q,1-q) = \frac{1}{3(1-\delta)} - \frac{\delta}{3(6+\delta)}$. Furthermore, we have:

$$s_1 + \delta v_1(\mathbf{s}) = s_2 + \delta v_2(\mathbf{s}) \tag{18}$$

since players 1 and 2 both accept the same allocation q. Thus we have formed four linear equations ((17a), (17b), (17c), and (18)) in four unknowns (μ_3^*, and $v_i(\mathbf{s})$, $i = 1, 2, 3$) that can be solved to obtain:

$$\mu_3^*[(0,q,1-q) \mid \mathbf{s}] = \frac{9 + 3\delta + \delta^2}{(3+2\delta)\delta} - \frac{3(\delta+6)s_2}{(3+2\delta)\delta} \tag{19}$$

[8] There are other (payoff equivalent) proposal strategies that are consistent with the equilibrium we characterize in these cases; the one we choose preserves the continuity of proposal strategies with respect to the status quo.

$$v_1(\mathbf{s}) = \frac{(15-\delta)}{6(1-\delta)(6+\delta)} - \frac{1-2s_2}{2\delta} \tag{20a}$$

$$v_2(\mathbf{s}) = \frac{(15-\delta)}{6(1-\delta)(6+\delta)} + \frac{1-2s_2}{2\delta} \tag{20b}$$

$$v_3(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{1}{(6+\delta)} \tag{20c}$$

<<Insert Figure 1 about here>>

With the above we have described equilibrium strategies and have derived continuation values for states $\mathbf{s} \in \Delta_{1,2}$. The Markov process over outcomes induced by these strategies within this subset of the two-dimensional simplex in $\Re^3$ is depicted graphically in Figure 1. In Figure 1a we depict the Markov process within the absorbing set Δ_2, while transitions from states in Δ_1 are depicted in Figure 1b. Figure 1b depicts a state $\mathbf{s}$ with $s_1 \geq s_2 > s_3 = 0$ as we analyzed in this subsection. Notice that transition to Δ_2 occurs whenever legislators 1 and 2 are recognized, *i.e.* there is only $\frac{1}{3}$ probability (when legislator 3 is recognized) that the decision remains in Δ_1 each period the state $\mathbf{s} \in \Delta_1$.

iii. Transient Allocations: $t = 1, \mathbf{s} \in \Delta_0$

If proposers never allocate a positive fraction of the dollar to more than one other legislator, allocations with all three legislators having a positive amount never prevail except perhaps for the very first period. For the latter cases, when the game happens to start with a state $\mathbf{s} = \mathbf{x}^0 \in \Delta_0$, equilibrium proposals are no different in nature than those analyzed so far, except for the additional complexity introduced by the various combinations of mixed and pure proposal strategies for various subsets of Δ_0 (seven subcases in total). In what follows we offer a brief description of equilibrium proposals in these cases.

First, the pattern of mixed proposal strategies described for states $\mathbf{s} \in \Delta_1$ is also a feature of the equilibrium whenever the difference in allocated amounts between any pair of players under the state $\mathbf{s} \in \Delta_0$ is small. This is the result of two effects. On the one hand, players with larger amount under $\mathbf{s}$ require higher compensation to overturn the status quo, *ceteris paribus*. On the other hand, these players are more likely to be excluded from coalitions in the future if the status quo is preserved, *ceteris paribus*. The first effect generates an incentive to preserve the status quo, while the second generates an incentive to overturn it. Similar, but opposite, incentives apply for players with small allocations under the status quo.

Thus, for a pair of players with small difference in allocations under **s**, *i.e.* when a condition analogous to (16) holds, the combination of the two effects implies that mixed strategies are required in equilibrium. By mixing in these cases the proposer ensures that coalition partners with a less favorable allocation under **s**, do not become too intransigent in their demands because they are guaranteed a position in the winning coalition under the status quo.

<<Insert Figure 2 about here>>

Figure 2a-d depicts the two-dimensional unit simplex in $\Re^3$ where the highlighted areas show cases when such mixing between coalition partners takes place for alternative values of the discount factor. Note that this happens for pairs of players with nearly equal allocations. From a comparison of these graphs it is apparent that mixing takes place in a smaller range of the state space as δ decreases. This is a direct consequence of the fact that the weight players put in the future benefit/cost of preserving the status quo associated with the probability of inclusion in future coalitions diminishes with δ. For $\delta = 0$ coalition building costs are solely determined by the share of the dollar under the status quo, and as a result only pure proposal strategies are played.

Another feature of the equilibrium that is implicit in the above discussion and is illustrated in Figures 2a-d is the fact that players are willing to vote *yes* on proposals that allocate them a smaller share of the dollar than what they obtain under the state **s**. In fact, there are areas – near the sides of the triangle – in which the player with the smallest share of the dollar accepts proposals that allocate her zero and the whole dollar to the proposer. In other words, there is the possibility of direct transition to Δ_2 from states in Δ_0.

In these cases, players take into account both the immediate loss in accepting a smaller amount compared to what they obtain under the status quo, **s**, as well as the *externality* that such proposals generate through a reduction in their coalition building costs in the future. By accepting such proposals these players are able to extract the whole dollar if recognized in the next period, while they would have to allocate a positive amount to one of the other players had they rejected the proposal and preserved the status quo. As is the case for mixed proposal strategies, the area of direct absorption to Δ_2 contracts with δ, since the value of future reduction in coalition building costs diminishes as well.

<<Insert Figure 3 about here>>

The Equilibrium induced Markov process described above is depicted graphically in Figure

3. While Proposition 1 in the next section contains the exact statement of the equilibrium, we provide here a brief summary that can serve as a guide for the analysis to follow.

Summary 1 *For any $\delta \in [0,1)$ there exists a symmetric MPNESUV that induces a Markov process over outcomes such that:*

- *Δ_2 is an irreducible absorbing set.*
- *For any state $\mathbf{s} \in \Delta_1$ there is probability $\frac{2}{3}$ of transition in Δ_2, and $\frac{1}{3}$ of remaining in Δ_1.*
- *For some $\mathbf{s} \in \Delta_0$ there is probability $\frac{2}{3}$ of transition in Δ_2 by a majority formed by the proposer and the player with minimum amount in $\mathbf{s}$, and probability $\frac{1}{3}$ of transition in Δ_1.*
- *In the remaining cases of states $\mathbf{s} \in \Delta_0$ there is probability 1 of transition in Δ_1.*
- *For $\mathbf{s} \in \Delta_0, \Delta_1$, proposers mix between possible coalition partners that have positive and nearly equal allocation under the state $\mathbf{s}$.*

4. EQUILIBRIUM

In this section we prove that the proposal and voting strategies we described above constitute part of a symmetric MPNESUV. Notice that we have already derived continuation values for the conjectured equilibrium for states in $\Delta_{1,2}$ in equations (14a) to (14c) and (20a) to (20c), so that we have a closed form expression for the expected utility of players for these legislative outcomes. It is critical in establishing the equilibrium that this expected utility function satisfies the following continuity and monotonicity property:

Lemma 1 *For all $\mathbf{x} = (x, 1-x, 0) \in \Delta$, (a) $U_i(\mathbf{x}), i = 1,2,3$ is continuous and piece-wise differentiable with respect to x, (b) $U_1(\mathbf{x})$ does not decrease with x, while $U_2(\mathbf{x})$ does not increase with x.*

Proof. From equations (14a) to (14c) and (20a) to (20c) we have

$$U_1(x, 1-x, 0) = \begin{cases} 1 + \frac{\delta}{3(1-\delta)} & if \quad x = 1 \\ x + \delta\left(\frac{1}{3(1-\delta)} - \frac{\delta(1-x)}{(9-\delta^2)}\right) & if \quad x \in \left[1 - \frac{(9-\delta^2)}{3(6+\delta)}, 1\right) \\ \frac{1}{2} + \frac{\delta(15-\delta)}{6(1-\delta)(6+\delta)} & if \quad x \in \left(\frac{(9-\delta^2)}{3(6+\delta)}, 1 - \frac{(9-\delta^2)}{3(6+\delta)}\right) \\ x + \delta\left(\frac{1}{3(1-\delta)} + \frac{x}{(3-\delta)}\right) & if \quad x \in \left(0, \frac{(9-\delta^2)}{3(6+\delta)}\right] \\ \frac{\delta}{3(1-\delta)} & if \quad x = 0 \end{cases} \tag{21}$$

hence $\lim_{x\longrightarrow 1}\left[x+\delta\left(\frac{1}{3(1-\delta)}-\frac{\delta(1-x)}{(9-\delta^2)}\right)\right] = 1+\frac{\delta}{3(1-\delta)}$, $\lim_{x\longrightarrow 0}\left[x+\delta\left(\frac{1}{3(1-\delta)}+\frac{x}{(3-\delta)}\right)\right] = \frac{\delta}{3(1-\delta)}$, and $U_1\left(1-\frac{(9-\delta^2)}{3(6+\delta)},\frac{(9-\delta^2)}{3(6+\delta)},0\right) = U_1\left(\frac{(9-\delta^2)}{3(6+\delta)},1-\frac{(9-\delta^2)}{3(6+\delta)},0\right) = \frac{1}{2}+\frac{\delta(15-\delta)}{6(1-\delta)(6+\delta)}$, which proves (a) for $i=1,2$ by symmetry. Continuity for $i=3$ follows since $\sum_{i=1}^{3} U_i(\mathbf{x}) = \frac{1}{1-\delta}$. For (b), we have $\frac{dU_1(\mathbf{x})}{dx} = 1+\frac{\delta^2}{(9-\delta^2)} > 0$ for $x \in \left[1-\frac{(9-\delta^2)}{3(6+\delta)},1\right)$, $\frac{dU_1(\mathbf{x})}{dx} = 0$ for $x \in \left(\frac{(9-\delta^2)}{3(6+\delta)},1-\frac{(9-\delta^2)}{3(6+\delta)}\right)$, and $\frac{dU_1(\mathbf{x})}{dx} = 1+\frac{\delta}{(3-\delta)} > 0$ for $x \in \left(0,\frac{(9-\delta^2)}{3(6+\delta)}\right]$, and symmetry completes the proof for $U_2(x,1-x,0)$. ■

