Competitive Policy Entrepreneurship[1]

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Abstract

The process of developing new policies in political organizations often involves multiple actors who craft competing proposals for consideration by a decisionmaker. These actors sometimes attempt to get their preferred policies enacted by making costly but productive up-front investments to improve their quality. We term this process "competitive entrepreneurship" and its participants "policy entrepreneurs," and develop a model to explore its implications for political institutions and policymaking. In particular, we consider how entrepreneurs' ideological extremism and their costs of developing quality affect patterns of competition, policy outcomes, and the welfare of the organizational decisionmaker. In equilibrium, ideologically extreme proposals are both higher quality and better for the decisionmaker. We also find that ideologically extreme entrepreneurs develop extreme proposals, force their opponents to generate moderate proposals, and result in larger overall average investments in quality. As a result, ideological extremism of policy entrepreneurs can be beneficial to the decisionmaker. A lower marginal cost of developing quality also results in greater quality investments and consequently benefits the decisionmaker, but some of these benefits are extracted by an entrepreneur in the form of ideological rents. We also show that when the contest is highly asymmetric, so that one entreprenuer almost always wins, the decisionmaker nonetheless can benefit substantially from the presence of competitive entrepreneurship. In our analysis, we develop techniques that can be applied to other contests in which individuals choose what policies to propose and try to get them enacted by exerting costly up-front effort, e.g., valence competition in elections and expenditures in lobbying contests.

1 Introduction

In political organizations, the process of developing new policies typically involves multiple competing actors. For example, when dealing with a particular issue, a legislature may consider bills drafted by different committees or interest groups. In bureaucratic politics, each subunit within a government agency may develop its own policy proposal for consideration by the agency head. Moreover, this pattern is not restricted to the public sector; on the contrary, many NGOs, universities, and firms have different factions that exert effort to craft competing proposals that they hope will be implemented.

We use the term "policy entrepreneur" to refer to an individual, faction, or interest group that takes the initiative to develop a policy, without any guarantee that it will be adopted. Of course, policy entrepreneurs often disagree – both with each other and with decisionmakers – about a variety of things. These disagreements may be ideological, or they may be about the organization's mission and the relative importance of different objectives. Yet despite their disagreements, members of a political organization usually have some interests in common. To the extent that there are overarching organizational goals, they (ceteris paribus) prefer policies that more effectively achieve them. When possible they prefer to save money, or to make money in the case of a for-profit firm. And, other things being equal, they prefer to enhance the organization's status and prestige.

To understand competition in such organizations, we model the development of high-quality policy alternatives in an organization as an *all pay contest* (Siegel 2009). Policies in the model have two components: an *ideological* dimension over which players have different preferences, and a *quality* dimension that is valued by all players. Two competing policy entrepreneurs simultaneously choose specific ideological locations at which to develop policies, and how much to invest in producing quality. Their investments are costly and cannot be combined or transferred to other policies. An organizational decisionmaker then chooses one of the entrepreneurs' proposals, a reservation policy, or any other ideological location for which no quality has been developed.

The canonical approach to modeling the endogenous development of high-quality policies is Crawford and Sobel's (1982) model in which policies and outcomes are ordered in a unidimensional space, and the link between policies and outcomes is a common additive shift. Information is thus *invertible* across policy options (Callander 2008), in the sense that knowing how to achieve a liberal outcome is also sufficient to know how achieve a conservative one. The canonical approach is well-suited to understanding the strategic use of expertise when the appropriate policy to enact depends on an unknown underlying factor, such as the number of Soviet nuclear missiles or the severity of global warming. The key strategic tension in such models is that privately informed experts worry that their information will be expropriated to implement outcomes that do not reflect their preferences.

Our model, in contrast, assumes that quality is *policy-specific* (Ting 2011; Hirsch and Shotts 2012); this is better suited to empirical domains where information and expertise are not readily transferable across different approaches to the same organizational problem.[1] For example, information about how to design an effective and equitable school voucher program cannot be used to improve the quality of public schools. Similarly, knowing how to design a negative campaign advertisement that voters do not find distasteful is useless for increasing the persuasiveness of positive messages that a campaign organization could adopt. Or, if we consider adoption of a "policy" to be the election of a particular party to control the government, then a party that makes productive investments in its own capacity to govern – e.g., by developing a well thought-out platform or by improving recruitment and training of its candidates and bureaucrats – knows that the benefits of its investments are only realized in the event that it actually wins office.

Because quality is policy-specific in our model, an entrepreneur does not need to worry about being expropriated, but rather attempts to *exploit* her investments to encourage the decisionmaker to select her policy. This effect is akin to Aghion and Tirole's (1997) "real authority," in that a decisionmaker who wishes to benefit from an entrepreneur's efforts must select her policy. However, the investments are wasted if the entrepreneur's policy is not selected.

A central feature of the model is how entrepreneurs resolve the tension between gaining the

[1] See Callander (2011a, 2011b) for models in which learning about one policy option provides information that is useful for small policy changes, but not necessarily for major ones.

support of the decisionmaker through productive quality investments, versus gaining it through ideological concessions. Inducing the decisionmaker to select a policy that reflects the entrepreneur's ideological preferences is the primary motivation for making productive investments in quality, but higher quality is necessary to gain support for more ideologically extreme policies. An important intermediate result of the analysis is that ideologically extreme policies are not bad for a centrist decisionmaker – in equilibrium, when more extreme policies are developed they are not only higher quality, but also strictly better for the decisionmaker.

