Managing Reputation with Litigation: Why Legal Sanctions Can Work Better than Market Sanctions*

Scott Baker
Washington University in St. Louis Law School

Albert Choi
University of Virginia Law School

August 20, 2013

Abstract

A long-lived firm sells a good to a sequence of consumers, where the good’s quality imperfectly depends on the firm’s unobservable effort. To solve the moral hazard problem, the firm can promise to pay damages (formal sanctions) or facilitate reputational punishment (informal sanctions). Formal sanctions engender litigation costs and possible court error while informal sanctions involve inefficient failures to trade. Using formal sanctions, however, are generally better because increasing damages induces more lawsuits (marginal deterrence) and makes existing lawsuits a stronger deterrent (infra-marginal effect). Increasing reputational sanctions lacks the second, infra-marginal effect.

*We would like to thank Ken Ayotte, Bob Ellickson, Nicola Persico, Kathy Spier, Abe Wickelgren, and workshop participants at University of Chicago Law School, UC Berkeley Law School, Washington University in St. Louis Law School, 2013 NBER Law and Economics Mid-year Meeting, and 2013 American Law and Economics Association Annual Meeting for many helpful comments and suggestions. Comments are welcome to sbaker@wulaw.wustl.edu and ahc4p@virginia.edu.
1 Introduction

Formal and informal incentives often co-operate in the real world. Restaurants that serve contaminated food can face both tort liability and a drop in customer traffic. Firms that defraud investors may be subject to civil or criminal penalties and limited future access to the capital markets. Contractors who breach their long term procurement agreements can face both private lawsuits and a loss of long-term customers. If concerns about attracting future business are a sufficient threat to control misbehavior, why are formal sanctions necessary, especially when they involve costly lawsuits and error-prone courts? Also, if the states of the world are divided into being either verifiable or non-verifiable, how can a single event—investor fraud, service of contaminated food, breach of procurement contract—lead to both formal and informal sanctions? We attempt to address these questions by combining costly verification with repeat play. More broadly, we examine the interaction between the two types of sanctions and how such interaction affects the optimal incentive scheme.

We construct a model where a long-lived firm sells a good (product or service) to a sequence of short-lived consumers. The firm’s choice of effort imperfectly translates into quality of the good in each period. Even though high effort is efficient, because effort choice is unobservable and high effort is more costly than low effort, the firm faces a commitment (moral hazard) problem. The firm can solve this problem one of two ways. It can promise to pay damages (formal sanction) and/or induce a boycott from future consumers (informal sanction), both of which are triggered when realized quality is low. Neither mechanism is perfect. Formal sanctions entail cost of dispute resolution and possibly erroneous judgment, two elements of verification cost. Court error, in particular, encourages nuisance suits (those that are filed even when realized quality is high) that impose litigation cost with no deterrence benefit. Informal sanctions, on the other hand, generate inefficiency from failures to trade.

The model reveals two insights. First, formal sanctions often provide more effective deterrence than informal sanction, so that even if consumers are perfectly aware of past quality
realizations while the court can err in quality determination, the firm will still rely on both
types of sanctions or even formal sanctions only. The reason stems from the presence
of what we label an infra-marginal effect. Increasing damages produces additional deter-
rence through two different channels: (1) more lawsuits being filed by marginal consumer-
plaintiffs; and (2) larger damages awarded to the infra-marginal consumer-plaintiffs, those
who would have filed even without the increase in damages. An increase in reputational
punishment, by contrast, increases deterrence at the margin, but does not make existing
periods of punishment more costly for the firm. Because reputational penalties lack the
infra-marginal effect, even with positive litigation cost, the firm does not opt completely out
of formal sanctions, unless court error is quite likely. The firm increases the damages until
the cost of nuisance suits overwhelms the infra-marginal benefit. Under certain conditions,
the firm completely “crowds out” reputational sanctions and relies exclusively on formal
sanctions.

Second, we consider the informational role of litigation. Litigation can often be a cata-
lyst for the imposition of reputational sanctions. For certain kinds of behavior (securities
fraud, for example), absent litigation, consumers are unlikely to be informed of the firm’s
wrongful behavior. We examine how the two types of sanctions interact when consumers
learn about the firm’s past behavior only through litigation. In that case, having some
formal sanctions is essential in solving the deterrence problem: formal sanctions “crowd
in” informal sanctions. Nuisance suits become more of a problem, however. Because
reputational sanctions are triggered by litigation outcomes, false convictions (i.e., a finding
that the quality was low even though high) now generate “nuisance” reputational sanctions.
Nuisance suits combined with nuisance reputational punishment make relying more on for-
mal sanctions less attractive. We uncover conditions (such as a monotone likelihood ratio
property on the probability of liability) under which relying more on formal sanctions still
remains advantageous. We also find that the firm is better off, and the welfare is higher, if
consumers learn about firm’s past performance from reliable third party sources (including
past consumers) rather than solely from litigation.
The model is developed in the context of consumer contracts, where the firm transacts with a new consumer every period and has complete freedom in setting damages. The results easily translate to a relational contract setting, where two long-lived players repeatedly interact with each other. The main difference is that with relational contracts, we need not worry about the players failing to learn about past quality realizations. Therefore, litigation has limited informational role. Further, the model can easily accommodate settings where liability is imposed through mandatory rules, such as in product or environmental liability and criminal penalties, and areas where the government plays the role of regulator and prosecutor, such as airline safety (FAA), consumer products (CPSC), pharmaceutical drug regulation (FDA), and financial markets (SEC). Since the firm acts as a residual claimant in our model, it stands in the same shoes as a social planner making choices about mandatory rules. The best legal regime creates deterrence at the lowest combined cost of formal and informal sanctions. In constructing product liability policy, for instance, the social planner might wish to cap (punitive and compensatory) damages to limit litigation costs and rely on informal sanctions to make up for the deterrence gap. The planner must also be aware of how consumers make inferences from litigation outcomes when other sources of information are unavailable or unreliable.\footnote{Polinsky and Shavell (2010) is particularly concerned about the cost, including litigation cost, of products liability system. From our model, if the social planner were to set damages too high, social welfare will only decrease from excessive litigation. In order to get the right level of damages, social planner must know the relevant parameters of the transaction. When this is not feasible, it may be better to leave the designing issue to the firm.}

The paper unfolds as follows. The next section reviews the related literature. Section 3 provides the basic setup of the model and lays out some benchmark results. The main results are in Section 4. Two information structures are considered. In the first, we assume that consumers observe both past quality realizations and litigation outcomes. In the second, we assume that consumers observe only the past litigation outcomes. Throughout, we uncover the conditions under which the firm may decide to rely only on formal sanctions and under which the firm may rely on both types of sanctions. Section 5 extends the discussion of the basic model, including the prospect of secret settlements. The last section
concludes. The appendix contains the proofs and analysis of alternate equilibria that do not rely on no trade (a boycott) as the punishment strategy. The analysis shows that the efficient, renegotiation-proof punishment strategy (that relies only on a price drop and subsequent rehabilitation of the relationship) requires the firm to be quite patient. For moderately patient firms, some reliance on formal sanctions is required.

2 Related Literature

This work relates primarily to the relational contracting literature. There, the typical assumption is that some metrics of the principal-agent relationship are verifiable (e.g., firm output) while others are non-verifiable (e.g., worker effort). The parties use a mixture of formal and informal self-enforcing contracts to induce desirable behavior by the principal and the agent (MacLeod and Malcomson (1989); Baker et. al. (1994); Baker et. al. (2002); Levin (2003); MacLeod (2003); MacLeod (2007)). Our paper extends this literature in two directions. First, rather than taking an all-or-nothing approach on verifiability, we allow costly verification through (1) litigation cost and (2) court error (Townsend (1979); Choi and Triantis (2008)). As noted, by interacting costly verification with repeat play, the model can explain how a single adverse event can lead to both formal and informal sanctions. This setup also allows us to analyze how the optimal mixture between formal and informal sanctions changes as verification costs change. Second, the relational contracting models look to the interaction between two long-run players, typically assuming complete knowledge of history. By contrast, our firm, by setting damages and controlling litigation frequency, can affect future consumers’ knowledge of past interactions.

2Bernheim and Whinston (1998) similarly argues that parties might deliberately fail to contract on verifiable metrics to enhance their ability to achieve a long-run cooperation. For example, by not binding itself to pay a bonus, the firm can threaten to take the bonus away if the worker shirks on observable, but unverifiable effort. Turning the bonus into a contractually mandated term eliminates this threat for the firm, making it more difficult to induce effort.

3When the states of the world are divided into being either verifiable or non-verifiable, formal and informal sanctions are similarly divided: verifiable event leads to a formal sanction while non-verifiable event triggers an informal sanction. In Baker, et. al. (1994), for instance, contracting parties rely on objective (verifiable) signal for formal incentive pay and subjective (non-verifiable) signal for informal incentives, such as bonus or termination.
Our framework for thinking about reputation follows the classic approach of Klein and Leffler (1981) and Shapiro (1983). If a firm provides low quality, it forfeits its reputational capital: the discounted stream of income associated with the ability to price above marginal cost in the future. Unlike these classic models, in our model, there is not a one-to-one correspondence between firm’s behavior and quality realizations. Given that the court determination on quality is not perfectly accurate and that there is no one-to-one mapping between effort and quality, our model falls within the class of repeated game models where inferences about past behavior are based on noisy public signals (Green and Porter (1984), Abreu et. al. (1990); Fudenberg et. al. (1994); and Mailath and Samuelson (2006)).