The significance of lemma 1 lies with the fact that we seek to establish an equilibrium with proposals that allocate a positive amount to at most two legislators. On the basis of the expected utility function in (21) we now state the definition of the equilibrium *demand* of a legislator:

Definition 2 *Legislator i's equilibrium demand when the state is* $\mathbf{s}$ *is the minimum amount* $d_i(\mathbf{s}) \in [0,1]$ *such that for a proposal* $\mathbf{x} \in \Delta_{1,2}$ *with* $x_i = d_i(\mathbf{s}), x_j = 1-d_i(\mathbf{s})$, $j \neq i$, *we have*

$$U_i(\mathbf{x}) \geq U_i(\mathbf{s}) \tag{22}$$

The continuity in lemma 1 ensures that the demand of legislator i for a state $\mathbf{s}$ exists as long as $U_i(\mathbf{s}) \leq 1+\frac{\delta}{3(1-\delta)}$. If the latter condition fails, then legislator i always prefers the status quo $\mathbf{s}$ over any proposal in $\Delta_{1,2}$[9]. Note that as a result of symmetry the demand does not depend on which of the two possible legislators $j \neq i$ receives $x_j = 1-d_i(\mathbf{s})$.

Assuming a demand $d_i(\mathbf{s}) \in [0,1]$ exists, we define a *minimum-winning-consistent* proposal as follows:

Definition 3 *A minimum winning consistent (mwc-)proposal,* $\mathbf{x}(j,d_i(\mathbf{s}))$, *for a proposer* j *and a legislator* $i \neq j$ *with demand* $d_i(\mathbf{s}) \in [0,1]$ *is an allocation* $\mathbf{x}(j,d_i(\mathbf{s})) \in \Delta_{1,2}$ *with coordinates* $x_i(j,d_i(\mathbf{s})) = d_i(\mathbf{s})$, $x_j(j,d_i(\mathbf{s})) = 1-d_i(\mathbf{s})$, *and* $x_h(j,d_i(\mathbf{s})) = 0$, $h \neq j,i$.

With a mwc-proposal $\mathbf{x}(j,d_i(\mathbf{s}))$, the proposer j allocates the demand, $d_i(\mathbf{s})$, to legislator i and retains the rest of the dollar. The next lemma follows immediately from these definitions and lemma 1:

Lemma 2 *A mwc-proposal* $\mathbf{x}(j,d_i(\mathbf{s}))$, $j \neq i$ *is such that*

$$\mathbf{x}(j,d_i(\mathbf{s})) \in \arg\max\{U_j(\mathbf{x}) \mid \mathbf{x} \in \Delta_{1,2}, U_i(\mathbf{x}) \geq U_i(\mathbf{s})\} \tag{23}$$

[9]This is never the case in the equilibrium we characterize.

Thus, according to the conjectured equilibrium, proposals for any state $\mathbf{s} \in \Delta$ take the form of mwc-proposals. In what follows, we will make use of the notion of equilibrium demands and mwc-proposals in order to state and establish the equilibrium. To reduce the notational burden, we omit the dependence of demands and mwc-proposals on the state $\mathbf{s}$ and write d_i and $\mathbf{x}(j, d_i)$ instead, unless otherwise necessary. We prove the following:

Proposition 1 *There exists a symmetric MPNESUV with the following demands, proposal strategies, and continuation values for all* $\mathbf{s} \in \Delta$*, where* $s_1 \geq s_2 \geq s_3$*:*

- *Case a:* $s_3 \leq \frac{3\delta}{(9-\delta^2)} s_2$*,* $s_3 \leq 1 - \frac{3(6+\delta)}{(9-\delta^2)} s_2$

$$d_1 = \begin{cases} 1 - s_2 - \frac{9-\delta^2}{9} s_3 & if \quad s_3 < \frac{3}{(6+\delta)} - \frac{9s_2}{(9-\delta^2)} \\ \frac{3-\delta}{3} - \frac{9s_2 + \left(9-\delta^2\right)s_3}{3(3+\delta)} & if \quad s_3 \geq \frac{3}{(6+\delta)} - \frac{9s_2}{(9-\delta^2)} \end{cases} \tag{24}$$

$$d_2 = s_2, d_3 = 0 \tag{25}$$

$$\mu_i^* \left[\mathbf{e}^i \mid \mathbf{s}\right] = \mu_3^* \left[\mathbf{x}(3, d_2) \mid \mathbf{s}\right] = 1, \;\; i = 1, 2 \tag{26}$$

$$v_1(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{\delta s_2}{(9-\delta^2)} \tag{27a}$$

$$v_2(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{s_2}{(3-\delta)} \tag{27b}$$

$$v_3(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{3s_2}{(9-\delta^2)} \tag{27c}$$

- *Case b:* $s_3 \leq \frac{\delta}{2(3+\delta)}$*,* $s_3 > 1 - \frac{3(6+\delta)}{(9-\delta^2)} s_2$

$$d_1 = d_2 = \frac{(s_1 + s_2)\left(9-\delta^2\right)}{3(6+\delta)}, d_3 = 0 \tag{28}$$

$$\mu_i^* \left[\mathbf{e}^i \mid \mathbf{s}\right] = 1, \;\; i = 1, 2 \tag{29}$$

$$\mu_3^* \left[\mathbf{x}(3, d_2) \mid \mathbf{s}\right] = 1 - \mu_3^* \left[\mathbf{x}(3, d_1) \mid \mathbf{s}\right] = \frac{9 + \delta(3+\delta)}{\delta(3+2\delta)} - \frac{3(6+\delta)s_2}{\delta(3+2\delta)(1-s_3)} \tag{30}$$

$$v_1(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{(3+\delta)s_2 - 3s_1}{\delta(6+\delta)} \tag{31a}$$

$$v_2(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{(3+\delta)s_1 - 3s_2}{\delta(6+\delta)} \tag{31b}$$

$$v_3(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{(s_1 + s_2)}{(6+\delta)} \tag{31c}$$

- *Case c:* $s_3 > \frac{3\delta}{(9-\delta^2)} s_2,\ s_3 \leq \frac{\left(9-2\delta^2\right)}{3(3+\delta)} s_2,\ s_2 \leq \frac{27-2\delta\left(9+3\delta-\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{3(1-\delta)(3+\delta)}{(18-9\delta-4\delta^2)} s_3$

$$d_1 = \begin{cases} 1 - \frac{\left(81-54\delta-27\delta^2+6\delta^3+2\delta^4\right)s_2}{3(27-18\delta-6\delta^2+2\delta^3)} - \frac{\left(81-27\delta-27\delta^2+3\delta^3+2\delta^4\right)s_3}{3(27-18\delta-6\delta^2+2\delta^3)} & if \quad \frac{27-18\delta-6\delta^2+2\delta^3}{(9-2\delta^2-3\delta)(6+\delta)} - \frac{\left(9-6\delta-2\delta^2\right)}{(9-3\delta-2\delta^2)} s_2 > s_3 \\ \frac{(3-\delta)}{3} - \frac{\left(9-3\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)} s_3 - \frac{\left(9-6\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)} s_2 & if \quad \frac{27-18\delta-6\delta^2+2\delta^3}{(9-2\delta^2-3\delta)(6+\delta)} - \frac{\left(9-6\delta-2\delta^2\right)}{(9-3\delta-2\delta^2)} s_2 \leq s_3 \end{cases} \tag{32a}$$

$$d_2 = \frac{\left(9-\delta^2\right)\left(\left(3-2\delta\right)s_2 - \delta s_3\right)}{27-2\delta\left(9+3\delta-\delta^2\right)} \tag{32b}$$

$$d_3 = \frac{\left(3-\delta\right)\left(\left(9-\delta^2\right)s_3 - 3\delta s_2\right)}{27-2\delta\left(9+3\delta-\delta^2\right)} \tag{32c}$$

$$\mu_1^*\left[\mathbf{x}\left(1,d_3\right) \mid \mathbf{s}\right] = \mu_2^*\left[\mathbf{x}\left(2,d_3\right) \mid \mathbf{s}\right] = \mu_3^*\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = 1 \tag{33}$$

$$v_1\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} + \frac{2\delta^2 s_2 - \left(9-2\delta^2\right)s_3}{27-2\delta\left(9+3\delta-\delta^2\right)} \tag{34a}$$

$$v_2\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} + \frac{\left(9-2\delta^2\right)s_2 - 3\left(3+\delta\right)s_3}{27-2\delta\left(9+3\delta-\delta^2\right)} \tag{34b}$$

$$v_3\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} + \frac{\left(18+3\delta-2\delta^2\right)s_3 - 9s_2}{27-2\delta\left(9+3\delta-\delta^2\right)} \tag{34c}$$