After developing the model and providing a general characterization of equilibrium, we analyze the case in which the two entrepreneurs have the same marginal cost of developing quality and have ideological ideal points that are symmetrically located on either side of the decisionmaker. This is the purest form of competitive entrepreneurship because neither side is advantaged. Despite these simplifications, the model is distinct from previous work on all-pay contests in several respects. First, unlike Siegel (2009) the entrepreneurs do not have fixed values for winning and losing – they are *policy motivated,* and thus care about the characteristics of the policy that is ultimately implemented. In this respect, our model is related to Baye, Kovenock, and de Vries (2012), who consider a symmetric all-pay contest where a competitor's bid enters a players' utility as an affine function that depends on the identity of the winner. However, strategies in our model are two dimensional, and the resulting spillovers are not a simple linear function of entrepreneurs' effort. Thus our model is better suited to political decisionmaking, where actors' utility depends on both the spatial location and the quality of the enacted policy. Much of our analysis focuses on the effects of intra-organizational polarization of entrepreneurs' policy preferences, a topic that is not addressed in previous models of all-pay contests.

The symmetric case of our model has a unique equilibrium in symmetric mixed strategies. In the equilibrium, each entrepreneur always develops a policy, and mixes uniformly over an interval of ideologies between herself and the decisionmaker. The entrepreneurs invest in identical levels of quality on policies that are equally distant from the decisionmaker, and produce higher quality on more ideologically-extreme policies. More ideologically-extreme policies are also better for the decisionmaker. Unsurprisingly, more ideologically extreme entrepreneurs produce policies that are more ideologically-extreme, in a first-order stochastic sense. However, somewhat surprisingly, they also produce policies that are first-order stochastically better for the decisionmaker. Their extremism gives them a greater incentive to make productive investments to capture ideological rents, and competition prevents them from fully extracting the benefits of their additional investments.

Decreasing the entrepreneurs' (symmetric) marginal cost of developing quality has a similar equilibrium effect as increasing their ideological extremism – policies become first-order stochastically more extreme, but the decisionmaker's utility nevertheless increases. Interestingly, decreasing costs has a non-monotonic effect on the entrepreneurs' equilibrium utility, because lower costs make it cheaper to compensate the decisionmaker for ideological losses, but also increase the intensity of competition. When cost are high to begin with, a competition effect dominates and decreases in the marginal cost make the entrepreneurs worse off. However, when cost levels become sufficiently low, a cost effect dominates and further decreases make the entrepreneurs better off.

We then extend the model to consider asymmetric preferences and asymmetric costs, and identify several interesting patterns of competition. For generic asymmetric parameters, one entrepreneur always enters, mixing over different proposals. The other entrepreneur sometimes sits out, and when she enters she too mixes over different proposals. The probability that the less-engaged entrepreneur sits out is a function of the two entrepreneurs' preferences and costs. For extremely asymmetric parameters, the less-engaged entrepreneur's probability of sitting out converges to 1. However, this does not imply that the model functions as if the less-engaged entrepreneur did not exist (which would mean that the more-engaged entrepreneur could extract all quality benefits for herself, in the form of ideological rents). Rather, the seldomly-realized threat of potential entry can induce the more-engaged entrepreneur to develop policies that benefit the decisionmaker.

We also show that the more-engaged entrepreneur may not dominate the contest. Rather, if she is more ideologically-motivated yet faces a sufficiently large cost disadvantage, her opponent is more likely to win the contest, despite being less likely to develop a policy proposal. On the other hand, if the more-engaged entrepreneur is both more ideologically-extreme and more cost-effective at developing quality, then she will develop policies that are (in a first order stochastic sense) more extreme and also better for the decisionmaker.

Our analysis also provides a variety of comparative statics. Each entrepreneur is worse off when her opponent's costs decrease. Lower costs make it cheaper to develop any given level of quality, and thus easier to realize ideological gains by investing in quality. As one entrepreneur's costs decrease, she develops more ideologically extreme policies, and her opponent develops more ideologically moderate ones. Eventually, the higher-cost entrepreneur is driven out of the contest – her probability of developing no policy increases, and her policies are on average worse for the decisionmaker. The effect of increasing one entrepreneur's ideological extremism is, for the most part, similar to decreasing her costs. Her policies become more ideologically extreme, her opponent's policies become more moderate, and her opponent is worse off.

The paper proceeds as follows. Section 2 introduces the model and Section 3 develops concepts, notation, and presents some general results. Sections 4 further characterizes equilibria. Section 5 considers the symmetric model, and Section 6 considers asymmetric variants. Section 7 concludes.

2 The Model

We analyze a two-stage game of policy development and choice played by a decisionmaker and two competing entrepreneurs.[2] Policies in the model have two components: *ideology* $y \in \mathbb{R}$ and quality $q \in [0, \infty) = \mathbb{R}^+$. Thus, a policy is a point in a subset of two-dimensional real space, $b = (y, q) \in \mathbb{R} \times \mathbb{R}^+ = \mathbb{B}$. Players' utility functions $U_i(b)$ over the two dimensions are additive and quality is valued equally by all players:

$$U_i(b) = q - (x_i - y)^2,$$

[2] We believe, but have not fully proved, that if there are N entrepreneurs with symmetric costs and distinct ideological preferences then the results of our model are robust, in the sense that there exists an equilibrium in which only the two most extreme entrepreneurs develop policies, while the others stay out of the contest by virtue of their lower incentive to invest in quality to realize ideological gains.

where x_i denotes the ideological ideal point of player i. We assume without loss of generality that the decisionmaker's ideal ideology is $x_D = 0$, and furthermore assume that the entrepreneurs are located on opposite sides of the decisionmaker, i.e., $sign\,(x_i) \neq sign\,(x_j)$.

In the *policy development* stage, each entrepreneur $i \in N = 2$ simultaneously develops a single policy $b_i = (y_i, q_i) \in \mathbb{B}$ with ideology y_i and quality $q_i \geq 0$. We assume for simplicity that the cost of developing a policy b_i with quality q_i is $c_i\,(q_i) = \alpha_i q_i$ where $\alpha_i > 1$. Thus, the cost is linear and independent of ideology y_i, and policies with 0-quality are "free" to develop. The *net benefit* of producing quality is equal to $(1 - \alpha_i)\,q_i < 0$, so an entrepreneur will only develop quality to increase the probability that her policy will be selected.