A few papers more expressly examine the interaction between formal and informal sanctions. Milgrom et. al. (1990) analyzes how a legal system (law merchant court in Medieval Europe) can generate information to facilitate reputational sanctions. That paper, however, limits the contractual choice of players and assumes a perfectly accurate court. Bakos and Dellarocas (2011) compares litigation and reputation in solving the moral hazard problem, but, like Milgrom et al. (1990) restricts the contracting choices available to the parties. That paper also does not allow for varying litigation cost or consider how litigation might generate information necessary for triggering reputational sanctions. Last, Ganuza et. al. (2012) also examines the interaction between formal and informal sanctions in a products liability context, but assumes zero litigation cost and exogenously capped formal sanctions, both of which limit the ability to compare the costs and benefits of changing the size of damages.

Finally, there is a sizable empirical literature on the interaction between legal sanctions and reputational sanctions. Mitchell and Maloney (1989) documents bigger stock price reaction to airline crashes due to pilot error. Bhagat et. al. (1998) presents a comprehensive analysis of stock price reaction to corporate litigation and finds that, among others, government-initiated suits and suits involving environmental, product liability, or securities law issues tend to generate bigger declines. Karpo et. al. (1999) examines stock price reaction to press reports of investigation of fraud, indictments, and suspensions in defense
procurement contracts and finds that the negative stock price reaction is smaller for larger, more influential companies. Alexander (1999) examines stock price reactions to federal criminal convictions and finds that those who are convicted of related-party crimes tend to experience a bigger decline in price. More recently, Atanasov et. al. (2012) finds that more reputable venture capital firms are less likely to be litigated and litigated venture capital firms suffer declines in future business compared to matched peer firms.

3 The Setup

A firm with a discount factor of \( \delta \in (0, 1) \) faces a sequence of single-period consumers in \( t \in \{1, 2, 3, \ldots \} \). In each period, the firm offers consumer a contract for the purchase of a single good (product or service). Denote the contract offer by \( \kappa = (p, d) \), where \( p \) stands for price and \( d \geq 0 \) for liquidated damages. For the sake of simplicity, we do not use a time subscript on the offer.\(^4\) Upon receiving the offer, the consumer decides to either accept or reject. If the consumer rejects the offer, both parties get a payoff of zero for that period. If the consumer accepts, the consumer pays the price \( p \) and the firm chooses the level of unobservable effort that affects the quality of the good. The firm’s effort can be high or low, \( e \in \{e_h, e_l\} \). The cost is given by \( c(e_i) = c_i \) where \( c_h > c_l \geq 0 \) (\( \Delta c \equiv c_h - c_l \)). Quality can also be either high or low: \( q \in \{q_h, q_l\} \).\(^5\) Effort does not perfectly translate to quality:

\[
\text{prob}(q_l|e_h) = \pi \quad \text{and} \quad \text{prob}(q_h|e_h) = 1 - \pi, \quad \text{where} \quad \pi \in (0, 1/2).
\]

The consumer attaches a value of \( v_h \equiv v(q_h) \) to high quality and a value of \( v_l \equiv v(q_l) \) to low quality, where \( v_h > v_l \geq 0 \). We assume that (1) choosing high effort is efficient \((E(v|e_h) - c_h > E(v|e_l) - c_l)\); (2) high quality generates a strictly positive surplus \((v_h > c_h)\); and (3) low quality generates a strictly negative surplus \((v_l < c_l)\). We also assume that

\(^4\)In equilibrium, the firm exerts high effort in each period which, in turn, determines the consumer’s willingness-to-pay at \( E(v|e_h) \). Since the firm has the power to make a take-it-or-leave-it offer, the firm will offer the same price-damages combination \((p, d)\) in each period so long as they are in the cooperation phase.

\(^5\)Our conceptions of quality \((q)\) and damages \((d)\) are flexible. Quality, for instance, can represent whether the good (product or service) is merchantable (contract or commercial law), defective (product liability law), or unlawful (criminal law). Similarly, damages can stand for warranty (contract or commercial law), compensatory and punitive damages (product liability law), or fines and penalties (criminal law).
\( \pi \) is sufficiently low so that \( E(v | e_h) - c_h > 0 \) and \( E(v | e_l) - c_l < 0 \). The following table represents the stage game payoffs without any sanctions. Without any sanctions, Low Effort is a weakly dominant strategy for the firm in the stage game, and the players are unable to reach efficient outcome (High Effort, Purchase).

<table>
<thead>
<tr>
<th></th>
<th>High Effort</th>
<th>Low Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Purchase</td>
<td>((0, 0))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>Purchase</td>
<td>((E(v</td>
<td>e_h) - p, p - c_h))</td>
</tr>
</tbody>
</table>

Table 1: Stage Game Payoffs without Formal Sanctions \((d = 0)\)

After the consumer receives the good, quality is realized and observed by both the consumer and the firm. Upon observing quality, the consumer can bring a lawsuit to recover liquidated damages of \(d\). The lawsuit imposes a litigation cost of \(k\) on the consumer, where \(k\) is distributed on \([\underline{k}, \overline{k}]\) with a strictly positive and continuously differentiable probability density function of \(f(\cdot)\) and a corresponding cumulative distribution function of \(F(\cdot)\). For convenience, we assume that \(\underline{k} = 0\) and \(\overline{k} \to \infty\). We also assume that \(k\) is realized after the purchase and observed privately by the consumer. Although both the firm and the consumer observe the realized quality, the court can err in its quality determination. Specifically, we let \(\rho = \text{prob}("q_h" | q_l) = \text{prob}("q_l" | q_h)\) where \(\rho \in (0, 1/2)\).\(^6\) Court’s judgments are put in quotes. Notice that, even when realized quality is high \((q = q_h)\), when damages are sufficiently generous \((\rho d \geq k)\), the consumer will bring a lawsuit to collect \(d\). This type of litigation involves “frivolous” or “nuisance” claims because the consumer is suing despite receiving high quality. On the other hand, when realized quality is low \((q = q_l)\), the consumer will bring a lawsuit if \((1 - \rho)d \geq k\). Since \(\rho \in (0, 1/2)\), the consumer is more

\(^6\)Court error \((\rho)\) and litigation cost \((k)\) are the two elements of verification cost. The conventional assumption of perfect verifiability can be replicated with \(\rho = 0\) and \(k = 0\). Similarly, if \(\rho \to 1/2\) and/or \(k \to \infty\), realized quality becomes non-verifiable. When \(k > 0\), the firm can eliminate nuisance suits by setting \(d \in \left(\frac{k}{1 - \rho}, \frac{k}{\rho}\right)\). When \(k\) is sufficiently small, however, this strategy will not be optimal since it requires the firm to rely heavily on informal sanctions. See Choi and Triantis (2008) for a more detailed discussion of costly verification.
likely to sue if he receives a low quality good. The reason: the court is more likely to award damages in that circumstance.

In addition to the formal sanctions, consumers can impose informal sanctions against the firm. The court’s judgment in period $t$ is public, meaning it is observed by all consumers in period $t' \geq t$. We consider two possible information structures: (1) consumers observe both past quality realizations and litigation outcomes and (2) consumers observe only past litigation outcomes. Under either information structure, when future consumers perceive that there was a low quality realization in period $t$, they can punish the firm by not purchasing from the firm from period $t+1$ for $T \geq 0$ periods.\(^7\) For analytical convenience, we assume that consumers can use mixed strategy in punishing the firm: $T \in \mathbb{R}^+$. The length of the punishment period depends on (1) size of the formal sanctions; (2) the information structure; and (3) the court’s judgment.

### 3.1 Two Benchmarks

Suppose the firm wishes to rely only on formal sanctions. For any $d \geq 0$, when $q = q_l$, the firm will be sued and be found liable with probability $\text{prob}(k \leq (1 - \rho)d) \cdot (1 - \rho) = F((1 - \rho)d)(1 - \rho)$. The comparable probability when $q = q_h$ is $F(\rho d)\rho$. To solve the

\(^7\)The equilibrium concept we are using is Perfect Public Equilibrium (PPE). See, e.g., Abreu et. al. (1990), Fudenberg and Tirole (1996), and Mailath and Samuelson (2006). During the punishment period, players revert to (Not Purchase, Low Efficiency) equilibrium. This can be sustained by switching the consumers’ beliefs about the firm’s behavior. Initially, the consumer in $t = 1$ starts with the belief that the firm will choose $e_h$. In any subsequent period when consumers observe an adverse outcome (either $q_l$ under the first information structure or an adverse judgment under the second), for $T$ periods, consumers believe that the firm is choosing $e_l$. Given this belief, since $E(v|e_l) - c_l < 0$, (Not Purchase, Low Effort) is a Nash equilibrium in the punishment phase. After $T$ periods, the initial belief of high effort is restored. One problem with this self-enforcing belief system is that, given that the firm is making an offer before consumers decide whether to accept, the firm can make the consumers’ belief of $e_l$ in the punishment phase irrational by offering $d = \overline{d}$. This can impose a limit on how much the consumers can punish the firm. Furthermore, given that the game is stationary and the parties maybe foregoing a potential surplus, they may have an incentive to renegotiate away from punishment. We discuss the subgame perfection and renegotiation-proofness issues in the appendix.
commitment problem with only formal sanctions, therefore, the firm must set \( d \) to satisfy

\[
p - c_h - (1 - \pi)F(\rho d)\rho + \pi F((1 - \rho)d)(1 - \rho)d \\
\geq p - c_l - (\pi F(\rho d)\rho + (1 - \pi)F((1 - \rho)d)(1 - \rho))d
\]