- *Case d:* $s_3 > \frac{\delta}{2(3+\delta)},\ s_3 \leq \frac{\left(9-2\delta^2\right)}{9(3+\delta)},\ s_2 > \frac{27-2\delta\left(9+3\delta-\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{3(1-\delta)(3+\delta)}{(18-9\delta-4\delta^2)} s_3$

$$d_1 = d_2 = \frac{\left(2\delta-3+3s_3\right)\left(9-\delta^2\right)}{3\left(4\delta^2+9\delta-18\right)} \tag{35a}$$

$$d_3 = \frac{\delta\left(3-\delta\right) - 2\left(9-\delta^2\right)s_3}{\left(4\delta^2+9\delta-18\right)} \tag{35b}$$

$$\mu_i^*\left[\mathbf{x}\left(i,d_3\right) \mid \mathbf{s}\right] = 1, i = 1,2 \tag{36}$$

$$\mu_3^*\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = 1 - \mu_3^*\left[\mathbf{x}\left(3,d_1\right) \mid \mathbf{s}\right] = \\ \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta\left(3+2\delta\right)\left(3-3s_3-2\delta\right)} - \frac{3\left(18-9\delta-4\delta^2\right)s_2 + 3\left(9-3\delta-\delta^2\right)s_3}{\delta\left(3+2\delta\right)\left(3-3s_3-2\delta\right)} \tag{37}$$

$$v_1\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} - \frac{\left(9-6\delta-2\delta^2\right) - \left(9-12\delta-4\delta^2\right)s_3}{\delta\left(18-9\delta-4\delta^2\right)} + \frac{s_2}{\delta} \tag{38a}$$

$$v_2\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} + \frac{\left(9-3\delta-2\delta^2\right) - 3\left(3+\delta\right)s_3}{\delta\left(18-9\delta-4\delta^2\right)} - \frac{s_2}{\delta} \tag{38b}$$

$$v_3\left(\mathbf{s}\right) = \frac{1}{3\left(1-\delta\right)} - \frac{3-\left(15+4\delta\right)s_3}{\left(18-9\delta-4\delta^2\right)} \tag{38c}$$

- *Case e:* $s_3 > \frac{\left(9-2\delta^2\right)}{3(3+\delta)} s_2$, $s_1 \geq \frac{9+2\delta(3+\delta)}{9(3+\delta)}$

$$d_1 = \begin{cases} 1 + \frac{\left(2\delta^3+6\delta^2-18\delta-54\right)s_2}{3(18+3\delta-2\delta^2)} + \frac{\left(6\delta^2+2\delta^3-18\delta-54\right)s_3}{3(18+3\delta-2\delta^2)} & if \quad s_3 < \frac{\left(18+3\delta-2\delta^2\right)}{2(3+\delta)(6+\delta)} - s_2 \\ \frac{(3-\delta)}{3} - \frac{(s_2+s_3)2\left(9-\delta^2\right)}{(18+3\delta-2\delta^2)} & if \quad \frac{18+3\delta-2\delta^2}{2(3+\delta)(6+\delta)} - s_2 \leq s_3 \end{cases} \tag{39a}$$

$$d_2 = d_3 = \frac{\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} \tag{39b}$$

$$\mu_1^*\left[\mathbf{x}(1,d_3) \mid \mathbf{s}\right] = 1 - \mu_1^*\left[\mathbf{x}(1,d_2) \mid \mathbf{s}\right] = \frac{3(3+\delta)s_2 - \left(9-2\delta^2\right)s_3}{\delta(3+2\delta)(s_2+s_3)} \tag{40}$$

$$\mu_2^*\left[\mathbf{x}(2,d_3) \mid \mathbf{s}\right] = \mu_3^*\left[\mathbf{x}(3,d_2) \mid \mathbf{s}\right] = 1 \tag{41}$$

$$v_1(\mathbf{s}) = \frac{1}{3(1-\delta)} - \frac{(3+2\delta)(s_2+s_3)}{(18+3\delta-2\delta^2)} \tag{42a}$$

$$v_2(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{3(3+\delta)s_3 - \left(9-2\delta^2\right)s_2}{\delta(18+3\delta-2\delta^2)} \tag{42b}$$

$$v_3(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{3(3+\delta)s_2 - \left(9-2\delta^2\right)s_3}{\delta(18+3\delta-2\delta^2)} \tag{42c}$$

- *Case f:* $s_2 \leq \frac{1}{3}$, $s_1 < \frac{9+2\delta(3+\delta)}{9(3+\delta)}$

$$d_i = \frac{3-\delta}{9}, i = 1,2,3. \tag{43}$$

$$\mu_1^*\left[\mathbf{x}(1,d_3) \mid \mathbf{s}\right] = 1 - \mu_1^*\left[\mathbf{x}(1,d_2) \mid \mathbf{s}\right] = 1 - \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \tag{44}$$

$$\mu_2^*\left[\mathbf{x}(2,d_3) \mid \mathbf{s}\right] = 1 - \mu_2^*\left[\mathbf{x}(2,d_1) \mid \mathbf{s}\right] = \frac{3(3+\delta)(3s_1-1)}{\delta(3+2\delta)} \tag{45}$$

$$\mu_3^*\left[\mathbf{x}(3,d_2) \mid \mathbf{s}\right] = 1 \tag{46}$$

$$v_i(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{1-3s_i}{3\delta}, i = 1,2,3. \tag{47}$$

- *Case g:* $s_2 > \frac{1}{3}$, $s_3 > \frac{\left(9-2\delta^2\right)}{9(3+\delta)}$

$$d_i = \frac{3-\delta}{9}, i = 1,2,3. \tag{48}$$

$$\mu_1^*\left[\mathbf{x}(1,d_3) \mid \mathbf{s}\right] = 1 \tag{49}$$

$$\mu_2^*\left[\mathbf{x}(2,d_3) \mid \mathbf{s}\right] = 1 - \mu_2^*\left[\mathbf{x}(2,d_1) \mid \mathbf{s}\right] = \frac{3(3+\delta)(1-3s_3)}{\delta(3+2\delta)} \tag{50}$$

$$\mu_3^*\left[\mathbf{x}(3,d_2) \mid \mathbf{s}\right] = 1 - \mu_3^*\left[\mathbf{x}(3,d_1) \mid \mathbf{s}\right] = 1 + \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \tag{51}$$

$$v_i(\mathbf{s}) = \frac{1}{3(1-\delta)} + \frac{1-3s_i}{3\delta}, i = 1,2,3. \tag{52}$$

Proof. Observe that cases a and b of Proposition 1 subsume the cases with state $\mathbf{s} \in \Delta_{1,2}$ we analyzed in sections 3.i-ii. It is tedious but straightforward to verify by direct application of (4) that if players play the proposal strategies in cases a to g and these proposals obtain a majority, then continuation values are as reported in proposition 1. Then, on the basis of the definition in (5), we obtain players' expected utility functions, U_i, and additional algebraic manipulation shows that the reported *demands* are in accordance with definition 2[10].

On the basis of the expected utility functions, U_i, we can then construct equilibrium voting strategies $A_i^*(\mathbf{s}) = \{\mathbf{x} \mid U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$, $i = 1, 2, 3$, for all $\mathbf{s} \in \Delta$. These voting strategies obviously satisfy equilibrium condition (8). Then, to prove proposition 1 it suffices to verify equilibrium condition (9). To do so, we make use of three additional lemmas. Lemma 3 establishes some properties of equilibrium demands, *i.e.* that they sum to less than unity and that they are (weakly) ordered in accordance with the ordering of allocations under the state $\mathbf{s}$. Lemma 4 then establishes that the proposal strategies for legislators $i = 1, 2, 3$ in Proposition 1 maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s}) \setminus \Delta_0$; these proposals would then maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s})$ if there is no $\mathbf{x} \in W(\mathbf{s}) \cap \Delta_0$ that accrues i higher utility. We establish that this is indeed the case in lemma 5.

Lemma 3 *For all* $\mathbf{s} \in \Delta$*, the demands reported in proposition 1 are such that (a)* $\sum_{i=1}^{3} d_i(\mathbf{s}) \leq 1$*, and (b)* $s_i \geq s_j \Longrightarrow d_i(\mathbf{s}) \geq d_j(\mathbf{s})$.

Proof. By symmetry it suffices to consider $\mathbf{s}$ such that $s_1 \geq s_2 \geq s_3$, whence part (b) reduces to showing $d_1 \geq d_2 \geq d_3$. We have the following cases:

Case a: To show (a) consider first the subcase when $s_3 < \frac{3}{(6+\delta)} - \frac{9s_2}{(9-\delta^2)}$. We have $\sum_{i=1}^{3} d_i(\mathbf{s}) = 1 - s_2 - \frac{9-\delta^2}{9}s_3 + s_2 \leq 1 \Longleftrightarrow -\frac{9-\delta^2}{9}s_3 \leq 0$. Now consider $s_3 \geq \frac{3}{(6+\delta)} - \frac{9s_2}{(9-\delta^2)}$; we have $\sum_{i=1}^{3} d_i(\mathbf{s}) = \frac{3-\delta}{3} - \frac{9s_2 + \left(9-\delta^2\right)s_3}{3(3+\delta)} + s_2 \leq 1 \Longleftrightarrow \left(9 - \delta^2\right)s_3 + 3\delta(1 - s_2) + \delta^2 \geq 0$, which completes (a). For (b), we first have that $d_2 = s_2 \geq d_3 = 0$, so it remains to show that $d_1 \geq d_2$. Notice that of the two possible values of d_1, we have $1 - s_2 - \frac{9-\delta^2}{9}s_3 \geq \frac{3-\delta}{3} - \frac{9s_2 + \left(9-\delta^2\right)s_3}{3(3+\delta)} \Longleftrightarrow 9(1 - s_2 - s_3) + \delta^2 s_3 + 3\delta \geq 0$. Thus it suffices to show $\frac{3-\delta}{3} - \frac{9s_2 + \left(9-\delta^2\right)s_3}{3(3+\delta)} \geq d_2 \Longleftrightarrow \frac{3-\delta}{3} - \frac{9s_2 + \left(9-\delta^2\right)s_3}{3(3+\delta)} \geq s_2 \Longleftrightarrow s_3 \leq 1 - \frac{3(6+\delta)}{(9-\delta^2)}s_2$, which completes part (b).

Case b: $\sum_{i=1}^{3} d_i(\mathbf{s}) = 2\frac{(s_1+s_2)\left(9-\delta^2\right)}{3(6+\delta)} \leq 1 \Longleftrightarrow s_1 + s_2 \leq \frac{(18+3\delta)}{(18-2\delta^2)} \Leftarrow \frac{(18+3\delta)}{(18-2\delta^2)} \geq 1$, for (a). For (b), $d_1 = d_2 \geq d_3 = 0$.