In the *policy choice* stage, the decisionmaker chooses from the set of newly developed policies $\mathbf{b} \in \mathbb{B}^N$ or a reservation policy b_0 equal to the decisionmaker's ideal ideology $(0, 0)$ with $0-$quality. These modeling choices reflect the implicit assumptions that the decisionmaker can choose freely from the $0-$quality policies, and that quality is *policy-specific* (Hirsch and Shotts 2012).

Literature With only one entrepreneur, our model would be technically similar to Snyder's (1991) model of vote-buying without price discrimination – the sole entrepreneur produces just enough quality to induce the decisionmaker to choose her policy over the reservation policy, and would have to balance the costs of developing quality against the ideological benefits of moving policy in her direction. (See, e.g., the single-proposer model of legislative policy choice that Hitt, Volden, and Wiseman (2011) use to analyze the effects of variation in the proposer's effectiveness or ability to craft high-quality proposals).

In contrast, we focus on competitive policy entrepreneurship, i.e., what happens when different entrepreneurs or factions can develop new policy proposals in a political organization. In our model, two entrepreneurs compete directly for the support of the decisionmaker by simultaneously making costly, quality-increasing investments that are specific to a particular ideology.[3] Because the cost

[3] Our assumption of simultaneous policy development contrasts with sequential models of vote-buying (Groseclose and Snyder 1996), valence-based electoral competition (Wiseman 2006), and judicial opinion-writing (Lax and Cameron 2007). Although sequential models have the benefit of being relatively analytically straightforward, in

of investing in quality is paid up-front, the game is an all-pay contest (Siegel 2009, 2010).

Our model has two primary differences from previous work on all-pay contests. First, entrepreneurs in our model are *policy motivated* rather than rent seeking (as in Tullock 1980 and Baye, Kovenock, and de Vries 1993). They care about which policies are implemented even if they lose the contest, and thus the contest features "spillovers" (Baye, Kovenock, and de Vries 2012). Our model is therefore better suited to analyzing policy entrepreneurship within political organizations, where potential entrepreneurs have both divergent ideological interests and common organizational interests. Second, in our model the investments made to gain influence are *productive*, and not simply transfers to the decisionmaker. Thus, ceteris paribus, an entrepreneur is less motivated to develop a policy when she expects her ideological opponent to develop a high-quality policy.

3 Preliminary Analysis

In this section we introduce notation, and provide general necessary and sufficient conditions for equilibrium as well as a general characterization. All proofs are in the Appendix.

A strategy for the decisionmaker $w(\mathbf{b}) : \mathbb{B}^N \to \Delta(N \cup 0)$ is a mapping from each profile of policies $\mathbf{b}$ to a probability distribution over the winning entrepreneur, where $w(\mathbf{b}) = 0$ denotes choosing the reservation policy b_0. We introduce additional notation and terminology to characterize decisionmaker strategies that are subgame perfect.

Definition 1 *Let the* ***score*** $s(y,q)$ *of a policy be the utility it gives to the decisionmaker, i.e.,*

$$s(y,q) = U_D(y,q) = q - y^2.$$

A decisionmaker strategy $w(\mathbf{b})$ *is subgame perfect i.f.f only policies with the highest score win, i.e.,*

$$\forall\, \mathbf{b} \text{ and } i,\ w_i(\mathbf{b}) > 0 \text{ i.f.f. } (y_i, q_i) \in \underset{(y,q)\in \mathbf{b}\cup b_0}{\arg\max} \{s(y_i, q_i)\}.$$

many general settings a simultaneous structure is more natural.

An entrepreneur i will therefore win the policy contest if and only if her policy gives the decisionmaker higher utility than both her opponent's policy and the reservation policy. In the event of ties, the decisionmaker may randomize arbitrarily. We borrow Siegel's (2009) terminology of "scores" to refer to the decisionmaker's utility, which plays a similar role in the analysis. First, developing a policy with a higher score is strictly worse for an entrepreneur conditional on winning the contest. Second, the entrepreneur who develops the policy with the highest score wins provided that the score is at least as high as the decisionmaker's utility from the reservation policy, i.e., $s(b_i) \geq s(0,0) = 0$.

Unlike Siegel (2009), however, a policy is more than just a score – there are a *continuum* of policies with different ideologies that lie on the same indifference curve for the decisionmaker, and thus generate the same score. These policies have different costs to develop; a policy with ideology y and score s must have quality equal to $s + y^2$, and thus entrepreneur i's cost to develop it is $\alpha_i(s + y^2)$. In addition, the policies are valued differently by different players; entrepreneur i's utility for a policy (s, y) is equal to $U_i(y, s + y^2) = -x_i^2 + s + 2x_i y$. It is therefore helpful to introduce notation for these quantities, which allow us to think of an entrepreneur's problem as the choice of a *score curve* s and an ideology y to develop along that score curve.

Definition 2 *Player i's utility for a policy (s, y) with score s and ideology y is*

$$V_i(s, y) = U_i\left(y, s + y^2\right) = -x_i^2 + s + 2x_i y.$$

The up-front cost to an entrepreneur of developing the policy herself is $-\alpha_i(s + y^2)$.

Figure 1 depicts the game in ideology-quality space for entrepreneurs who are located equidistant from the decisionmaker on either side. The decisionmaker's indifference curves – i.e., the policies with equal score – are depicted by the green lines.

3.1 Necessary and Sufficient Conditions

An entrepreneur's pure strategy b_i is a two-dimensional element (y_i, q_i) of $\mathbb{B}$ consisting of an ideology and a level of quality; a mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$. A strategy profile is $(\sigma, w(\mathbf{b}))$, a strategy for every entrepreneur and a decisionmaker decision rule $w(\mathbf{b})$ satisfying Definition 1.