The left hand side is the firm’s payoff from high effort, recalling that high effort leads to high quality with probability \((1 - \pi)\). The right hand side is the firm’s payoff from low effort, recalling that low effort leads to high quality with probability \(\pi\). Rearranging provides the expression for the minimum necessary damages the firm must promise to effectively commit to high effort

\[
d \geq \frac{\Delta c}{(1 - 2\pi)(F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho)} \equiv \phi(d)
\]

The function \(\phi(d)\) represents total sanctions necessary to solve the commitment problem. Although, a priori, \(\text{sign}(\phi'(d))\) is not certain, we assume that

\[
\frac{\partial}{\partial d} (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho) = f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2 > 0
\]

so that \(\phi'(d) < 0\). The assumption ensures that higher damages produce larger deterrence. The assumption can be satisfied when \(\rho\) is sufficiently small, \(f(\cdot)\) is not decreasing too rapidly, or both. Since \(\lim_{d \to 0} \phi(d) = \infty\), \(\exists \bar{d} < \infty\) such that \(\bar{d} = \phi(\bar{d})\).\(^8\) When \(d = \bar{d}\), therefore, formal sanctions alone solve the commitment problem. For most of the analysis, we also assume that with \(d = \bar{d}\), the per-period surplus from trade, net of expected litigation cost, is still positive.\(^9\) This assumption allows using formal sanctions only \((d = \bar{d})\) as a viable alternative for the firm.

Now consider the other extreme benchmark: the firm sets \(d = 0\) and relies only on informal sanctions. Suppose also that the consumers observe past quality realizations and

---

\(^8\)The condition also assumes that there is no legal limit on how high the liquidated damages can be. An upper limit on liquidated damages may stem from the anti-penalty doctrine in contract law which prohibits liquidated damages from being more than compensatory. Our formulation replicates the familiar result that such limitations can prevent efficient exchanges.

\(^9\)Formally, we assume that \(E(v|c_h) - c_h - (1 - \pi)F(\rho \bar{d})E(k|k \leq \rho \bar{d}) - \pi F((1 - \rho)\bar{d})E(k|k \leq (1 - \rho)\bar{d}) \geq 0\).
employ the grim-trigger punishment strategy \((T = \infty)\). Given this strategy, if the consumer observes \(q = q_l\), they never purchase from the firm again. For the firm to choose high effort, we need

\[
p - c_h + (1 - \pi)\delta V^+(0) \geq p - c_l + \pi \delta V^+(0)
\]

where \(V^+(0)\) represents the present value of the firm’s all future profits conditional on \(e = e_h\) and \(d = 0\). Since the firm will set \(p = E(v|e_h)\) in each period, we must have \(V^+(0) = E(v|e_h) - c_h + (1 - \pi)\delta V^+(0)\). After simplification, we get \(V^+(0) = \frac{E(v|e_h) - c_h}{(1 - \delta) + \pi \delta}\).

Given this identity, the incentive condition becomes

\[
\delta \frac{E(v|e_h) - c_h}{(1 - \delta) + \pi \delta} \geq \frac{\Delta c}{1 - 2\pi}
\]

We will assume that \(\delta\) is sufficiently high and \(\pi\) is sufficiently low so that this inequality is satisfied: \(E(v|e_h) - c_h > \frac{\pi \Delta c}{1 - 2\pi}\) and \(\delta \geq \frac{\Delta c}{1 - \pi \delta}(1 - \delta)(1 - \Delta c/\Delta (1 - 2\pi)) \equiv \delta \in (0, 1)\). This means that the firm has the option of relying only on informal sanctions, given that consumers observe past quality realizations.\(^{10}\)

4 Deploying Formal and Informal Sanctions

We now examine conditions under which the firm would want to rely on both types of sanctions to solve the commitment problem. As previously noted, we consider two information structures. In case 1, consumers observe both past quality realizations and litigation outcomes. In case 2, consumers only observe past litigation outcomes: they must infer firm’s conduct based on court judgment. Thus, in case 1, even without any litigation, consumers have sufficient information to impose reputational punishment. By contrast, in case 2, consumers must rely on litigation outcomes to impose reputational sanctions.

\(^{10}\) With \(d > 0\), firms with \(\delta < \delta\) can still solve the commitment problem. As \(\delta\) decreases, \(d\) has to increase. This leads to the conventional account of how more patient firms rely on informal sanctions while less patient firms rely on formal sanctions. The more interesting claim of the paper is that firms who could achieve deterrence through informal sanctions only will decide not to do so, as explored in proposition 1.
4.1 Case 1: When Consumers Observe Both Past Litigation Outcomes and Quality Realizations

In this section, we assume that the consumers observe past quality realizations—for instance, from independent sources such as Consumer Reports or Angie’s List, and their knowledge (market information) is perfectly accurate. The court, on the other hand, can err: it might impose damages even when the firm delivered high quality or relieve the firm from paying damages even when delivered quality was low. Given that the market information is perfectly accurate and the court is prone to error, formal sanctions seem an inefficient method of solving the commitment problem. The following proposition, however, demonstrates that, unless court error is quite likely, it is in the firm’s interest to use formal sanctions.

Proposition 1 Suppose the consumers observe both past quality realizations and litigation outcomes. When \( \rho < (\sqrt{\pi^2 + \pi - \pi}) \in (0, \frac{\sqrt{3} - 1}{2}) \), the firm sets \( d > 0 \). As \( \rho \to 0 \), \( d \to d^* \) but as \( \pi \to 0 \) or \( \rho \to 1/2 \), \( d \to 0 \).

In equilibrium, the firm capitalizes both the expected cost of litigation and the deadweight loss from reputational sanctions. Its long-run profit can be written as

\[
\frac{1}{1 - \delta} \times \left\{ E \left( \begin{array}{c} \text{Gross Surplus from Trade} \\ \text{Litigation Cost} \\ \text{Deadweight Loss from Reputational Sanctions} \end{array} \right) - E \left( \begin{array}{c} \text{Litigation Cost} \\ \text{Deadweight Loss from Reputational Sanctions} \end{array} \right) \right\}
\]

A marginal increase in damages produces three distinct effects: two marginal and one infra-marginal. First, with possible court error \( (\rho > 0) \), additional damages create an uptick in nuisance suits—suits arising even though the consumer received high quality. These

---

\(^{11}\)For simplicity, we are assuming here that (1) consumers observe past history with probability one; and (2) their knowledge is accurate. We can relax these assumptions by (1) allowing quality realization to become observable to future consumers with probability less than one and (2) consumers knowledge is less than perfectly accurate, i.e., even if quality realization is high, due to a faulty signal, future consumers believe that quality was low. The main results, we suspect, will not change. Particularly with respect to the latter, when reputational sanctions can be faulty, the firm will have a stronger incentive to rely more on formal sanctions.
nuisance suits produce no deterrence benefit and only impose litigation costs on the firm. Second, as damages rise, some consumers who received low quality who wouldn’t have sued before will now do so. These marginal lawsuits bring extra deterrence and extra litigation costs. The extra deterrence means that the punishment period can be shortened, benefiting the firm. At the same time, the extra lawsuits impose litigation cost, which reduces the long run profit. These two effects cancel each other out. Third, the increase in damages makes the existing lawsuits a stronger deterrent. Since these are lawsuits that would have been filed without any increase in damages, they produce extra infra-marginal deterrence benefit at no additional cost to the firm.\textsuperscript{12}

To maximize profit, the firm trades off the additional cost from nuisance suits against the additional benefit from infra-marginal deterrence. So long as \( \rho \) is not too large, the infra-marginal effect outweighs the cost of additional nuisance suits and the firm incorporates formal sanctions in its optimal deterrence scheme. Not surprisingly, as the court becomes more accurate in determining realized quality (smaller \( \rho \)), the firm is more likely to rely on formal sanctions. At the extreme, when the court judgment is error-free (\( \rho = 0 \)), the firm no longer needs to worry about nuisance suits and relies only on formal sanctions: informal sanctions are crowded out altogether. The opposite trade-off takes place as the court becomes more error-prone (larger \( \rho \)). Finally, as the realized quality becomes a more accurate signal of the firm’s effort (smaller \( \pi \)), since faulty reputational sanctions become less likely, the firm relies less on formal sanctions (\( d \) decreases).\textsuperscript{13}

\textsuperscript{12}In this model, increasing damages is costly because it attracts additional lawsuits. An alternative modeling choice would be to have the existing lawsuits become more expensive as damages rise (as litigants spend more when the benefit of winning gets higher). In this alternative framework, the firm would balance the increased cost of the existing lawsuits against the increased benefit from the infra-marginal effect on deterrence. We suspect some level of formal sanctions would still be optimal.