Case c: Of the two possible values for d_1 we have $1 - \frac{\left(81-54\delta-27\delta^2+6\delta^3+2\delta^4\right)s_2}{3(27-18\delta-6\delta^2+2\delta^3)} - \frac{\left(81-27\delta-27\delta^2+3\delta^3+2\delta^4\right)s_3}{3(27-18\delta-6\delta^2+2\delta^3)} \geq \frac{(3-\delta)}{3} - \frac{\left(9-3\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)}s_3 - \frac{\left(9-6\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)}s_2 \Longleftrightarrow 9\delta s_2 - 6\delta^2 s_2 - 3\delta^2 s_3 + \left(27 - 18\delta - 6\delta^2 + 2\delta^3\right)(1 - s_2 - s_3)$

[10] All the above calculations are available upon request.

≥ 0, which is true since both $\left(9\delta s_2 - 6\delta^2 s_2 - 3\delta^2 s_3\right)$ and the remaining term are positive. Thus, to show (a) it suffices to consider $d_1 = 1 - \frac{\left(81-54\delta-27\delta^2+6\delta^3+2\delta^4\right)s_2}{3(27-18\delta-6\delta^2+2\delta^3)} - \frac{\left(81-27\delta-27\delta^2+3\delta^3+2\delta^4\right)s_3}{3(27-18\delta-6\delta^2+2\delta^3)}$. We have $\sum_{i=1}^{3} d_i(\mathbf{s}) = 1 - \frac{\left(81-54\delta-27\delta^2+6\delta^3+2\delta^4\right)s_2}{3(27-18\delta-6\delta^2+2\delta^3)} - \frac{\left(81-27\delta-27\delta^2+3\delta^3+2\delta^4\right)s_3}{3(27-18\delta-6\delta^2+2\delta^3)} + \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} + \frac{(3-\delta)\left(\left(9-\delta^2\right)s_3-3\delta s_2\right)}{27-2\delta(9+3\delta-\delta^2)} \leq 1 \iff -\frac{1}{3}\left(\frac{\left(27-18\delta-3\delta^2+2\delta^3\right)\delta}{(27-18\delta-6\delta^2+2\delta^3)}s_3 + \frac{\delta\left(27-27\delta+2\delta^3\right)}{(27-18\delta-6\delta^2+2\delta^3)}s_2\right) \leq 0$, hence (a) holds. For (b) it suffices to consider $d_1 = \frac{(3-\delta)}{3} - \frac{\left(9-3\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)}s_3 - \frac{\left(9-6\delta-2\delta^2\right)(3-\delta)}{(27-18\delta-6\delta^2+2\delta^3)}s_2$; we have $d_1 \geq d_2 \iff \frac{3-\delta}{3} + \frac{(3-\delta)\left(3\delta s_2-\left(9-3\delta-2\delta^2\right)(s_2+s_3)\right)}{27-2\delta(9+3\delta-\delta^2)} \geq \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} \iff s_2 \leq \frac{27-2\delta\left(9+3\delta-\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{3(1-\delta)(3+\delta)}{(18-9\delta-4\delta^2)}s_3$, and $d_2 \geq d_3 \iff \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} \geq \frac{(3-\delta)\left(\left(9-\delta^2\right)s_3-3\delta s_2\right)}{27-2\delta(9+3\delta-\delta^2)} \iff s_3 \leq \frac{(9-2\delta^2)}{3(3+\delta)}s_2$.

Case d: $\sum_{i=1}^{3} d_i(\mathbf{s}) = 2\frac{(2\delta-3+3s_3)\left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)} + \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)} = 1 - \frac{\delta}{3} \leq 1 \iff \delta \geq 0$, for (a). For (b) $d_1 = d_2 \geq d_3 \iff \frac{(2\delta-3+3s_3)\left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)} \geq \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)} \iff s_3 \leq \frac{9-2\delta^2}{9(3+\delta)}$.

Case e: For the two possible values of d_1 we have $1 + \frac{\left(2\delta^3+6\delta^2-18\delta-54\right)s_2}{3(18+3\delta-2\delta^2)} + \frac{\left(6\delta^2+2\delta^3-18\delta-54\right)s_3}{3(18+3\delta-2\delta^2)} \geq \frac{(3-\delta)}{3} - \frac{(s_2+s_3)2\left(9-\delta^2\right)}{(18+3\delta-2\delta^2)} \iff \left(18-2\delta^2\right)(1-s_2-s_3)+3\delta \geq 0$. Thus to show (a) it suffices to consider $d_1 = 1 + \frac{\left(2\delta^3+6\delta^2-18\delta-54\right)s_2}{3(18+3\delta-2\delta^2)} + \frac{\left(6\delta^2+2\delta^3-18\delta-54\right)s_3}{3(18+3\delta-2\delta^2)}$. We then have $\sum_{i=1}^{3} d_i(\mathbf{s}) = 1 + \frac{\left(2\delta^3+6\delta^2-18\delta-54\right)s_2}{3(18+3\delta-2\delta^2)} + \frac{\left(6\delta^2+2\delta^3-18\delta-54\right)s_3}{3(18+3\delta-2\delta^2)} + 2\frac{\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} \leq 1 \iff \left(18\delta-2\delta^3\right)(s_2+s_3) \geq 0$ which proves (a). For (b) it suffices to consider $d_1 = \frac{(3-\delta)}{3} - \frac{(s_2+s_3)2\left(9-\delta^2\right)}{(18+3\delta-2\delta^2)}$ and we have $d_1 \geq d_2 = d_3 \iff \frac{3-\delta}{3} - \frac{2\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} \geq \frac{\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} \iff \frac{\left(18+3\delta-2\delta^2\right)}{9(3+\delta)} - s_3 \geq s_2$.

Cases f,g: We have $\sum_{i=1}^{3} d_i(\mathbf{s}) = 3\frac{3-\delta}{9} = 1 - \frac{\delta}{3} \leq 1$, for (a), while (b) holds trivially. ■

We now show that equilibrium proposals are optima over feasible alternatives in $\Delta_{1,2}$.

Lemma 4 $\mu_i[\mathbf{z} \mid \mathbf{s}] > 0 \Longrightarrow \mathbf{z} \in \arg\max\{U_i(\mathbf{x}) \mid \mathbf{x} \in W(\mathbf{s}) \setminus \Delta_0\}$, *for all* $\mathbf{z}, \mathbf{s} \in \Delta$.

Proof. All equilibrium proposals take the form of mwc-proposals $\mathbf{x}(i, d_j)$. Also, whenever $\mu_i[\mathbf{x}(i,d_j) \mid \mathbf{s}] > 0$ and $\mu_i[\mathbf{x}(i,d_h) \mid \mathbf{s}] > 0, h \neq j$ we have $d_h = d_j$ so that $U_i(\mathbf{x}(i,d_j)) = U_i(\mathbf{x}(i,d_h))$. Thus, in view of lemma 2 it suffices to show that if $\mu_i[\mathbf{x}(i,d_j) \mid \mathbf{s}] = 1$, $j \neq i$, then $U_i(\mathbf{x}(i,d_j)) \geq U_i(\mathbf{x}(i,d_h))$, $h \neq i, j$, *i.e.* proposer i has no incentive to coalesce with player h instead of j. To show $U_i(\mathbf{x}(i,d_j)) \geq U_i(\mathbf{x}(i,d_h))$, $h \neq i \neq j$ it suffices to show $d_h \geq d_j$ by part (b) of lemma 1. We can trivially check that $d_h \geq d_j$ is true in proposition 1 since we have $s_1 \geq s_2 \geq s_3$ and (by part (b) of lemma 3) $d_1 \geq d_2 \geq d_3$ and: when $d_2 \neq d_3$ we have $\mu_1[\mathbf{x}(1,d_3) \mid \mathbf{s}] = 1$; when $d_1 \neq d_3$ we have $\mu_2[\mathbf{x}(2,d_3) \mid \mathbf{s}] = 1$; and when $d_1 \neq d_2$ we have $\mu_3[\mathbf{x}(3,d_2) \mid \mathbf{s}] = 1$. ■

We conclude the proof by showing that optimum proposal strategies cannot belong in Δ_0. In particular, we show that if an alternative in Δ_0 beats the status quo by majority rule, then for any player i we can find another alternative in $\Delta_{1,2}$ that is also majority preferred to the status quo and improves i's utility.

Lemma 5 *Assume* $\mathbf{x} \in (W(\mathbf{s}) \cap \Delta_0)$*; then for any* $i = 1, 2, 3$ *we can find* $\mathbf{y} \in W(\mathbf{s}) \setminus \Delta_0$ *such that* $U_i(\mathbf{y}) \geq U_i(\mathbf{x})$*.*

Proof. Consider first the case $\mathbf{x} \in A_i^*(\mathbf{s})$. Then, $\mathbf{x}$ is weakly preferred to $\mathbf{s}$ by a majority of (at least) i and some $j \neq i$. Now set $\mathbf{y} = \mathbf{x}(i, d_j(\mathbf{x}))$, where $d_j(\mathbf{x})$ is the applicable demand from proposition 1. We have $U_j(\mathbf{x}(i, d_j(\mathbf{x}))) \geq U_j(\mathbf{x})$, by the definition of demand. From part (a) of Lemma 3 we have $d_i(\mathbf{x}) + d_j(\mathbf{x}) \leq 1$ and as a result $x_i(i, d_j(\mathbf{x})) = 1 - d_j(\mathbf{x}) \geq d_i(\mathbf{x})$; hence, $U_i(\mathbf{x}(i, d_j(\mathbf{x}))) \geq U_i(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(i, d_j(\mathbf{x})) \in W(\mathbf{s})$ by a majority of i and j, and we have completed the proof for this case. Now consider the case $\mathbf{x} \notin A_i^*(\mathbf{s})$, i.e. $U_i(\mathbf{s}) > U_i(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_i(\mathbf{s}) + d_j(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(i, d_j(\mathbf{s}))$ has $x_i(i, d_j(\mathbf{s})) \geq d_i(\mathbf{s})$. Then, $U_i(\mathbf{y}) \geq U_i(\mathbf{s}) > U_i(\mathbf{x})$, $U_j(\mathbf{y}) \geq U_j(\mathbf{s})$, and $\mathbf{y} \in W(\mathbf{s}) \setminus \Delta_0$. The above hold for arbitrary i, and we have completed the proof. ■