We first establish that in any equilibrium, there is 0 probability that there are two distinct available policies over which the decisionmaker is indifferent. The absence of "score ties" is an intuitive consequence of the all-pay nature of investing in quality – if an entrepreneur knew that her policy might tie with her opponent's policy or the reservation policy, she could invest up front in ε-more quality to eliminate the tie.[4]

Lemma 1 *In equilibrium, the probability the entrepreneurs develop new policies $b_i \neq b_0$ with the same score as the reservation policy $(s(b_i) = s(b_0))$ or each other $(s(b_i) = s(b_j))$ is 0.*

Lemma 1 allows us to solve for two-dimensional equilibrium strategies by applying a simple substitution method to an entrepreneur's choice (s_i, y_i) of score and ideology. The reason for applying a substitution method to (s_i, y_i) rather than ideology and quality (y_i, q_i) is as follows – given the opponent's strategy σ_{-i}, two different possible policies (s_i, y_i') and (s_i, y_i'') with the same score must generically win the policy contest with the same probability. This is 0 if $s_i < 0$ is worse than the reservation policy, and if $s_i > 0$ it is the probability $P(s(b_{-i}) \leq s_i)$ that her opponent $-i$ produces a lower-score policy. This property generates the following essential Lemma.

Lemma 2 *Let $F_i(s)$ denote the CDF of $\max\{0, s(b_i)\}$. At any score $s_i > 0$ where the score CDF $F_{-i}(\cdot)$ of i's opponent has no atom, developing the policy $(s_i, y_i^*(s_i))$ is strictly better than developing any other policy (s_i, y_i), where*

$$y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}.$$

Lemma 2 states that for almost every score $s_i > 0$ better than the reservation policy, entrepreneur i's best combination of ideology y_i and quality q_i to generate that score is unique. Crucially, the combination does not does not depend on the specific policies that her opponent develops. Instead, it

[4] Proving this property is more complex than in all-pay contests without spillovers because the utility from tying can be a complicated function of the opponent's policies and the decisionmaker's decision rule.

is simply $F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}$, the weighted average of the entrepreneur and decisionmaker's ideal ideologies, multiplied by the probability $F_{-i}(s_i)$ that her opponent develops a lower-score policy.

Lemmas 1 and 2 jointly imply that in equilibrium, a player i can compute her expected utility as if her opponent always develops policies of the form $\left(s_{-i}, y^*_{-i}(s_{-i})\right)$. Thus, entrepreneur i's utility in equilibrium from developing any (s_i, y_i) with $s_i > 0$ where her opponent's score CDF F_{-i} has no atom (or if a tie would be broken in her favor) can be written as

$$\Pi^*_i(s_i, y_i; F) = \underbrace{-\alpha_i\left(s_i + y_i^2\right)}_{\text{quality cost}} + \underbrace{F_{-i}(s_i) \cdot V_i(s_i, y_i)}_{\text{Pr win} \cdot \text{utility if win}} + \underbrace{\int_{s_i}^{\infty} V_i\left(s_{-i}, y^*_{-i}(s_{-i})\right) dF_{-i}}_{\text{utility when lose}}. \tag{1}$$

Her utility from developing the *best* policy with score $s_i > 0$ must then be $\Pi^*_i(s_i, y^*_i(s_i); F)$. We henceforth denote this quantity $\Pi^*_i(s_i; F)$ and use it to characterize necessary and sufficient conditions for equilibrium.

Lemma 3 *A profile $(\sigma, w(\mathbf{b}))$ is a SPNE i.f.f. it satisfies three conditions.*

1. ***(No Ties)*** *In equilibrium, the probability the entrepreneurs develop new policies $b_i \neq b_0$ with the same score as the reservation policy $(s(b_i) = s(b_0))$ or each other $(s(b_i) = s(b_j))$ is 0.*

2. ***(Ideological Optimality)*** *With probability 1, each entrepreneur develops policies that either*

   - *generate score $s(y_i, q_i) < 0$ and have quality $q_i = 0$, or*
   - *generate score $s(y_i, q_i) \geq 0$ and satisfy $y_i = y^*_i(s(y_i, q_i))$.*

3. ***(Score Optimality)*** *For all i and s_i in the support of F_i, $s_i \in \arg\max_{s_i}\{\Pi^*_i(s_i; F)\}$.*

The Lemma provides necessary and sufficient conditions for equilibrium that do not depend on decisionmaker's decision rule $w(\mathbf{b})$. Entrepreneurs cannot be developing new, high-quality policies that are no better than the reservation policy or that tie each other with strictly positive probability. They must generate policies that are ideologically optimal. Finally, a score s_i can be in the support of i's score CDF F_i if and only if developing the ideologically-optimal policy for that score would maximize i's utility when a tie would be broken in her favor.

3.2 Equilibrium Characterization

Lemma 3 provides a straightforward necessary and sufficient condition on the score CDFs $\{F_i, F_{-i}\}$ for equilibrium. In this section we apply this condition to complete the equilibrium characterization.

We say that an entrepreneur is *active* when she develops a policy with strictly positive score and hence positive quality (note that no ties rules out positive-quality policies with a score of zero).[5] It is easy to see that in any equilibrium, both entrepreneurs must be active with strictly positive probability. If one entrepreneur were inactive (say $-i$), then her score CDF would be $F_{-i}(s) = 1, \forall s \geq 0$. Her opponent would thus develop $\left(0, \frac{x_i}{\alpha_i}\right)$, i.e., a new policy that has the same score as the reservation policy. This violates no ties.

In addition, all equilibria must be in mixed strategies. No ties implies that in any pure strategy profile one entrepreneur's policy (y_i, q_i) must have a strictly lower score $(s(y_i, q_i) < s(y_{-i}, q_{-i}))$ and hence lose the policy contest for sure, which means that i would therefore be strictly better off developing no policy. Thus, in equilibrium both entrepreneurs mix over both the ideological locations and qualities of the policies they develop, according to a strategy profile σ that generates no ties, is ideologically optimal, and induces score CDFs $(F_i(\cdot), F_{-i}(\cdot))$ satisfying score optimality. While characterizing score-optimal CDFs seems potentially complex, the next Proposition states that all such profiles satisfy simple conditions.