\textsuperscript{13}The appendix shows that show very patient firms can rely solely on informal sanctions and efficient punishment to achieve commitment (i.e., no boycott in the punishment period, but rather simply a price cut to marginal cost). That said, the appendix demonstrates that for slightly less patient firms such an approach is not feasible for achieving commitment, while the boycott equilibrium discussed in the text does still exist.
4.2 Case 2: When Consumers Only Observe Past Litigation Outcomes

When consumers do not observe past quality realizations and must infer the firm’s conduct through litigation, the presence of some litigation is essential in allowing them to engage in reputational sanctions – securities fraud is one prominent example here. Furthermore, damages must be sufficiently large for the firm to solve the commitment problem. If damages are too small \( (d = \varepsilon) \), even with the maximal informal sanctions \( (T = \infty) \), the firm will have insufficient incentive to exert high care, because litigation (and the subsequent reputational punishment) will arise too infrequently. In the presence of these challenges, the firm has two choices: (1) either promise sufficiently high damages \( (d = \overline{d}) \) and rely only on formal sanctions or (2) rely on both formal and informal sanctions with damages large enough for the consumers to observe court judgment with sufficient frequency \( (0 < d \leq d < \overline{d}) \).

**Proposition 2** Suppose consumers only observe past litigation outcomes. In equilibrium, the firm sets \( d \in [\underline{d}, \overline{d}] \) where \( 0 < \underline{d} < \overline{d} < \infty \), and the consumers impose reputational sanctions when the firm is found liable. If \( \frac{\partial}{\partial d} \left( \frac{F((1-\rho)d)(1-\rho)}{F(\rho d)\beta} \right) \geq 0 \ \forall d \in [\underline{d}, \overline{d}] \), the firm will rely only on formal sanctions: \( d = \overline{d} \). As \( \rho \to 0 \), \( d \to \overline{d} \).

In choosing optimal damages, there are two primary differences when compared to the case where consumers observed both past quality realizations and litigation outcomes. First, because reputational sanctions kick in when the firm is found liable, increase in damages \( (d) \) will lead not only to a higher probability of nuisance suits but also to a more frequent faulty reputational sanctions (both of which occur when \( q = q_h \)). This will, at the margin, make increasing damages relatively unattractive. On the other hand, because larger damages lead to shorter reputational sanctions, the firm gains when inevitable faulty reputational sanctions lasts a shorter period of time. These two effects point in opposite directions. Thus, even though the benefit from infra-marginal deterrence still arises, it is unclear, a priori, whether relying more on formal sanctions is beneficial for the firm.
Notwithstanding the uncertainty, there are two sufficient conditions under which reliance on legal sanctions dominate. The first is when the court becomes more accurate in quality determination. As \( \rho \) get smaller, the effects of both frivolous litigation and faulty reputational sanctions become smaller and, given the infra-marginal effect of increasing damages, it is in the firm’s interest to rely more on formal sanctions.\(^{14}\) Second, when the deterrence effect from increasing damages is sufficiently strong, that is, when
\[
\frac{\text{Prob} \left( \text{liability} | d, q_l \right)}{\text{Prob} \left( \text{liability} | d, q_h \right)} = \frac{F \left( (1-\rho)d(1-\rho) \right)}{F \left( \rho d \right)}
\]
is (at least weakly) increasing with respect to \( d \), the firm will rely more on formal sanctions.\(^{15}\) When this condition is satisfied, increasing damages will (weakly) reduce the expected faulty sanctions \( (F(\rho d)\rho \phi(d)) \), which, in turn, renders increasing damages unconditionally attractive.

### 4.3 Comparison of Two Information Regimes

Having analyzed the optimal deterrence system in the respective information regimes, in this subsection, we compare the respective outcomes. We address two specific questions: (1) whether the firm will prefer the regime where consumers observe both past quality realizations and litigation outcomes; and (2) whether the firm will rely more on formal sanctions when information available to consumers is more limited. The first question deals more broadly with consumers’ knowledge of history in general. The following corollary demonstrates that (1) the firm will always prefer the regime where consumers observe both past quality realizations and litigation outcomes; but (2) the firm may or may not rely more on formal sanctions when the consumers’ knowledge of history is limited to litigation outcomes.

**Corollary 1** Unless it is optimal for the firm to rely only on formal sanctions \((d = \overline{d})\)

---

\(^{14}\)In contrast to the previous case, the optimal damages as \( \rho \to 1/2 \) cannot be readily determined unless a stronger assumption on the distribution function, in particular, on \( f'(\cdot) \), is made.

\(^{15}\)We can roughly translate this as a version of (weak) monotone likelihood ratio property. The conditional probability, \( p(\text{liability}|d, q_i) \), is an aggregation of (1) probability of conviction; and (2) probability that the litigation cost is lower than the expected return. Note that the condition that \( \frac{p(\text{liability}|d, q_l)}{p(\text{liability}|d, q_h)} \) is (weakly) increasing with respect to \( d \) is stronger than the condition that \( p(\text{liability}|d, q_l) - p(\text{liability}|d, q_h) \) is increasing with respect to \( d \), which we have assumed to ensure that \( \phi'(d) < 0 \).
in both regimes, the firm’s equilibrium expected profit is strictly higher when consumers observe both past quality realizations and litigation outcomes than when consumers observe only litigation outcomes. However, the firm may or may not rely more on formal sanctions when consumers observe only litigation outcomes.

When the consumers have to rely on litigation outcomes to unleash reputational sanctions, the firm suffers from both false convictions and faulty reputational sanctions, both of which occur despite $q = q_h$. By contrast, when the consumers independently observe past quality realizations, they impose reputational sanctions only when $q = q_l$. Given that the firm captures all the surplus from trade, since faulty reputational sanctions reduce the firm’s profit without providing any deterrence benefit, it is only natural that the firm strictly prefers the latter regime. In fact, the firm has an incentive to provide (or encourage the market to produce) more accurate information about its past history to the consumers so as to prevent faulty reputational sanctions from being triggered from nuisance suits. The only exception to this preference ordering is when relying only on formal sanctions ($d = \bar{d}$) is optimal in both regimes, in which case the firm is indifferent between the two regimes.

While the firm’s long-run profit will be strictly higher when the consumers observe both past quality realizations and litigation outcomes, it is unclear whether the firm will rely more on formal sanctions (bigger $d$) when the consumers’ knowledge of history is limited to litigation outcomes. Even though the infra-marginal effect is present in both cases (which makes the formal sanctions more attractive), the limited information case, as noted before, has two additional, opposing effects with respect to changes in damages: higher damages induce more faulty reputational sanctions but reduces their magnitude. Since it is uncertain which effect will dominate, the firm may or may not rely more on reputational sanctions when consumers’ knowledge of history is more limited.
5 Extensions

Our model analyzes the interaction between formal and informal sanctions in a setting where one long-term player is facing a sequence of short-term players. The analysis can be easily extended to a setting where two long-term players are facing each other, a setup frequently invoked in relational contracting literature. One primary difference will be that, because long-term players can be presumed to observe all past outcomes (history) in the relationship, the extension will be quite similar to the setting where the consumers observe both past quality realizations and litigation outcomes. Our analysis suggests that litigation mechanism, though costly, can be quite useful (due to its infra-marginal effect) even for long-term players trying to sustain cooperation. If litigation provides additional information about the effort choice (for instance, when the court has to determine whether or not one party was “negligent”), litigation will play an even more useful role. Under certain circumstances, long-term players may decide to completely “crowd out” reputational sanctions and rely only on formal sanctions.

In our model, when a consumer files (or threatens to file) suit, the firm may have a strong incentive to secretly settle with the consumer so as to eliminate the litigation cost \( k \) that engenders deadweight loss. In case 2, when adverse judgments also lead to reputational sanctions, firm’s incentive to secretly settle will be even higher. There are two problems with such secret settlements. First, even if all lawsuits are secretly settled, once consumers rationally expect this, the firm will no longer be able to sustain a high effort equilibrium. To prevent this unraveling, the firm has an incentive to commit to a more public release of litigation outcomes. Perhaps this can explain why firms often allow consumers to bring lawsuits in court (a public medium) rather than through informal arbitration, even though arbitration is perceived to be (much) less costly.\(^{16}\) Second, even if the firm were to attempt to secretly settle with consumers, the fact that the consumers are privately informed of secret settlement also may not be feasible when government is producing information to the public and bringing (criminal or civil) claim against the firm. To the extent that the government is concerned about social welfare, the government-prosecutor can commit not to settle cases and release information to the public to facilitate reputational sanctions.

\(^{16}\)
their litigation costs will prevent all lawsuits from settling. The presence of asymmetric information actually helps the players in achieving necessary deterrence.\textsuperscript{17}

6 Concluding Remarks

Economics and legal scholars have long recognized that formal and informal sanctions play an important role in deterring inefficient or undesirable behavior. This paper has examined how the two sanctions can work together in providing the optimal deterrence. The paper has recognized that formal sanctions engender cost of dispute resolution while informal sanctions can lead parties to forego beneficial trade. The optimal regime makes a trade-off between these two costs. At the same time, the paper has identified that because marginal increase in formal sanctions also make existing lawsuits more effective (infra-marginal effect), relying on formal sanctions is generally more advantageous than relying on informal sanctions. While the primary focus of the paper as been on the design of the optimal deterrence system by private parties, the trade-offs identified extend to circumstances when the liability regime is mandated by law, as in products liability or criminal/regulatory sanctions.