As a result of lemmas 4 and 5, equilibrium proposals are optima over the entire range of feasible alternatives. It then follows that proposal strategies in cases a to g of Proposition 1 satisfy equilibrium condition (9) which completes the proof except it remains to show that the reported (non-degenerate) mixing probabilities are well defined in the applicable range of the state, $\mathbf{s}$. Specifically,

Case b: $\mu_3^*[\mathbf{x}(3, d_2) \mid \mathbf{s}] \geq 0 \iff \frac{9+\delta(3+\delta)}{\delta(3+2\delta)} - \frac{3(6+\delta)s_2}{\delta(3+2\delta)(1-s_3)} \geq 0 \iff (9 + \delta(3+\delta))(s_1 + s_2) \geq 3(6+\delta)s_2 \iff \left(9 + 3\delta + \delta^2\right)s_1 \geq \left(9 + \delta^2\right)s_2$; also $\mu_3^*[\mathbf{x}(3, d_2) \mid \mathbf{s}] < 1 \iff \frac{9+\delta(3+\delta)}{\delta(3+2\delta)} - \frac{3(6+\delta)s_2}{\delta(3+2\delta)(1-s_3)} < 1 \iff \left(9 - \delta^2\right)(1 - s_3) < 3(6+\delta)s_2 \iff .s_3 > 1 - \frac{3(6+\delta)}{(9-\delta^2)}s_2$.

Case d: $\mu_3^*[\mathbf{x}(3, d_2) \mid \mathbf{s}] \geq 0 \iff \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta(3+2\delta)(3-3s_3-2\delta)} - \frac{3\left(18-9\delta-4\delta^2\right)s_2+3\left(9-3\delta-\delta^2\right)s_3}{\delta(3+2\delta)(3-3s_3-2\delta)} \geq 0 \iff \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{\left(9-3\delta-\delta^2\right)}{(18-9\delta-4\delta^2)}s_3 \geq s_2 \Longleftarrow \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{\left(9-3\delta-\delta^2\right)}{(18-9\delta-4\delta^2)}s_3 \geq \frac{1-s_3}{2}$, since s_2 is less or equal to $\frac{1-s_3}{2}$. But $\frac{27-\delta\left(9+6\delta+2\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{\left(9-3\delta-\delta^2\right)}{(18-9\delta-4\delta^2)}s_3 \geq \frac{1-s_3}{2} \iff s_3 \leq 1 - \frac{2}{3}\delta$, which is true. Also $\mu_3^*[\mathbf{x}(3, d_2) \mid \mathbf{s}] \leq 1 \iff \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta(3+2\delta)(3-3s_3-2\delta)} - \frac{3\left(18-9\delta-4\delta^2\right)s_2+3\left(9-3\delta-\delta^2\right)s_3}{\delta(3+2\delta)(3-3s_3-2\delta)} \leq 1 \iff \frac{27-2\delta\left(9+3\delta-\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{3(1-\delta)(3+\delta)}{(18-9\delta-4\delta^2)}s_3 \leq s_2$.

Case e: $\mu_1^*[\mathbf{x}(1, d_3) \mid \mathbf{s}] \geq 0 \iff \frac{3(3+\delta)s_2-\left(9-2\delta^2\right)s_3}{\delta(3+2\delta)(s_2+s_3)} \geq 0 \iff s_2 \geq \frac{\left(9-2\delta^2\right)}{3(3+\delta)}s_3$ which is true since $\frac{\left(9-2\delta^2\right)}{3(3+\delta)} \leq 1$; also $\mu_1^*[\mathbf{x}(1, d_3) \mid \mathbf{s}] \leq 1 \iff \frac{3(3+\delta)s_2-\left(9-2\delta^2\right)s_3}{\delta(3+2\delta)(s_2+s_3)} \leq 1 \iff s_3 \geq \frac{\left(9-2\delta^2\right)}{3(3+\delta)}s_2$.

Case f: $\mu_1^*[\mathbf{x}(1, d_3) \mid \mathbf{s}] \geq 0 \iff 1 - \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \geq 0 \iff s_2 \geq \frac{\left(9-2\delta^2\right)}{9(3+\delta)} \iff 1 - \frac{\left(9-2\delta^2\right)}{9(3+\delta)} - s_3 \geq s_1 \Longleftarrow 1 - \frac{\left(9-2\delta^2\right)}{9(3+\delta)} - s_3 \geq \frac{9+2\delta(3+\delta)}{9(3+\delta)} \iff \frac{1}{3} \geq s_3$; and $\mu_1^*[\mathbf{x}(1, d_3) \mid \mathbf{s}] \leq 1 \iff 1 - \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \leq 1 \iff s_2 \leq \frac{1}{3}$. Also, $\mu_2^*[\mathbf{x}(2, d_3) \mid \mathbf{s}] \geq 0 \iff \frac{3(3+\delta)(2-3(s_2+s_3))}{\delta(3+2\delta)} \geq 0 \iff \frac{2}{3} \geq s_2 + s_3$; and $\mu_2^*[\mathbf{x}(2, d_3) \mid \mathbf{s}] \leq 1 \iff \frac{3(3+\delta)(2-3(s_2+s_3))}{\delta(3+2\delta)} \leq 1 \iff \frac{18+3\delta-2\delta^2}{9(3+\delta)} \leq s_3 + s_2 \iff . \frac{9+6\delta+2\delta^2}{9(3+\delta)} \leq s_1$.

Case g: $\mu_2^* [\mathbf{x}(2, d_3) \mid \mathbf{s}] \geq 0 \iff \frac{3(3+\delta)(1-3s_3)}{\delta(3+2\delta)} \geq 0 \iff s_3 \leq \frac{1}{3}$ and $\mu_2^* [\mathbf{x}(2, d_3) \mid \mathbf{s}] \leq 1 \iff \frac{3(3+\delta)(1-3s_3)}{\delta(3+2\delta)} < 1 \iff \frac{9-2\delta^2}{9(3+\delta)} < s_3$. Also $\mu_3^* [\mathbf{x}(3, d_2) \mid \mathbf{s}] \geq 0 \iff 1 + \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \geq 0 \iff s_2 \geq \frac{9-2\delta^2}{9(3+\delta)}$ which is true since $\frac{9-2\delta^2}{9(3+\delta)} < \frac{1}{3}$; lastly $\mu_3^* [\mathbf{x}(3, d_2) \mid \mathbf{s}] \leq 1 \iff \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} \leq 0 \iff s_2 \geq \frac{1}{3}$. ■

It is easy to show that the MPNESUV in proposition 1 is not unique. The multiplicity of equilibria arises from the fact that the expected utility function $U_i(x, 1-x, 0)$ in equation (21) is constant for $x \in \left[\frac{(9-\delta^2)}{3(6+\delta)}, 1 - \frac{(9-\delta^2)}{3(6+\delta)}\right]$, $i = 1, 2$. Thus, proposer h and coalition partner j are indifferent among all proposals with $x_h = 1 - z$, $x_j = z$ where $z \in \left[\frac{(9-\delta^2)}{3(6+\delta)}, 1 - \frac{(9-\delta^2)}{3(6+\delta)}\right]$, whenever the demand of j is $d_j = \frac{(9-\delta^2)}{3(6+\delta)}$. But all the additional equilibria we obtain exploiting this feature of equilibrium expected utility are payoff equivalent to the one we establish in proposition 1. Thus, it remains an open question whether the class of MPNESUV for this game are payoff equivalent, in analogy to the result of Eraslan [13] for the Baron-Ferejohn model.

Among the multiple payoff equivalent equilibria we can establish, the one we report in proposition 1 has the added feature that proposal strategies, $\mu_i[\cdot \mid \mathbf{s}]$, are weakly continuous in the status quo, $\mathbf{s}$. To be precise, we show that:

Proposition 2 *Equilibirum proposal strategies $\mu_i^* : S \longrightarrow \wp(\Delta)$ are such that for any sequence $\mathbf{s}_n \in \Delta$ with $\mathbf{s}_n \longrightarrow \mathbf{s}^*$, $\mu_i^* [\cdot \mid \mathbf{s}_n]$ converges weakly to $\mu_i^* [\cdot \mid \mathbf{s}^*]$.*

Proof. The equilibrium is such that $\mu_i^* [\cdot \mid \mathbf{s}]$ has mass on at most two points, $\mathbf{x}(i, d_j(\mathbf{s}))$ and $\mathbf{x}(i, d_h(\mathbf{s}))$, $i \neq j, h, j \neq h$. It suffices to show that these proposals (when played with positive probability) and associated mixing probabilities are continuous in $\mathbf{s}$. Then clearly

$$\sum_{j \neq i} \mu_i^* [\mathbf{x}(i, d_j(\mathbf{s}_n)) \mid \mathbf{s}_n] f(\mathbf{x}(i, d_j(\mathbf{s}_n))) \longrightarrow \sum_{j \neq i} \mu_i^* [\mathbf{x}(i, d_j(\mathbf{s}^*)) \mid \mathbf{s}^*] f(\mathbf{x}(i, d_j(\mathbf{s}^*)))$$

for any bounded, continuous f, which establishes weak convergence (Billingsley [7]). Continuity holds in the interior of cases a to g in proposition 1, so it remains to check the boundaries of these cases. In order to distinguish the various applicable functional forms we shall write $d_h^w(\mathbf{s})$ and $\mu_i^{*w}[\cdot \mid \mathbf{s}]$ where $w \in \{a, b, c, d, e, f, g\}$ identifies the case for which the respective functional form applies. We have:

- Boundary of cases a and b: At the boundary we have $s_3 = 1 - \frac{3(6+\delta)}{(9-\delta^2)} s_2$; then $\mu_3^{*b}[\mathbf{x}(3, d_2) \mid \mathbf{s}] = \frac{9+\delta(3+\delta)}{\delta(3+2\delta)} - \frac{3(6+\delta)s_2}{\delta(3+2\delta)(1-s_3)} = 1 = \mu_3^{*a}[\mathbf{x}(3, d_2) \mid \mathbf{s}]$. Also $d_2^b = \frac{(s_1+s_2)(9-\delta^2)}{3(6+\delta)} = s_2 = d_2^a$. Clearly $\mu_i^{*a}[\mathbf{x}(i, d_3^a) \mid \mathbf{s}] = \mu_i^{*b}\left[\mathbf{x}\left(3, d_3^b\right) \mid \mathbf{s}\right] = 1$ and $d_3^a = d_3^b = 0$.