Proposition 1 *A profile of CDFs F satisfies score optimality i.f.f. it satisfies the following boundary conditions and differential equations.*

Boundary Conditions: $F_k(0) > 0$ *for at most one* $k \in \{L, R\}$*, and* $\min\left\{F_i^{-1}(1)\right\} = \bar{s}\ \forall i$.

Differential Equations: *For all* $i \in N$ *and* $s \in [0, \bar{s}]$

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(\left(\frac{x_i}{\alpha_i}\right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)\right).$$

Proposition 1 implies that equilibria of the model all have the following straightforward form.

First, at least one entrepreneur is *always* active – thus, in equilibrium competition is always strictly beneficial for the decisionmaker. The other entrepreneur may also always be ac-

[5] An inactive entrepreneur can develop the reservation policy or another 0-quality policy that won't be accepted.

tive ($F_i(0) = 0$) or be inactive with strictly positive probability ($F_i(0) > 0$). Second, when either entrepreneur i is active (which occurs with probability $1 - F_i(0)$), she mixes smoothly over the ideologically-optimal policies $(s, y_i^*(s)) = \left(s, \frac{x_i}{\alpha_i} F_{-i}(s)\right)$ with scores in the interval $[0, \bar{s}]$ according to the CDF $F_i(s)$.

The differential equations that generate equilibrium score CDFs arise intuitively from the requirement that both entrepreneurs be indifferent over developing all ideologically-optimal policies with scores in the interval $[0, \bar{s}]$. The left hand side of each differential equation is $i's$ net marginal cost of producing a policy with a higher score given a fixed probability $F_{-i}(s)$ of winning the contest. Specifically, the entrepreneur pays marginal cost $\alpha_i > 1$ for sure, but with probability $F_{-i}(s)$ her policy will be chosen and she will enjoy a marginal benefit of 1 (because she too values quality). The right hand side represents $i's$ marginal ideological benefit of producing a higher score. Doing so increases by $f_{-i}(s)$ the probability that her policy will win the contest, which changes the ideological outcome from her opponent's optimal ideology $y_{-i}^*(s) = \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)$ at score s to her own optimal ideology $y_i^*(s) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s)$.

Figures 2a and 2b summarize a mixed strategy equilibrium of the game with symmetrically located entrepreneurs ($-x_L = x_R$) and a cost advantage for the right entrepreneur ($\alpha_L > \alpha_R$). Figure 2a depicts the entrepreneurs' equilibrium score CDFs. The right entrepreneur is always active by virtue of her cost advantage ($F_R(0) = 0$), whereas the left entrepreneur is inactive with strictly positive probability ($F_L(0) > 0$). Moreover, the right entrepreneur's policies are better for the decisionmaker in a first-order stochastic sense; we later show that this property is a general feature of the game with symmetric ideologies and asymmetric costs.

Figure 2b depicts the ideological locations and quality of the policies over which each entrepreneur mixes, which is a parametric plot of $\left(y_i^*(s), s + (y_i^*(s))^2\right)$ for $s \in [0, \bar{s}]$. The ideological locations of each entrepreneur i's policies range from 0 to $\frac{x_i}{\alpha_i}$, which is the policy she would develop absent competition. In the equilibrium, the right entrepreneur exploits her cost advantage to develop more ideologically extreme policies at every score, and overall her policies are first-order stochastically more extreme. This too is a general feature of symmetric ideologies paired with asymmetric costs.

A notable feature of the equilibrium is that more extreme policies are not merely higher-quality than less extreme ones – they also have higher scores, so their additional quality *overcompensates* the decisionmaker for his ideological losses. As a result, the decisionmaker prefers the more ideologically extreme policies in the support of each entrepreneur's strategy to the less ideologically extreme ones. This is a general property of the model that follows immediately from ideological optimality. For two policies (y_i', s_i') and (y_i'', s_i'') with scores $s_i' < s_i'' \in [0, \bar{s}]$, the higher-score policy necessarily wins the policy contest with strictly higher probability $F_{-i}(s_i'') > F_{-i}(s_i')$, because both score CDFs are continuous. Thus, the ideology $y_i'' = y_i^*(s_i'') = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i'')$ of the higher-score policy is necessarily more extreme than the ideology $y_i' = y_i^*(s_i') = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i')$ of the lower-score policy. Intuitively, a policy that gives greater utility to the decisionmaker must be paired with a more extreme ideology because it has a higher chance of being selected, and thus the entrepreneur developing it will find it more worthwhile to pay the sure costs of developing quality for the uncertain benefits of ideological change. As noted in the following Corollary, a surprising substantive implication of this result is that when competing factions choose to develop more extreme policies, such policies are better for the organizational decisionmaker.

Corollary 1 *For two policies $(y_i, q_i), (y_i', q_i')$ in the support of i's strategy σ_i, the more ideologically extreme policy $(y_i' > y_i)$ is both higher-quality $(q_i' > q_i)$ and preferred by the decisionmaker $(s(y_i', q_i') > s(y_i, q_i))$.*

4 Analytical Characterization

Proposition 1 can be used to numerically compute all equilibrium score CDFs for a given set of parameter values.[6] However, our results thus far ensure neither existence nor uniqueness. In

[6] We focus on analytical results in this paper. However, it's worth noting a procedure that can be used for variants of our model in which analytical results cannot be obtained. Posit one entrepreneur $-k$ to always be active (i.e., $\hat{F}_{-k}(0) = p_{-k} = 0$), search over all potential starting values $\hat{F}_k(0) = \hat{p}_k \in [0, 1)$ for the other entrepreneur,

this section we provide an analytical characterization of the equilibrium score CDFs that ensures both, and identify some straightforward properties. Further analysis of the equilibria is deferred to Sections 6 and 7, where we develop a number of special cases of substantive interest.