\textsuperscript{17}When $q = q_b$ ($q = q_l$), consumers with $k > \rho d$ ($k > (1 - \rho)d$) will have a negative value suit. If the firm were to settle with all consumers, even those with negative value suits will threaten to sue. The firm, in turn, would not want to settle with all consumers. In equilibrium, not all lawsuits (or threats) will be settled and future consumers will observe some litigation outcomes.
Appendix A: Proofs

Proof of Proposition 1. Suppose, in equilibrium, \( e = e_h \). The consumer’s reservation value is given by
\[
E(v|e_h) + (1 - \pi)[F(\rho d)\rho d - F(\rho d)E(k|k \leq \rho d)] + \pi[F((1 - \rho)d)(1 - \rho)d - F((1 - \rho)d)E(k|k \leq (1 - \rho)d)].
\]
It consists of three terms: (1) the expected value of the good conditional on \( e = e_h \); (2) the expected net recovery from litigation when \( q = q_h \); and (3) the expected net recovery from litigation when \( q = q_l \). The firm’s discounted stream of payoffs from high care can be represented recursively as
\[
V_H^+ = p - c_h + (1 - \pi)\{\delta V_H^+ - F(\rho d)\rho d\} + \pi\{\delta V_H^- - F((1 - \rho)d)(1 - \rho)d\}
\]  
(1)

The subscript \( H \) stands for the assumption that the consumers observe the past quality realizations (“history”). Since the consumers observe all past qualities and lawsuits do not generate additional information about the firm’s behavior, reputational sanctions will be triggered whenever consumers observe low quality realization. Let \( V_H^- = \delta^T V_H^+ \). The firm’s payoff from deviating and providing low effort is
\[
p - c_l + \pi\{\delta V_H^+ - F(\rho d)\rho d\} + (1 - \pi)\{\delta V_H^- - F((1 - \rho)d)(1 - \rho)d\}
\]

The equilibrium requires that this deviation is not profitable. The firm will put in high care if
\[
p - c_h + (1 - \pi)\{\delta V_H^+ - F(\rho d)\rho d\} + \pi\{\delta V_H^- - F((1 - \rho)d)(1 - \rho)d\}
\geq p - c_l + \pi\{\delta V_H^+ - F(\rho d)\rho d\} + (1 - \pi)\{\delta V_H^- - F((1 - \rho)d)(1 - \rho)d\}
\]

which reduces to
\[
\delta(1 - \delta^T)V_H^+ \geq \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho d)
\]
When we solve equation (1) for \( V^+_H \), we get

\[
V^+_H = \frac{p - c_h - (1 - \pi)F(\rho d)\rho d - \pi F((1 - \rho)d)(1 - \rho)d}{1 - \delta + \pi \delta (1 - \delta^T)}
\]  

(2)

In equilibrium, the firm will set the price equal to the consumer’s reservation value: \( p = E(v|e_h) + (1 - \pi)[F(\rho d)\rho d - F(\rho d)E(k|k \leq \rho d)] + \pi[F((1 - \rho)d)(1 - \rho)d - F((1 - \rho)d)E(k|k \leq (1 - \rho)d)] \). The incentive compatibility condition becomes

\[
\delta(1 - \delta^T)\frac{E(v|e_h) - c_h - (1 - \pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1 - \rho)d)E(k|k \leq (1 - \rho)d)}{(1 - \delta) + \pi \delta (1 - \delta^T)} \geq \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho d)
\]

To simplify the expressions, let

\[
A(d) \equiv E(v|e_h) - c_h - (1 - \pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1 - \rho)d)E(k|k \leq (1 - \rho)d)
\]

\[
B_H(d) \equiv \max \left( \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho d), 0 \right)
\]

Now, the incentive compatibility condition can be written as

\[
\delta(1 - \delta^T)\frac{A(d)}{(1 - \delta) + \pi \delta (1 - \delta^T)} = B_H(d)
\]

In this expression, \( A(d) \) represents the firm’s per-period profit and \( B_H(d) \) represents the size of informal sanctions, if any. It will be useful to note that when we differentiate \( A(d) \) with respect to \( d \), we get

\[
A'(d) = \frac{\partial}{\partial d} \left( E(v|e_h) - c_h - (1 - \pi) \int_{0}^{\rho d} k f(k)dk - \pi \int_{0}^{(1 - \rho)d} k f(k)dk \right)
\]

\[
= -[(1 - \pi)f(\rho d)\rho^2 + \pi f((1 - \rho)d)(1 - \rho)^2]d
\]

The firm’s program involves setting \( d \) and the length punishment period, \( T \), to maximize \( V^+_H \), subject to the incentive compatibility condition. We can turn this constrained maximization
problem into an unconstrained maximization problem by solving the incentive compatibility condition for $\delta(1 - \delta^T)$ and plugging the result into equation (2). Doing so yields

$$V_H^+(d) = \frac{1}{1 - \delta} \{ A(d) - \pi B_H(d) \}$$

When we differentiate $V_H^+(d)$ with respect to $d$, assuming $B_H(d) > 0$, we get

$$V_H^{+'}(d) = \frac{1}{1 - \delta} \{ A'(d) - \pi B'_H(d) \}$$

$$= \frac{1}{1 - \delta} \left\{ A'(d) - \frac{\partial}{\partial d} \left( \frac{\pi \Delta_c}{1 - 2 \pi} - \pi(F((1 - \rho)d)(1 - \rho) - F(\rho d) \rho d) \right) \right\}$$

$$= \frac{1}{1 - \delta} \left\{ -[1 - \pi]f(\rho d)\rho^2 + \pi f((1 - \rho)d)(1 - \rho)^2 \right\}$$

$$= \frac{1}{1 - \delta} \left\{ +\pi f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2 \right\}$$

$$= \frac{1}{1 - \delta} \left\{ +\pi F((1 - \rho)d)(1 - \rho) - F(\rho d) \rho \right\}$$

When $B_H(d) = 0$, $V_H^{+'}(d) = \frac{-f(\rho d)\rho^2 d}{1 - \delta} < 0$: thus increasing damages above $\overline{d}$ is strictly dominated. Differentiating with respect to $d$ again yields

$$V_H^{+''}(d) = \frac{1}{1 - \delta} \left\{ -f'(\rho d)\rho^3 d - f(\rho d)\rho^2 + \pi f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2 \right\}$$

First, let $(\rho, \pi) \in (0, 1/2)^2$ and consider $d = 0$. We have $V_H^{+'}(0) = 0$ but $V_H^{+''}(0) = \frac{1}{1 - \delta} f(0)\rho^2(1 - \rho)^2 - \pi \rho^2$. At $d = 0$, the firm’s long-run profit is strictly increasing with respect to $d$ if $\pi > \frac{\rho^2}{1 - 2\rho}$ or, equivalently, $\rho < \sqrt{\pi^2 + \pi} - \pi$. Under that condition, setting $d = 0$ locally minimizes the firm’s long-run profit. The firm, therefore, will set $d > 0$. Let $d \in (0, \overline{d}]$ and consider changes in $(\rho, \pi)$. For any $\pi > 0$, as $\rho \to 0$, $V_H^{+'}(d) \to \frac{1}{1 - \delta} \pi F(d) > 0$. In contrast, as $\pi \to 0$ or $\rho \to 1/2$, $V_H^{+'}(d) \to \frac{1}{1 - \delta} \{-f(\rho d)\rho^2 d\} < 0$. Therefore, as $\rho \to 0$, $d \to \overline{d}$ but as $\pi \to 0$ or $\rho \to 1/2$, $d \to 0$. ■

Proof of Proposition 2. Suppose, in equilibrium, $e = e_h$. Each consumer’s reservation value is given by $E(v|e_h) + (1 - \pi)[F(\rho d)\rho d - F(\rho d)E(k|k \leq \rho d)] + \pi[F((1 - \pi) - \frac{1}{1 - \delta} \pi F(\rho d)\rho d \leq \rho d)]$.
\[ (1 - \rho)d - F((1 - \rho)d)E(k|k \leq (1 - \rho)d), \] and the firm, in equilibrium, will set the price equal to the reservation value. The firm’s discounted stream of payoffs from high care can be represented recursively as

\[
V^+_{N_1} = p - c_h + (1 - \pi)\{F(\rho d)\rho(\delta V^-_{N_1} - d) + F(\rho d)(1 - \rho)\delta V^-_{N_2} + (1 - F(\rho d))\delta V^+_{N_2}\} \\
+ \pi\{F((1 - \rho)d)(1 - \rho)(\delta V^-_{N_1} - d) + F((1 - \rho)d)\rho\delta V^-_{N_2} + (1 - F((1 - \rho)d))\delta V^+_{N_2}\}
\]

where the subscript \( N \) represents the assumption that the consumers do not observe past quality realizations (“no quality history”). Since the consumers observe all lawsuits filed, they can impose reputational sanctions against the firm even when there is no liability judgment against the firm, which happens with probability \( F(\rho d)(1 - \rho) \) when \( q = q_h \) and with probability \( F((1 - \rho)d)\rho \) when \( q = q_l \). Let \( V^-_{N_1} = \delta^{T_1} V^+_{N} \) and \( V^-_{N_2} = \delta^{T_2} V^+_{N} \). The firm’s payoff from providing low care (deviation) is

\[
p - c_l + \pi\{F(\rho d)\rho(\delta V^-_{N_1} - d) + F(\rho d)(1 - \rho)\delta V^-_{N_2} + (1 - F(\rho d))\delta V^+_{N_2}\} \\
+ (1 - \pi)\{F((1 - \rho)d)(1 - \rho)(\delta V^-_{N_1} - d) + F((1 - \rho)d)\rho\delta V^-_{N_2} + (1 - F((1 - \rho)d))\delta V^+_{N_2}\}
\]