- Boundary of cases a and c: At the boundary we have $s_3 = \frac{3\delta}{(9-\delta^2)} s_2$. Then $d_3^c = \frac{(3-\delta)\left(\left(9-\delta^2\right)s_3-3\delta s_2\right)}{27-2\delta(9+3\delta-\delta^2)} = 0 = d_3^a$. Also $d_2^c = \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} = s_2 = d_2^a$.

- Boundary of cases b and d: At the boundary we have $s_3 = \frac{\delta}{2(3+\delta)}$. Thus $\mu_3^{*b}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = \frac{9+\delta(3+\delta)}{\delta(3+2\delta)} - \frac{3(6+\delta)s_2}{\delta(3+2\delta)\ \ 1-\frac{\delta}{2(3+\delta)}} = \frac{\left(9+3\delta+\delta^2\right)-3(6+2\delta)s_2}{\delta(3+2\delta)}$. Likewise, $\mu_3^{*d}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta(3+2\delta)\ \ 3-3\frac{\delta}{2(3+\delta)}-2\delta} - \frac{3\left(18-9\delta-4\delta^2\right)s_2+3\left(9-3\delta-\delta^2\right)\frac{\delta}{2(3+\delta)}}{\delta(3+2\delta)\ \ 3-3\frac{\delta}{2(3+\delta)}-2\delta} = \frac{\left(9+3\delta+\delta^2\right)-3(6+2\delta)s_2}{\delta(3+2\delta)}$. With regard to demands, we have $d_2^b = \frac{(s_1+s_2)\left(9-\delta^2\right)}{3(6+\delta)} = \frac{1-\frac{\delta}{2(3+\delta)}\ \ \left(9-\delta^2\right)}{3(6+\delta)} = \frac{3-\delta}{6}$, and $d_2^d = \frac{2\delta-3+3\frac{\delta}{2(3+\delta)}\ \ \left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)} = \frac{3-\delta}{6}$. Also $d_3^d = \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)} = \frac{\delta(3-\delta)-2\left(9-\delta^2\right)\frac{\delta}{2(3+\delta)}}{(4\delta^2+9\delta-18)} = 0 = d_3^b$.

- Boundary of cases c and d: At the boundary $s_2 = \frac{27-2\delta\left(9+3\delta-\delta^2\right)}{3(18-9\delta-4\delta^2)} - \frac{3(1-\delta)(3+\delta)}{(18-9\delta-4\delta^2)} s_3$. We have $\mu_3^{*d}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta(3+2\delta)(3-3s_3-2\delta)} - \frac{3\left(18-9\delta-4\delta^2\right)s_2+3\left(9-3\delta-\delta^2\right)s_3}{\delta(3+2\delta)(3-3s_3-2\delta)} = 1$. Also $d_2^d = \frac{(2\delta-3+3s_3)\left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)}$ while $d_2^c = \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} = \frac{(2\delta-3+3s_3)\left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)}$. Finally, $d_3^d = \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)}$ while $d_3^c = \frac{(3-\delta)\left(\left(9-\delta^2\right)s_3-3\delta s_2\right)}{27-2\delta(9+3\delta-\delta^2)} = \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)}$.

- Boundary of cases c and e: At the boundary we have $s_3 = \frac{\left(9-2\delta^2\right)}{3(3+\delta)} s_2$. Then $\mu_1^{*e}\left[\mathbf{x}\left(1,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)s_2-\left(9-2\delta^2\right)s_3}{\delta(3+2\delta)(s_2+s_3)} = 1$. Also $d_3^c = \frac{(3-\delta)\left(\left(9-\delta^2\right)s_3-3\delta s_2\right)}{27-2\delta(9+3\delta-\delta^2)} = \frac{(3-\delta)}{3} s_2$, and $d_2^c = \frac{\left(9-\delta^2\right)((3-2\delta)s_2-\delta s_3)}{27-2\delta(9+3\delta-\delta^2)} = \frac{(3-\delta)}{3} s_2$, while $d_2^e = d_3^e = \frac{\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} = \frac{(3-\delta)}{3} s_2$.

- Boundary of cases d and g: At the boundary we have $s_3 = \frac{\left(9-2\delta^2\right)}{9(3+\delta)}$. Then, $\mu_3^{*d}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = \frac{27-\delta\left(9+6\delta+2\delta^2\right)}{\delta(3+2\delta)(3-3s_3-2\delta)} - \frac{3\left(18-9\delta-4\delta^2\right)s_2+3\left(9-3\delta-\delta^2\right)s_3}{\delta(3+2\delta)(3-3s_3-2\delta)} = 1 + \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} = \mu_3^{*g}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right]$. Also, $\mu_2^{*g}\left[\mathbf{x}\left(2,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)(1-3s_3)}{\delta(3+2\delta)} = 1$. Finally, $d_2^d = \frac{(2\delta-3+3s_3)\left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)} = \frac{2\delta-3+3\frac{\left(9-2\delta^2\right)}{9(3+\delta)}\ \ \left(9-\delta^2\right)}{3(4\delta^2+9\delta-18)} = \frac{3-\delta}{9} = d_2^g$, and $d_3^d = \frac{\delta(3-\delta)-2\left(9-\delta^2\right)s_3}{(4\delta^2+9\delta-18)} = \frac{3-\delta}{9} = d_3^g$.

- Boundary of cases e and f: At the boundary we have $s_1 = \frac{9+2\delta(3+\delta)}{9(3+\delta)}$. Then, $\mu_1^{*e}\left[\mathbf{x}\left(1,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)s_2-\left(9-2\delta^2\right)s_3}{\delta(3+2\delta)(s_2+s_3)} = \frac{3(3+\delta)s_2-\left(9-2\delta^2\right)\ \ 1-s_2-\frac{9+2\delta(3+\delta)}{9(3+\delta)}}{\delta(3+2\delta)\ \ 1-\frac{9+2\delta(3+\delta)}{9(3+\delta)}} = 1-\frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)}$, while $\mu_1^{*f}\left[\mathbf{x}\left(1,d_3\right) \mid \mathbf{s}\right] = 1-\frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)}$. Also, $\mu_2^{*f}\left[\mathbf{x}\left(2,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)(3s_1-1)}{\delta(3+2\delta)} = 1$. Lastly, $d_2^e = d_3^e = \frac{\left(9-\delta^2\right)(s_2+s_3)}{(18+3\delta-2\delta^2)} = \frac{\left(9-\delta^2\right)\ \ 1-\frac{9+2\delta(3+\delta)}{9(3+\delta)}}{(18+3\delta-2\delta^2)} = \frac{3-\delta}{9} = d_2^f = d_3^f$.

- Boundary of cases f and g: At the boundary we have $s_2 = \frac{1}{3}$. Clearly $d_i^f = d_i^g$. With regard to mixing probabilities we have $\mu_1^{*f}\left[\mathbf{x}\left(1,d_3\right) \mid \mathbf{s}\right] = 1-\frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} = 1$. Also, $\mu_2^{*f}\left[\mathbf{x}\left(2,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)(3s_1-1)}{\delta(3+2\delta)}$ and $\mu_2^{*g}\left[\mathbf{x}\left(2,d_3\right) \mid \mathbf{s}\right] = \frac{3(3+\delta)(1-3s_3)}{\delta(3+2\delta)} = \frac{3(3+\delta)\left(1-3\left(1-s_1-\frac{1}{3}\right)\right)}{\delta(3+2\delta)} = \frac{3(3+\delta)(3s_1-1)}{\delta(3+2\delta)}$. Finally, $\mu_3^{*g}\left[\mathbf{x}\left(3,d_2\right) \mid \mathbf{s}\right] = 1 + \frac{3(3+\delta)(1-3s_2)}{\delta(3+2\delta)} = 1$.