Proposition 2 *Define the following notation.*

- *Let* $\epsilon_i(p) = \left(\frac{\alpha_i - p}{\alpha_i - 1}\right)^{|x_i|} = e^{\left(\int_p^1 \frac{|x_i|}{\alpha_i - p} dp\right)}$ *denote entrepreneur* i*'s* ***engagement*** *at probability* p*, which is a decreasing function that ranges from* $\epsilon_i(0) = \left(\frac{\alpha_i}{\alpha_i - 1}\right)^{|x_i|}$ *to* $\epsilon_i(1) = 1$*.*
- *Denote* i*'s engagement at probability* 0 *as* ϵ_i*, and let* k *denote the less engaged entrepreneur at probability* 0*.*
- *Let* $p_i(\epsilon) = \epsilon_i^{-1}(\epsilon) = \alpha_i - (\alpha_i - 1)\,\epsilon^{\frac{1}{|x_i|}}$ *be the unique probability such that* i*'s engagement is equal to* ϵ*.*

Then the unique score CDFs satisfying Proposition 1 are

$$F_i^*(s) = p_{-i}(\epsilon^*(s))$$

where $\epsilon^*(s)$ *is the inverse of*

$$s^*(\epsilon) = 2\sum_i |x_i| \cdot \left(\ln\left(\frac{\epsilon_k}{\epsilon}\right) - \left|\frac{x_i}{\alpha_i}\right| \cdot (p_i(\epsilon) - p_i(\epsilon_k))\right)$$

The unique equilibrium score CDFs $\left(F_i^*, F_{-i}^*\right)$ can be understood through the function $\epsilon_i(p) = \left(\frac{\alpha_i - p}{\alpha_i - 1}\right)^{|x_i|}$, which we call entrepreneur i's *engagement at probability* p. Essentially, this quantity captures an entrepreneur's willingness to develop policies whose probability of winning the contest is $\geq p$; it is mononotically decreasing in p and thus and minimized at 1. An entrepreneur's overall willingness to participate in the contest is captured by $\epsilon_i(0) = \epsilon_i$.t.

When entrepreneur i develops a policy of score s, the probability that she will win the contest is $F_{-i}(s)$. Thus, her willingness to develop policies with score $\geq s$ is equal to $\epsilon_i(F_{-i}(s))$, which is

numerically compute the unique candidate score CDFs $\left(\hat{F}_k, \hat{F}_{-k}\right) \mid (\hat{p}_k, \hat{p}_{-k})$ satisfying the differential equations and starting conditions, and verify whether they satisfy the boundary condition $\hat{F}_k^{-1}(1) = \hat{F}_{-k}^{-1}(1) = \bar{s}$.

decreasing in s. The key property of equilibrium is that at each score $s \in [0, \bar{s}]$, the entrepreneurs must be *equally engaged*, i.e. $\epsilon_i \left(F^*_{-i}(s)\right) = \epsilon_{-i}\left(F^*_i(s)\right) \iff$

$$\left(\frac{\alpha_{-i} - F^*_i(s)}{\alpha_{-i} - 1}\right)^{|x_{-i}|} = \left(\frac{\alpha_i - F^*_{-i}(s)}{\alpha_i - 1}\right)^{|x_i|} \tag{2}$$

Thus, in equilibrium every score $s \in [0, \bar{s}]$ is associated with a unique level of engagement $\epsilon^*(s)$, a function whose inverse is characterized analytically in the Proposition. The function is uniquely pinned down by the boundary conditions on $\left(F^*_i, F^*_{-i}\right)$, and is necessarily decreasing in s, because higher scores must be associated with a greater probability of winning the cost and hence lower engagement.

The main equilibrium quantities are then easily derived from these functions. The probability $F^*_i(s)$ that entrepreneur i develops a policy with score $\leq s$ must be unique probability of winning the contest $p_{-i}(\epsilon^*(s))$ such that her competitor $-i$'s engagement at score s is equal to $\epsilon^*(s)$. Since i's optimal ideology is a linear function of her opponent's score CDF $F_{-i}(s)$, i.e., $y^*_i(s) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s)$, her unique optimal ideologies at each score s must then be equal to $y^*_i(s) = \left(\frac{x_i}{\alpha_i}\right) p_i(\epsilon^*(s))$.

4.0.1 Equilibrium Likelihood of Activity

Proposition 2 yields a closed form characterization of the likelihood that each side will be active in the policy contest. Thus, the model can be used to analyze how the ideological extremism and the costs of two competing factions in an organization jointly determine the likelihood that each faction will develop a policy proposal. In particular, it allows us to consider how one faction's activity will react to changes in the costs and ideological extremism of the other.

It is easy to verify from the inverse function $s^*(\epsilon)$ that the engagement associated with score $s = 0$ is ϵ_k, the engagement of the less engaged entrepreneur k. Thus, the probability that an entrepreneur i is inactive is $F^*_i(0) = p_{-i}(\epsilon_k)$, which gives the following corollary.

Corollary 2

- *The more-engaged entrepreneur $-k$ is always active* $\left(F^*_{-k}(0) = p_k(\epsilon_k(0)) = 0\right)$.

- *The less-engaged entrepreneur k is active with probability*

$$1 - p_{-k}(\epsilon_k) = (\alpha_{-k} - 1)\left(\epsilon_k^{\frac{1}{|x_{-k}|}} - 1\right) \text{ which is} < 1 \text{ when } \epsilon_k < \epsilon_{-k}.$$

The probability that the less engaged entrepreneur is active is strictly increasing in her engagement ϵ_k, strictly increasing in the more-engaged entrepreneur's costs α_{-k}, and strictly decreasing in the more-engaged entrepreneur's extremism $|x_{-k}|$.