The incentive compatibility condition, after some simplification, can be written as

\[
(1 - 2\pi) \begin{cases} 
(F((1 - \rho)d) - F(\rho d))\delta V^+_{N_2} \\
-(F((1 - \rho)d)\rho - F(\rho d)(1 - \rho))\delta V^-_{N_2} \\
+(F((1 - \rho)d) - F(\rho d))(d - \delta V^-_{N_1})
\end{cases} \geq \Delta c
\]

Comparing \( F((1 - \rho)d) - F(\rho d) \) with \( F((1 - \rho)d)\rho - F(\rho d)(1 - \rho) \), we see that

\[
[F((1 - \rho)d) - F(\rho d)] - [F((1 - \rho)d)\rho - F(\rho d)(1 - \rho)] = F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho > 0
\]

This implies that lawsuit and liability judgment is a stronger signal of low effort than lawsuit and no liability finding. Hence, the consumers will set \( T_1 \geq 0 \) but \( T_2 = 0 \). For a slight abuse of notation, let \( T_1 = T \) and \( V^-_{N_1} = \delta^T V^+_{N_1} \). With these additional simplifications, the
incentive compatibility condition can now be written as

$$\delta(1-\delta^T) V_N^+ \geq \frac{\Delta c}{(1-2\pi)(F((1-\rho)d)(1-\rho) - F(\rho d)\rho)} - d = \phi(d) - d$$

Like in the proof of proposition 1, we can solve equation (??) for \( V_N^+ \). Doing so and plugging in the reservation value of the consumer for the price gives us

$$V_N^+ = \frac{E(v|e_h) - c_h - (1-\pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1-\rho)d)E(k|k \leq (1-\rho)d)}{(1-\delta) + \delta((1-\pi)F(\rho d)\rho + \pi F((1-\rho)d)(1-\rho))}$$

(3)

When we plug this expression into the incentive compatibility condition and let \( T \to \infty \), the inequality converges to

$$\frac{\delta E(v|e_h) - c_h - (1-\pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1-\rho)d)E(k|k \leq (1-\rho)d)}{(1-\delta) + \delta((1-\pi)F(\rho d)\rho + \pi F((1-\rho)d)(1-\rho))} \geq \phi(d) - d$$

Since (a) \( \phi(\bar{d}) - \bar{d} = 0 \); (b) \( \phi(0) = \infty \); (c) \( E(v|e_h) - c_h - (1-\pi)F(\rho \bar{d})E(k|k \leq \rho \bar{d}) - \pi F((1-\rho)\bar{d})E(k|k \leq (1-\rho)\bar{d}) \geq 0 \), and (d) \( \frac{\delta}{\delta d} \left( \frac{E(v|e_h) - c_h - (1-\pi)F(\rho \bar{d})E(k|k \leq \rho \bar{d}) - \pi F((1-\rho)\bar{d})E(k|k \leq (1-\rho)\bar{d})}{(1-\delta) + \delta((1-\pi)F(\rho \bar{d})\rho + \pi F((1-\rho)\bar{d})(1-\rho))} \right) < 0 \), \( \exists d \in (0, \bar{d}) \) where the inequality is satisfied. Let \( \bar{d} \) be the minimum \( d \) that satisfies the inequality. Also, define

$$A(d) \equiv E(v|e_h) - c_h - (1-\pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1-\rho)d)E(k|k \leq (1-\rho)d)$$

$$B_N(d) \equiv \max \left( \frac{\Delta c}{(1-2\pi)(F((1-\rho)d)(1-\rho) - F(\rho d)\rho)} - d, 0 \right) = \max (\phi(d) - d, 0)$$

When we differentiate \( A(d) \) with respect to \( d \), we get

$$A'(d) = \frac{\partial}{\partial d} \left( E(v|e_h) - c_h - (1-\pi) \int_0^{\rho d} kf(k)dk - \pi \int_0^{(1-\rho)d} kf(k)dk \right)$$

$$= -[(1-\pi)f(\rho d)^2 + \pi f((1-\rho)d)(1-\rho)^2]d$$

Note that \( (1-\pi)f(\rho d)^2 + \pi f((1-\rho)d)(1-\rho)^2 = \frac{\partial}{\partial d} [(1-\pi)F(\rho d)\rho + \pi F((1-\rho)d)(1-\rho)] \).
Similarly, assuming that \( B_N(d) > 0 \),

\[
B'_N(d) = \phi'(d) - 1 = -\frac{\triangle c \cdot (f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2)}{(1 - 2\pi) \cdot (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho^2)} - 1 < 0
\]

The inequality is satisfied since we have assumed that \( f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2 > 0 \).

Now, we rewrite the incentive condition as

\[
\delta(1 - \delta^T) = \frac{A(d)}{(1 - \delta)(1 - \delta^T)((1 - \pi)F(\rho d)\rho + \pi F((1 - \rho)d)(1 - \rho))} = B_N(d)
\]

Solving for \( \delta(1 - \delta^T) \), we get

\[
\delta(1 - \delta^T) = \frac{(1 - \delta)B_N(d)}{A(d) - ((1 - \pi)F(\rho d)\rho + \pi F((1 - \rho)d)(1 - \rho))B_N(d)}
\]

Plugging this value from the constraint into equation (3) eliminates the constraint and simplifies \( V_N^+(d) \) to

\[
V_N^+(d) = \frac{1}{1 - \delta} \left\{ A(d) - [(1 - \pi)F(\rho d)\rho + \pi F((1 - \rho)d)(1 - \rho)]B_N(d) \right\}
\]

\[
= \frac{1}{1 - \delta} \left\{ A(d) - F(\rho d)\rho B_N(d) - \pi [F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho]B_N(d) \right\}
\]

\[
= \frac{1}{1 - \delta} \left\{ A(d) - F(\rho d)\rho [\phi(d) - d] - \frac{\pi \triangle c}{(1 - 2\pi)} + \pi [F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho]d \right\}
\]

The firm’s objective is to choose \( d \in [\underline{d}, \bar{d}] \) to maximize \( V_N^+(d) \). When \( B_N(d) = 0 \), \( V_N^+(d) = \frac{A(d)}{1 - \delta} \) and \( V_N^+(d) = \frac{-(1 - \pi) f(\rho d) \rho^2 + \pi f((1 - \rho)d)(1 - \rho)^2 d}{1 - \delta} < 0 \). When \( B_N(d) > 0 \), we have

\[
V_N^{+}(d) = \frac{1}{1 - \delta} \left\{ -[(1 - \pi) f(\rho d) \rho^2 + \pi f((1 - \rho)d)(1 - \rho)^2 d] - f(\rho d)\rho^2 B_N(d) - F(\rho d)\rho B'_N(d) + \pi [F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho] \right. \\
\left. + \pi [f((1 - \rho)d)(1 - \rho)^2 - f(\rho d)\rho^2]d \right\}
\]
where

$$V_N^+(d) = \frac{1}{1 - \delta} \left\{ \left[ (1 - \pi) F(pd) \rho + \pi F((1 - \rho)d)(1 - \rho) \right] - \frac{\partial}{\partial d} \left( \frac{\pi \Delta c}{1 - 2\pi} + F(pd) \rho \cdot \phi(d) \right) \right\}$$

$$= \frac{1}{1 - \delta} \left\{ \left[ (1 - \pi) F(pd) \rho + \pi F((1 - \rho)d)(1 - \rho) \right] + \frac{\Delta c}{1 - 2\pi} \cdot \frac{F(pd) \rho f((1 - \rho)d)(1 - \rho) - F((1 - \rho)d)(1 - \rho) f(pd) \rho^2}{(F((1 - \rho)d)(1 - \rho) - F(pd) \rho)^2} \right\}$$

Under the assumption that $$\frac{\partial}{\partial d} \left( \frac{F((1 - \rho)d)(1 - \rho)}{F(pd) \rho} \right) \geq 0$$ we get $$F(pd) \rho f((1 - \rho)d)(1 - \rho) - F((1 - \rho)d)(1 - \rho) f(pd) \rho^2 \geq 0$$ and $$V_N^+(d) > 0 \ \forall d$$. In that case, the firm will set $$d = \bar{d}$$. Note that $$\frac{\partial}{\partial d} \left( \frac{F((1 - \rho)d)(1 - \rho)}{F(pd) \rho} \right) \geq 0$$ implies that $$\frac{\partial}{\partial d} (F((1 - \rho)d)(1 - \rho) - F(pd) \rho) \geq 0$$, our initial assumption throughout the analysis. The reverse, however, is not necessarily true. The monotone ratio property is a stronger condition. As $$\rho \to 0$$, $$V_N^+(d) \to \frac{1}{1 - \delta} \pi F(d) > 0 \ \forall d \in [d, \bar{d}]$$. The firm will set $$d = \bar{d}$$.