■

Thus, in equilibrium a small change in the status quo implies a small change in proposal strategies $\mu_i^*[\cdot \mid \mathbf{s}]$ and, by extension, to the equilibrium transition probabilities $Q[\mathbf{x} \mid \mathbf{s}]$ defined in equation (3). An immediate implication of the continuity of transition probabilities is the fact that continuation functions $v_i(\mathbf{x})$ and expected utility $U_i(\mathbf{x})$ are continuous. It is interesting to ask whether $U_i(\mathbf{x})$ inherits other properties of the stage utility function $u_i(\mathbf{x})$. Unfortunately, the answer is negative:

Proposition 3 *The expected utility, $U_i(\mathbf{x})$, induced by the equilibrium in proposition 1, (a) is continuous, (b) is not quasi-concave, (c) induces thick indifference contours.*

Proof. (a) follows from proposition 2. To show (b), consider $\mathbf{x}_1 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$, $\mathbf{x}_2 = \left(\frac{1}{3}, 0, \frac{2}{3}\right)$, and define convex combinations $\mathbf{x}(\lambda) = \lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$. Since $\frac{1}{3} < \frac{(9-\delta^2)}{3(6+\delta)}$, we have $U_1(\mathbf{x}_1) = U_1(\mathbf{x}_2) = \frac{1}{3} + \delta\left(\frac{1}{3(1-\delta)} + \frac{1}{3(3-\delta)}\right) = \frac{1}{3(1-\delta)} + \frac{\delta}{3(3-\delta)}$. Also, $U_1\left(\mathbf{x}\left(\frac{1}{2}\right)\right) = U_1\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3} + \delta\frac{1}{3(1-\delta)} = \frac{1}{3(1-\delta)}$. Clearly, $U_1\left(\mathbf{x}\left(\frac{1}{2}\right)\right) < \frac{1}{2}U_1(\mathbf{x}_1) + \frac{1}{2}U_1(\mathbf{x}_2)$. Finally, for (c) $U_i(\mathbf{x}) = \frac{1}{3(1-\delta)}$, $i = 1, 2, 3$ for $\mathbf{x}$ in cases f and g of proposition 1. ■

<<Insert Figure 4 about here>>

In Figure 4 we depict the indifference contours induced by the equilibrium expected utility $U_i(\mathbf{x})$ for various values of the discount factor δ. Figure 4a corresponds to the case the discount factor is zero, and effectively depicts the indifference contours for the stage utility of players, $u_i(\mathbf{x})$. For positive discount factors the indifference maps become non-standard inducing acceptance sets, $A_i^*(\mathbf{x})$, that are non-convex. Also, $U_i(\mathbf{x})$, takes constant values and induces 'thick' indifference contours in cases f and g of Proposition 1. These are represented in Figure 4b-4d by the hexagon that forms part of the indifference contour at the center of the unit simplex. Thus, our analysis rules out convexity of acceptance sets, $A_i^*(\mathbf{x})$, or lower-hemicontinuity of the acceptance sets correspondence, $A_i^*(\mathbf{x})$, as general properties of such bargaining games.

We conclude this section with a brief discussion of two additional features of the equilibrium. First, we note that the equilibrium absorbing set, Δ_2, is the only subset of the entire simplex Δ that does not belong to the uncovered set[11]. Despite the fact that the set of covered alternatives has measure zero in the space of possible allocations Δ, equilibrium outcomes fall in that set with probability one in the long run.

[11]We use the following definition of the covering relation: y covers x if $y \succ x$ and $z \succ y \Longrightarrow z \succ x$, where $\succ$ is the (strong) majority preference relation. See Epstein [12], and Maggie Penn [21].

Second, convergence of the equilibrium distribution of legislative decisions is fast. The long-run distribution of policy outcomes is a natural focus in such dynamic games, but this focus is less justified if convergence to the steady state distribution is slow. If that were the case for our equilibrium – and depending on the initial allocation of the dollar – legislative policy decisions might concentrate in an area of relatively equitable allocations for a significant period of time before eventual absorption. This is not the case, since (except perhaps for the very first period) there is probability $\frac{2}{3}$ of absorption into Δ_2, from any equilibrium allocation not in that set. As a result, it is straightforward to show that the maximum expected time before absorption to Δ_2 is 2.5 periods.

5. CONCLUSIONS

We analyzed a three-player majority rule bargaining game with a recurring decision over a divide-the-dollar policy space and an endogenous reversion point or default alternative. We provided a complete characterization of a Markov Perfect Nash equilibrium for this dynamic game. The equilibrium is such that in the long run and irrespective of the discount factor or the initial allocation of the dollar the proposer obtains the whole dollar with probability one.

Besides establishing existence, we showed that the equilibrium expected utility of players is continuous in current period's decision while proposal strategies are weakly continuous in the state variable, *i.e.* the status quo. While equilibrium is well behaved in that respect, players's expected utility is not quasi-concave and induces thick indifference contours.

The non-standard form of equilibrium expected utility implies that we cannot rely on the lower-hemicontinuity of the proposers' feasible set in the equilibrium analysis of this class of games. It remains an open question whether it is possible to show existence of Markov equilibrium in these dynamic legislative bargaining games with more general policy spaces.

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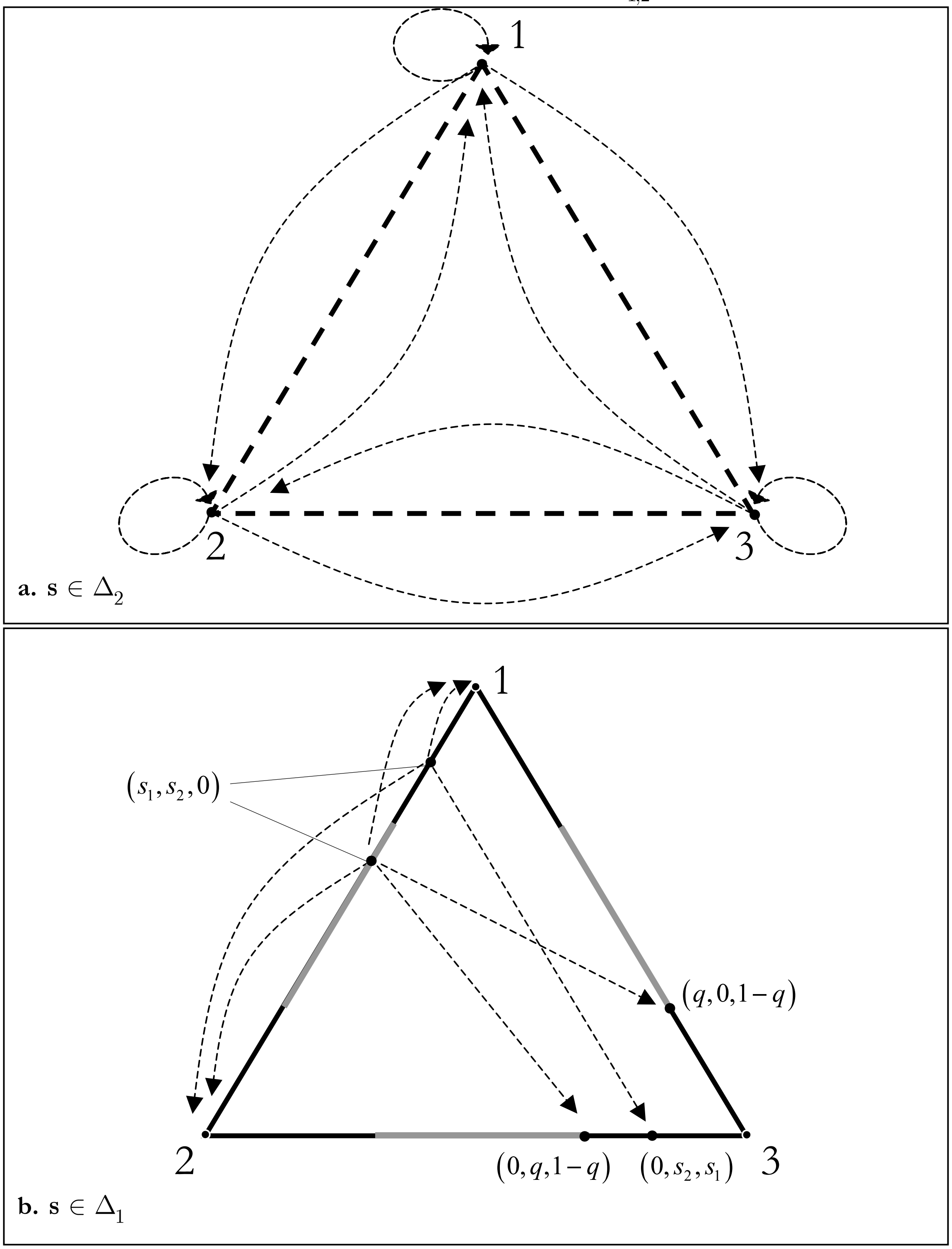
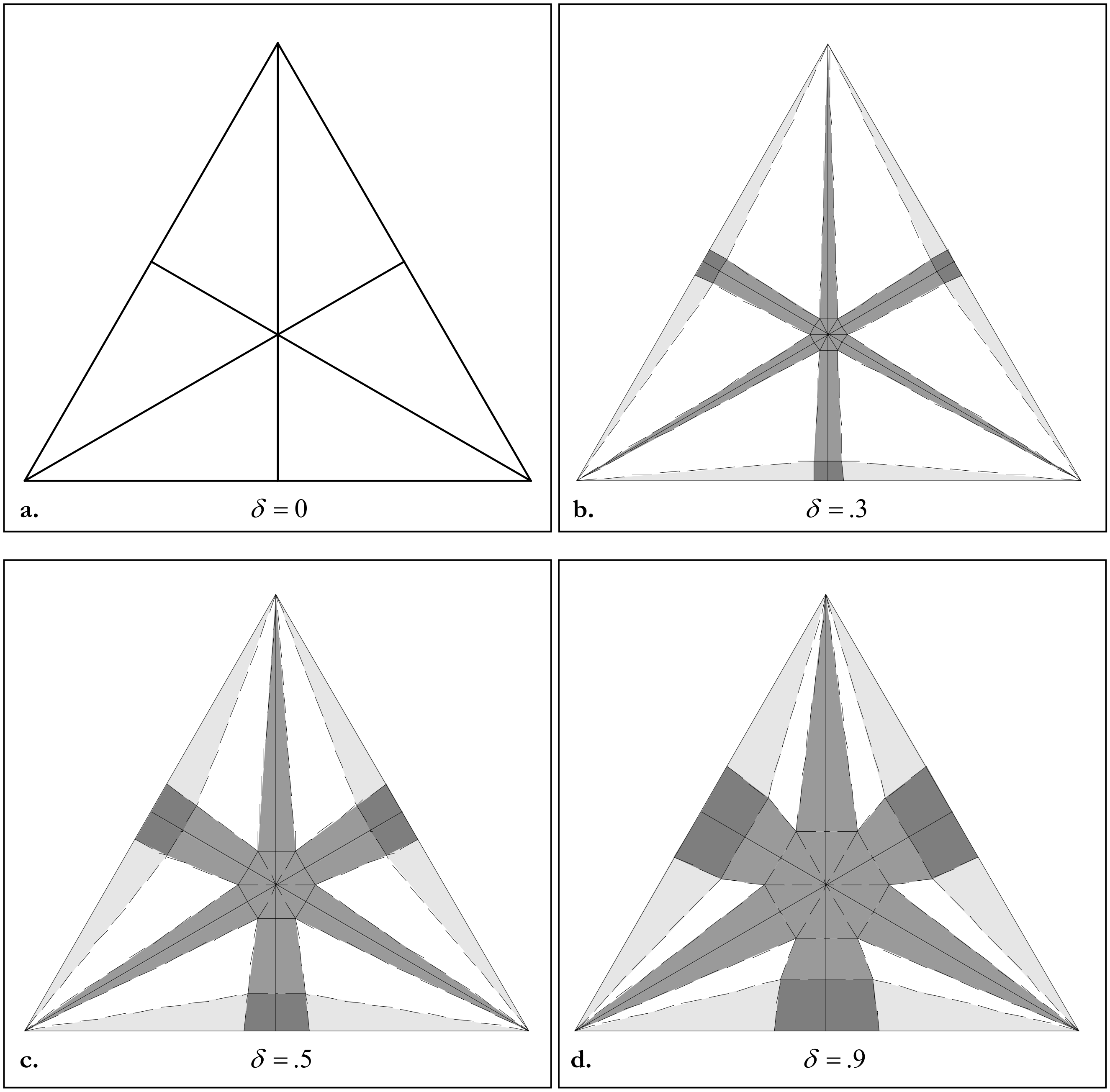
Figure 1: Equilibrium Induced Markov Process -- $\mathbf{s} \in \Delta_{1,2}$