Figure 3 is a contour plot of the probability that the less-engaged entrepreneur is active as a function of the ideology x_R and costs α_R of the right entrepreneur – the left entrepreneur's parameters (x_L, α_L) are held fixed. The white curve depicts where the two entrepreneurs are equally engaged, and hence always active. In the purple region, the right entrepreneur is less engaged. Here, both decreases in her costs α_R and/or increases in her ideological extremism x_R increase her engagement ϵ_R with the contest, and thus the probability $1 - p_L(\epsilon_R) < 1$ that she will develop a policy. In the blue region, in contrast, the right entrepreneur is more engaged and thus is always active. However, her parameters (α_R, x_R) influence the probability $1 - p_R(\epsilon_L)$ that the left entrepreneur will choose to be active. Decreases in the right entrepreneur's costs and/or increases in her extremism further accentuate the imbalance in engagement, and the consequence is a decrease in the probability that the left entrepreneur will develop a proposal. This comparative static is somewhat surprising given that (as we later show) more ideological entrepreneurs develop more ideologically extreme policies, which could theoretically give the less-engaged entrepreneur a greater incentive to develop a competing proposal.

Overall, the probability of observing direct competition is largely a function of how evenly the two entrepreneurs are engaged in the contest. As their engagement becomes increasingly asymmetric, the less-engaged entrepreneur increasingly drops out of the contest.

4.0.2 Relative Strength

Proposition 2 and the implied equation (2) also allows us to characterize the equilibrium likelihood that each entrepreneur will win the policy contest.

Corollary 3 *The probability that the more engaged entrepreneur $-k$ wins the policy contest is,*

$$\int_0^{\bar{s}} \frac{\partial F^*_{-k}(s)}{\partial s} F^*_k(s)\, ds = \int_0^{\bar{s}} \frac{\partial F^*_{-k}(s)}{\partial s} p_{-k}\left(\epsilon_k\left(F^*_{-k}(s)\right)\right) ds = \int_0^1 p_{-k}\left(\epsilon_k(p)\right) dp$$

which is strictly increasing in her ideological extremism $|x_{-k}|$ and her opponent's costs α_k, and strictly decreasing in her costs α_{-k} and her opponent's ideological extremism $|x_k|$.

Thus, each entrepreneur's probability of victory responds naturally to changes in the underlying model parameters – as either entrepreneur becomes more ideologically motivated *or* better able to develop quality, her probability of winning the contest increases and her opponent's correspondingly decreases.

We can also apply the relationship in Equation (2) to characterize when the more-engaged entrepreneur *score-dominates* the policy contest, in the sense of developing policies that are first-order stochastically better for the decisionmaker.

Lemma 4 *The more engaged entrepreneur $-k$ score-dominates the policy contest, i.e. $F_{-k}(s) < F_k(s)\ \forall s \in [0, \bar{s})$, if and only if she is more engaged at every probability p, i.e., $\epsilon_{-k}(p) > \epsilon_k(p)\ \ \forall p \in [0, 1)$.*

Being more engaged at probability 0 – and thus more likely to enter the contest – is therefore necessary but not sufficient for entrepreneur $-k$ to score-dominate the policy contest. Intuitively, the reason is that the entrepreneurs actually place some value on the quality that they invest in to gain influence. Relative cost advantages therefore become magnified when the entrepreneurs develop higher score policies, because they are more likely to be chosen and thus the entrepreneur is more likely to enjoy the intrinsic benefits of her quality investment. Mathematically, if entrepreneur $-k$ has higher costs ($\alpha_{-k} > \alpha_k$), then greater engagement at probability 0, i.e., $\left(\frac{\alpha_{-k}-1}{\alpha_{-k}}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-1}{\alpha_k}\right)^{|x_k|}$, is an easier hurdle to satisfy than greater engagement at higher probabilities, i.e., $\left(\frac{\alpha_{-k}-1}{\alpha_{-k}-p}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-1}{\alpha_k-p}\right)^{|x_k|}$.

4.0.3 Equilibrium Ideologies

A key question is how *ideologically extreme* are the policies that the entrepreneurs develop in equilibrium; Proposition 2 also generates an analytical characterization of the probability distribution over the ideological extremism of each entrepreneur's policies. Thus, the model predicts how both factions in an organization alter the ideology of the proposals that they develop in response to changes in the underlying structural parameters of competition.

Observing that $y_i^*(s) = F_{-i}^*(s)\frac{x_i}{\alpha_i} \iff y_i^{-1}(y_i) = F_{-i}^{-1}\left(\frac{y_i}{x_i/\alpha_i}\right)$ and exploiting the previously noted relationships, the probability that i develops a policy with ideological extremism $\leq |y_i|$ is equal to the probability that she develops a lower score than that associated with y_i, which is $F_i\left(y_i^{-1}(y_i)\right) = p_{-i}\left(\epsilon^*\left(y_i^{-1}(y_i)\right)\right) = p_{-i}\left(\epsilon_i\left(F_{-i}\left(F_{-i}^{-1}\left(\frac{y_i}{x_i/\alpha_i}\right)\right)\right)\right) = p_{-i}\left(\epsilon_i\left(\frac{y_i}{x_i/\alpha_i}\right)\right)$. This yields the following Corollary.

Corollary 4 *The ideological extremism $|y_i|$ of entrepreneur i's policies is distributed according to*

$$G_i(|y_i|) = p_{-i}\left(\epsilon_i\left(\frac{y_i}{x_i/\alpha_i}\right)\right) = \alpha_{-i} - (\alpha_{-i} - 1)\left(\frac{x_i - y_i}{x_i - x_i/\alpha_i}\right)^{\left|\frac{x_i}{x_{-i}}\right|}$$

The distribution is first-order stochastically increasing in i's ideological extremism $|x_i|$, decreasing in her costs α_i, decreasing in her opponent's ideological extremism $|x_{-i}|$, and increasing in her opponent's costs α_i.

Unsurprisingly, when an entrepreneur's extremism $|x_i|$ increases or her costs α_i decrease, she reacts by increasing the ideological extremism of her policies – in the former case she is more motivated to exploit quality to realize ideological gains, and in the latter case she is better able to do so.