**Proof of Corollary 1.** Recall that when consumers observed both past quality and litigation outcomes, the firm’s long-run profit was given by

$$V_H^+(d) = \frac{1}{1 - \delta} \{ A(d) - \pi B_H(d) \}$$

where

$$A(d) = E(v|e_h) - c_h - (1 - \pi) F(pd) E(k|k \leq pd) - \pi F((1 - \rho)d) E(k|k \leq (1 - \rho)d)$$

$$B_H(d) = \max \left( \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(pd) \rho) d, 0 \right)$$

When the consumers only observed past litigation outcomes, by contrast, we have

$$V_N^+(d) = \frac{1}{1 - \delta} \{ A(d) - [(1 - \pi) F(pd) \rho + \pi F((1 - \rho)d)(1 - \rho)] B_N(d) \}$$

where

$$B_N(d) = \max \left( \frac{\Delta c}{(1 - 2\pi)(F((1 - \rho)d)(1 - \rho) - F(pd) \rho) - d, 0} \right)$$
Recall also that regardless of the information assumption, conditional on \(d\), the firm’s per-period profit \((A(d))\) is the same. Assuming that \(d < \overline{d}\), the value functions can be rewritten as

\[
V_H^+(d) = \frac{1}{1-\delta} \left[ A(d) - \frac{\pi \Delta c}{1-2\pi} + \pi (F((1-\rho)d)(1-\rho) - F(\rho d)\rho d) \right]
\]

\[
V_N^+(d) = \frac{1}{1-\delta} \left[ A(d) - \frac{\pi \Delta c}{1-2\pi} + \pi (F((1-\rho)d)(1-\rho) - F(\rho d)\rho d - F(\rho d)\phi(d) - d) \right]
\]

Note that the term, \(F(\rho d)\rho(\phi(d) - d)\), represents the deadweight loss from faulty reputational sanctions (reputational sanctions that kick in when the firm is sued and found liable even though \(q = q_h\)). By comparison, we see that \(V_H^+(d) > V_N^+(d) \forall d < \overline{d}\). Hence, if we let \((d_H, d_N)\) to represent optimal damages under the respective information regime, we must have \(V_H^+(d_H) \geq V_H^+(d_N) > V_N^+(d_N)\) whenever \(d_N < \overline{d}\).

The respective first derivatives, with respect to \(d\), are:

\[
V_H^{+'}(d) = \frac{1}{1-\delta} \left\{ -f(\rho d)\rho^2 d + \pi [F((1-\rho)d)(1-\rho) - F(\rho d)\rho] \right\}
\]

\[
V_N^{+'}(d) = \frac{1}{1-\delta} \left\{ -f(\rho d)\rho^2 \phi(d) - F(\rho d)\rho B_N'(d) + \pi [F((1-\rho)d)(1-\rho) - F(\rho d)\rho] \right\}
\]

Conditional on \(d\), we can have \(V_H^{+'}(d) \geq V_N^{+'}(d)\), because \(-f(\rho d)\rho^2 \phi(d) < -f(\rho d)\rho^2 d\) but \(-F(\rho d)\rho B_N'(d) > 0\). Hence, \(d_H \geq d_N\).
Appendix B: Alternate Reputational Punishment Strategies
(For Online Publication)

Throughout the paper, we have assumed that when consumers observe low quality realization (or judgment against the firm), they can engage in reputational punishment against the firm by not purchasing from the firm for $T \geq 0$ periods. Although this type of punishment strategy is commonly used in applied game theory and relational contracting literature, the strategy is not renegotiation-proof and also may not be subgame perfect. Since the consumers rationally believe that the firm has chosen $e = e_h$ in equilibrium and that there is potential surplus from trade, when the no-trade punishment is to kick in, the consumers and the firm will have an incentive to renegotiate out of the punishment phase. Furthermore, the firm may be able to preempt such punishment by making a contract offer with sufficiently high damages. When the reputational punishment is to start but the firm, in response, makes a contract offer with $d = d$, consumers should know that formal sanctions are sufficient to solve the commitment problem and should be willing to purchase from the firm. Reputational punishment based on no purchase may not be subgame perfect.

To address these issues, in this appendix, we examine two other types of reputational punishment strategies: one that relies on maximal formal sanctions ($d = \overline{d}$) but is inefficient (due to high litigation cost) and the other that relies on no formal sanctions ($d = 0$) but is efficient. For the sake of brevity, we assume, throughout this appendix, that the consumers observe both past quality realizations and litigation outcomes. If the consumers only observe past litigation outcomes, the analysis for the maximal formal sanctions strategy will be analogous. On the other hand, because consumers cannot engage in reputational sanctions without litigation, efficient (no formal sanctions) strategy will not be feasible.

Alternative 1: Reputational Punishment with Maximal Damages

Consider the following strategy. The firm initially starts with $d \in (0, \overline{d})$. If consumers observe $q = q_l$ in any period, during the punishment phase that would last for $T \geq 0$
subsequent periods, the firm makes a contract offer with $d = \bar{d}$. While in the punishment phase, consumers believe that for any offer with $d < \bar{d}$, the firm is choosing $c_l$ and do not purchase from the firm. After $T$ punishment period, the players revert to the original equilibrium with $d \in (0, \bar{d})$. The following proposition demonstrates that the firm faces the identical trade-off as in the case where the consumers are using boycott as reputational punishment.

**Proposition 3** Suppose, during the punishment phase that lasts $T \geq 0$ periods, the firm makes an offer with $d = \bar{d}$. The firm’s maximization problem is identical to the one where consumers use boycott (no trade) as punishment device.

**Proof.** In the punishment phase, since the consumers know that the firm is choosing $e_h$ with $d = \bar{d}$, they are willing to pay upto $E(v|e_h) + (1 - \pi)[F(\rho \bar{d}) \rho \bar{d} - F(\rho \bar{d}) E(k|k \leq \rho \bar{d})] + \pi[F((1 - \rho) \bar{d})(1 - \rho) \bar{d} - F((1 - \rho) \bar{d}) E(k|k \leq (1 - \rho) \bar{d})]$. The firm earns, in each punishment period,

$$A(\bar{d}) = E(v|e_h) - c_h - (1 - \pi)F(\rho \bar{d}) E(k|k \leq \rho \bar{d}) - \pi F((1 - \rho) \bar{d}) E(k|k \leq (1 - \rho) \bar{d})$$

when it sets price equal to the consumer’s reservation value. Given that the punishment phase will last for $T$ periods, the present value of the punishment phase profit, as of the beginning of the punishment phase, is $\frac{1 - \delta^T}{1 - \delta} A(\bar{d})$.

In the cooperation phase, the firm’s discounted stream of payoffs from $e_h$ is given by

$$V^+ = p - c_h + (1 - \pi)\{\delta V^+ - F(\rho d) \rho d\} + \pi \{\delta V^- - F((1 - \rho) d)(1 - \rho) d\}$$

where $V^- = \frac{1 - \delta^T}{1 - \delta} A(\bar{d}) + \delta^T V^+$. The firm’s payoff from providing low care (deviation) is

$$p - c_l + \pi \{\delta V^+ - F(\rho d) \rho d\} + (1 - \pi)\{\delta V^- - F((1 - \rho) d)(1 - \rho) d\}$$
The firm will put in high care if

\[ p - c_h + (1 - \pi)\{\delta V^+ - F(\rho d)\rho d\} + \pi\{\delta V^- - F((1 - \rho)d)(1 - \rho)d\} \geq p - c_t + \pi\{\delta V^+ - F(\rho d)\rho d\} + (1 - \pi)\{\delta V^- - F((1 - \rho)d)(1 - \rho)d\} \]

which can be written as

\[ \delta(1 - \delta^T) \left( V^+ - \frac{A(d)}{1 - \delta} \right) \geq \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho d) \]

From \( V^+ = p - c_h + (1 - \pi)\{\delta V^+ - F(\rho d)\rho d\} + \pi\{\delta V^- - F((1 - \rho)d)(1 - \rho)d\}, V^- = \frac{1 - \delta^T}{1 - \delta} A(d) + \delta^T V^+ \), and \( p = E(v|e_h) + (1 - \pi)[F(\rho d)\rho d - F(\rho d)E(k|k \leq \rho d)] + \pi[F((1 - \rho)d)(1 - \rho)d - F((1 - \rho)d)E(k|k \leq (1 - \rho)d)], \) we get

\[ V^+(d) = \frac{\pi \delta(1 - \delta^T)A(d)}{(1 - \delta)((1 - \delta) + \pi \delta(1 - \delta^T))} + \frac{A(d)}{(1 - \delta) + \pi \delta(1 - \delta^T)} \]

where \( A(d) \equiv E(v|e_h) - c_h - (1 - \pi)F(\rho d)E(k|k \leq \rho d) - \pi F((1 - \rho)d)E(k|k \leq (1 - \rho)d). \)