Key: a. Δ_2 is an irreducible absorbing set; **b.** player 3 mixes between $(0, q, 1-q)$ and $(q, 0, 1-q)$, $q = \frac{(9-\delta^2)}{3(6+\delta)}$**,** when difference between s_1 and s_2 is small (allocations marked with ▬).

Sources: Constructed by author on basis of Proposition 1.

Figure 2: Demands and Equilibrium Proposal Strategies vs. Discount Factor

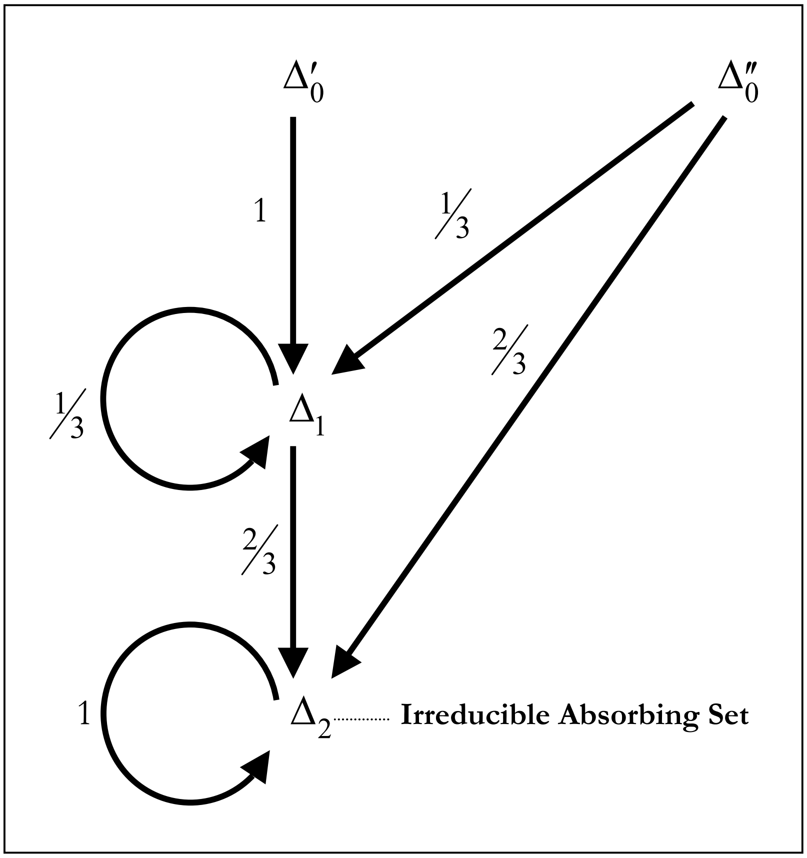


Key: Allocations for which mixed proposal strategies are played and/or legislators demand zero, expand with larger discount factor.

- Legislator with min. amount demands zero & plays mixed proposal strategy (Prop. 1, cases b).
- Proposer(s) play mixed proposal strategies (Prop. 1, cases d-g).
- Legislator with min. amount demands zero. Proposers play pure strategies (Prop. 1, case a).
- All legislators demand positive amount. Proposers play pure strategies (Prop. 1, case c).

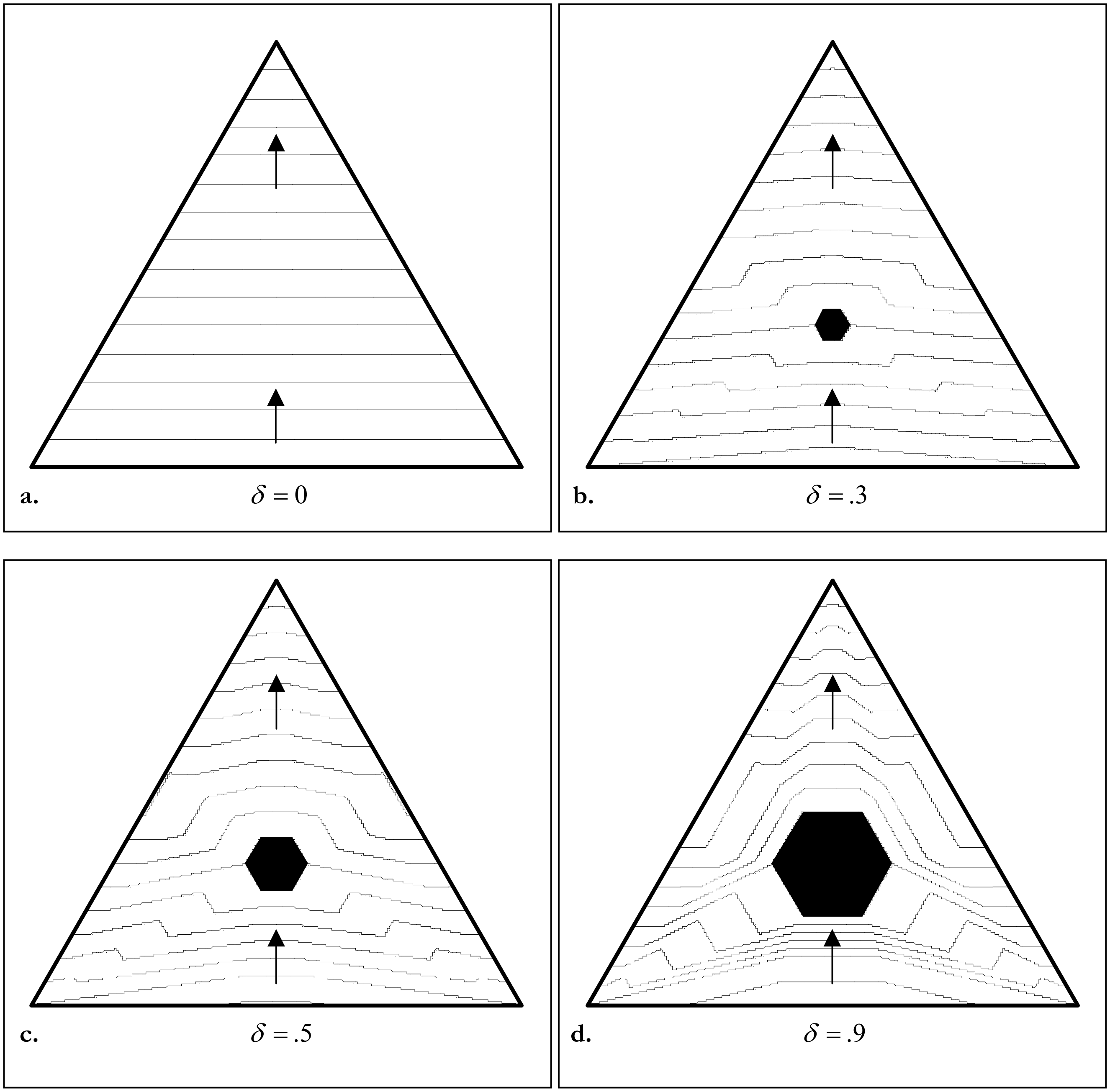
Sources: Constructed by author on basis of Proposition 1.

Figure 3: Equilibrium Induced Markov Process



Key: Δ_0', cases 3d-h of Proposition 1. Δ_0'', cases 3a-c of Proposition 1. Δ_1, cases 2a-b of Proposition 1. Δ_2, case 1 of Proposition 1.
Sources: Constructed by author on basis of Proposition 1.

Figure 4: Equilibrium Expected Utility Indifference Contours vs. Discount Factor



Key: Grapshs a to d depict the expected utility $U_i(\mathbf{x})$ of the player whose stage utility satiation point corresponds to the top corner of each triangle. Arrows indicate direction of increasing utility. Each indifference contour indicates an increase in utility by 1/15-th.. The hexagons indicate areas of constant expected utility.

Equilibrium expected utility fails quasi-concavity and induces non-convex acceptance sets. Furthermore, the acceptance sets correspondance fails lower-hemicontinuity.

Sources: Constructed by author on basis of Proposition 1.