With this expression, the incentive compatibility condition becomes

\[ \frac{\delta(1 - \delta^T)}{(1 - \delta) + \pi \delta(1 - \delta^T)} \{ A(d) - A(d) \} = B(d) \]

where \( B(d) \equiv \max \left( \frac{\Delta c}{1 - 2\pi} - (F((1 - \rho)d)(1 - \rho) - F(\rho d)\rho d, 0) \right). \) When we solve for \( \delta(1 - \delta^T), \) we get

\[ \delta(1 - \delta^T) = \frac{(1 - \delta)B(d)}{A(d) - \pi B(d) - A(d)} \]
When we use this expression to solve for $V^+(d)$, we get

$$V^+(d) = \frac{1}{(1-\delta) + \pi \delta(1-\delta^T)} \left( \frac{\pi \delta (1-\delta^T) A(d) + A(d)}{(1-\delta)} \right)$$

$$= \frac{1}{(1-\delta) + \frac{\pi (1-\delta) B(d)}{A(d) - \pi B(d) - A(d)}} \left( \frac{\pi (1-\delta) B(d)}{(1-\delta)} A(d) + A(d) \right)$$

$$= \frac{A(d) - \pi B(d) - A(\bar{d})}{(1-\delta)(A(d) - A(\bar{d}))} \left( \frac{\pi B(d)}{A(d) - \pi B(d) - A(d)} A(\bar{d}) + A(d) \right)$$

$$= \frac{A(d) - \pi B(d)}{(1-\delta)}$$

which is identical to the one we get when consumers are using boycott as reputational sanctions. Hence, the optimization problem for the firm is the same as before. □

This invariance result can be explained as follows. Given that the firm extracts all the consumer surplus and becomes the residual claimant, for any punishment strategy that involves additional deadweight loss, the size of the punishment is equal to the size of the additional deadweight loss. Hence, whether the additional punishment comes from boycott (no trade) or from additional litigation cost (when $d = \bar{d}$), best reputational punishment will set the additional deadweight loss just enough to make up for the sanctions shortfall ($B(d)$). The only difference is that with the maximal damages reputational sanctions, because the firm is making a positive profit ($A(\bar{d})$) even during the punishment phase, the length of the punishment period ($T$) is longer.

**Alternative 2: Efficient Reputational Punishment**

One main drawback of relying on either consumer boycott (as done in the main body) or maximal damages (in the previous case) as reputational punishment mechanism is not neither are efficient. Both involve certain amount of deadweight loss during the punishment phase, and this may provide an incentive for the players to renegotiate away from using
the stipulated punishment strategies and eliminate the deadweight loss. In fact, to achieve maximal efficiency, the firm should not rely on any formal sanctions since litigation cost reduces the surplus from trade, unless litigation provides additional information to the future consumers about the firm’s behavior (as in case 2). To address these issues, we examine an alternate punishment strategy that involves neither litigation nor boycott.

Suppose the firm always sets \( d = 0 \) and the players coordinate on the following punishment strategy. Initially, the firm offers a “high” price \( p^+ = E(v|e_h) \) and the consumers purchase. If consumers observe \( q = q_h \), they believe the firm is choosing \( e_h \) and keep purchasing from the firm so long as \( p^+ \leq E(v|e_h) \). When they observe \( q = q_l \), on the other hand, they form the belief that unless the firm offers a “low” price \( p^- < p^+ \), the firm is choosing \( e_l \) and do not purchase from the firm. The firm, in the punishment phase, offers the good at \( p^- \in [c_h, E(v|e_h)] \) and induce the consumers to purchase. While in the punishment phase, if consumers observe \( q = q_h \), they allow the firm to revert back to the original equilibrium (of \( p^+ = E(v|e_h) \)) with probability \( \eta \in (0, 1) \).\(^{18}\) If they observe \( q = q_l \), on the other hand, the equilibrium remains in punishment phase. The reversion probability \( \eta \) is chosen so as to provide effort incentive to the firm during the punishment phase. The following proposition demonstrates that while this punishment strategy is efficient, it imposes (substantial) restriction on the discount factor.

**Proposition 4** Suppose \( d = 0 \) and the players use the efficient punishment strategy, with the reversion probability of \( \eta \in (0, 1) \) in the punishment phase. \( \exists \tilde{\delta} \in (0, 1) \) such that the firm solves the commitment problem with the efficient punishment strategy for any \( \delta \geq \tilde{\delta} \). However, \( \tilde{\delta} > \delta \) and if \( \delta \in [\delta, \tilde{\delta}) \), the firm has to rely on at least some formal sanctions to solve the commitment problem.

\(^{18}\) This rehabilitation method is similar to that in Cai and Obara (2009). One main difference is that, unlike more conventional repeated game theory models, which allows firms to make an offer that only consists of a price, our firm can credibly commit to choose \( e_h \) by setting \( d = \tilde{d} \). This problem is briefly discussed at the end.
Proof. Suppose the players use the efficient punishment strategy. Consider the cooperation phase. For the firm to choose $e_h$, we need

$$p^+ - c_h + (1 - \pi)\delta V^+ + \pi \delta V^- \geq p^+ - c_l + \pi \delta V^+ + (1 - \pi)\delta V^-$$

$$\iff \delta (V^+ - V^-) \geq \frac{\Delta c}{1 - 2\pi}$$

where $V^+$ ($V^-$) stands for the firm’s long-run profit in the cooperation (punishment) phase and $p^+$ stands for the price the firm charges in the cooperation period. The comparable condition in the punishment phase, with the reversion probability of $\eta \in (0, 1)$, can be written as

$$p^- - c_h + (1 - \pi)(\eta \delta V^+ + (1 - \eta)\delta V^-) + \pi \delta V^- \geq p^- - c_l + \pi(\eta \delta V^+ + (1 - \eta)\delta V^-) + (1 - \pi)\delta V^-$$

$$\iff \eta \delta (V^+ - V^-) \geq \frac{\Delta c}{1 - 2\pi}$$

Since $\eta \delta (V^+ - V^-) < \delta (V^+ - V^-) \forall \eta \in (0, 1)$, we can focus only on the punishment phase incentive condition.

Using $V^+ = p^+ - c_h + (1 - \pi)\delta V^+ + \pi \delta V^-$ and $V^- = p^- - c_h + (1 - \pi)(\eta \delta V^+ + (1 - \eta)\delta V^-) + \pi \delta V^-$, we get

$$V^+ = \frac{\pi \delta (p^- - c_h) + ((1 - \delta) + (1 - \pi)\eta \delta)(p^+ - c_h)}{(1 - \delta)(1 - (1 - \pi)(1 - \eta)\delta)}$$

$$V^- = \frac{(1 - (1 - \pi)\delta)(p^- - c_h) + (1 - \pi)\eta \delta (p^+ - c_h)}{(1 - \delta)(1 - (1 - \pi)(1 - \eta)\delta)}$$

Now the (punishment phase) incentive condition can be written as

$$\frac{\eta \delta \triangle p}{(1 - (1 - \pi)(1 - \eta)\delta)} \geq \frac{\Delta c}{1 - 2\pi}$$

where $\triangle p \equiv p^+ - p^-$. Assuming that the players will choose the maximum $\eta$ feasible to
satisfy the inequality, we get
\[
\eta(\delta) = \frac{\Delta c/1-2\pi}{\delta \left[ \Delta p - (1-\pi) \frac{\Delta c}{1-2\pi} \right]}
\]
Note that \( \eta'(\delta) < 0 \), \( \eta(1) = \frac{\pi \Delta c/(1-2\pi)}{\Delta p - (1-\pi) \frac{\Delta c}{1-2\pi}} \), and \( \lim_{\delta \to 0} \eta(\delta) = \infty \). When \( \eta = 1 \), we get \( \delta = \frac{\Delta c/(1-2\pi)}{\Delta p \Delta p} \equiv \tilde{\delta} \). For the efficient reputational punishment to work, we need \( \delta \geq \tilde{\delta} \). Recall that, if the consumers were to use grim-trigger strategy, to induce the firm to choose \( e_h \), we needed \( \delta E(v|e_h) - c_h \geq \Delta c/1-2\pi \), which can be written as \( \delta \geq \frac{\Delta c/(1-2\pi)}{E(v|e_h) - c_h + \Delta c/(1-2\pi)} \equiv \tilde{\delta} \in (0,1) \).

When \( p^+ = E(v|e_h) \) and \( p^- = c_h \), \( \hat{\delta} = \frac{\Delta c/(1-2\pi)}{E(v|e_h) - c_h} > \tilde{\delta} \). As \( p^- \) rises, for instance to \( c_h + A(\hat{d}) \) so as to guarantee the firm the profit from setting \( d = \tilde{d} \), \( \hat{\delta} \) rises. When \( \delta \in [\tilde{\delta}, \hat{\delta}) \), efficient punishment strategy will not be feasible while relying on either boycott or maximal damages as reputational punishment remains feasible.

There are two reasons why the efficient punishment strategy requires a more patient firm. First, to provide effort incentive to the firm during the punishment phase, consumers must allow some chance of reversion (\( \eta > 0 \)) to the high price state. This is tantamount to having an upper bound on the punishment period \( T \). With \( \eta \) bounded away from zero (or \( T \) from \( \infty \)), the firm must be sufficiently patient for the efficient punishment strategy to work. The second reason is that the firm may need to earn some profit in the punishment state. Suppose \( p^- = c_h \). If the firm were to make an offer with \( d = \tilde{d} \), consumers should rationally believe that the firm is choosing \( e_h \). The firm can earn \( A(\tilde{d}) \) in each punishment period through this deviation. To deal with this subgame perfection issue, the efficient punishment strategy should allow the firm to set \( p^- = c_h + A(\tilde{d}) \). With this additional restriction, the firm will have to be even more patient (\( \hat{\delta} \) higher) for the efficient punishment strategy to be feasible.
References