More interestingly, each entrepreneur reacts to increases in her *opponent's* ideological extremism $|x_i|$ and decreases in her opponent's costs α_i by *moderating* the ideological location of her own policies. In the former case, entrepreneur $-i$ becomes relatively less willing to develop quality to realize ideological gains, and in the latter case she is relatively less able. Thus, the pattern of policy development under competitive entrepreneurship is one in which increased ideological extremism by one faction is necessarily accompanied by greater moderation from the competing faction.

4.0.4 Equilibrium Payoffs of the Entrepreneurs

Finally, Proposition 2 yields a closed form characterization of the maximum score $\bar{s}$, which can be used to compute the entrepreneurs' equilibrium utility. Because $F_i^*(\bar{s}) = 1 = p_{-i}(1)$, the maximum score is simply the score $s^*(1)$ associated with an engagement of 1, implying the following.

Lemma 5 *The maximum equilibrium score $\bar{s}$ is*

$$\bar{s} = s^*(1) = 2\sum_i |x_i| \cdot \left(\ln(\epsilon_k) - \left| \frac{x_i}{\alpha_i} \right| (1 - p_i(\epsilon_k)) \right),$$

and is increasing in ideological extremism $|x_i|$ and decreasing in costs α_i $\forall i$.

Entrepreneur i's utility is

$$\Pi_i^*(\bar{s}; F^*) = -\left(\frac{\alpha_i - 1}{\alpha_i} \right) x_i^2 - (\alpha_i - 1)\,\bar{s},$$

which is decreasing in her opponent's extremism $|x_{-i}|$ and increasing in her opponent's costs α_{-i}.

Each entrepreneur's equilibrium utility is written in terms of two components. The first component $-\left(\frac{\alpha_i-1}{\alpha_i}\right) x_i^2$ depends solely on her own parameters, and represents what her utility *would be* if she could engage in entrepreneurship absent competition. The second component $-(\alpha_i - 1)\,\bar{s}$ is the cost generated by competition, which forces her to develop policies that are strictly better for the decisionmaker than the reservation policy in order to maintain her influence. This cost is increasing in i's own marginal cost α_i of developing quality, and increasing in the intensity of competition as captured by the maximum score $\bar{s}$.

The intensity of competition $\bar{s}$ is affected by the entire profile of parameters in a natural way – it increases if either entrepreneur becomes more extreme, and decreases if either entrepreneur's costs of developing quality increase. An interesting implication is that an entrepreneur is unambiguously *worse off* if her opponent's parameters change in a manner that makes her more willing or able to compete. In particular, an entrepreneur is harmed if her opponent becomes more efficient at developing quality, *even though it is a fully common value dimension.* The reason is that cost of her opponent's greater ideological aggression outweighs the benefit to her of the additional investments.

5 Symmetric Competition

In this section, we analyze equilibrium outcomes and welfare in the special case of symmetric competition. We set $x \equiv |x_i|$ so that the entrepreneurs are equidistant from the decisionmaker, and $\alpha \equiv |\alpha_i|$ so that they face the same marginal cost of developing quality. A model with symmetric competition is an important subcase of our general model for two reasons. First, varying the extremity of the entrepreneurs is a natural way to analyze the effects of *polarization of preferences* in an organization. Second, we can characterize how the marginal cost of developing quality α affects decisionmaking. Although quality benefits all members of the organization, its welfare effects are nonobvious because the entrepreneurs also exploit quality to realize ideological gains.

We begin by taking advantage of symmetry to characterize the equilibrium in a form that is simpler than Proposition 2.

Lemma 6 *If $x \equiv |x_i|$ and $\alpha \equiv |\alpha_i|$, then the unique equilibrium is in symmetric mixed strategies. The entrepreneurs develop policies of the form $(y_i, s(|y_i|) + y_i^2)$, where*

1. *the ideological extremity $|y_i|$ of each entrepreneur's policies is uniform on $[0, \frac{x}{\alpha}]$*
2. *the score of a policy with ideology y_i is $s^*(|y_i|) = 4x\left(x \ln\left(\frac{x}{x-|y_i|}\right) - |y_i|\right)$*
3. *the maximum score is $\bar{s} = 4x^2\left(\ln\left(\frac{\alpha}{\alpha-1}\right) - \frac{1}{\alpha}\right)$, and each entrepreneur's expected utility is*
$$-\left(1 - \frac{1}{\alpha}\right)x^2 - (\alpha - 1)\bar{s} = -4x^2(\alpha - 1)\left(\ln\left(\frac{\alpha}{\alpha - 1}\right) - \frac{3}{4\alpha}\right)$$
4. *the decisionmaker's expected utility is*
$$4x^2\left(\left(\alpha + \frac{1}{2} - \frac{2}{3\alpha}\right) - \left(\alpha^2 - 1\right)\ln\left(\frac{\alpha}{\alpha - 1}\right)\right).$$

A pair of figures summarize the equilibrium of the symmetric game. Figure 4a depicts the entrepreneurs' equilibrium (identical and atomless) score CDFs; both entrepreneurs are always active. Figure 4b depicts equilibrium policies – the ideological distance of each entrepreneur's policies from the decisionmaker is uniformly distributed on $\left[0, \frac{x}{\alpha}\right]$.

The key simplification produced by symmetry is that the ideological extremity $|y_i^*(s)|$ of the entrepreneurs' optimal ideologies, and hence their score CDFs, must be identical at every score. This implies that the ideologies of each entrepreneur's policies are uniformly distributed – because $P(|y_i| \leq |y|) = \frac{y_{-i}^*\left(y_i^{*-1}(y)\right)}{x_{-i}/\alpha_{-i}} = \frac{|y|}{x/\alpha}$ – which allows us to easily characterize the equilibrium in terms of the score as a function of ideology y_i.

Policy Outcomes and Decisionmaker Utility Lemma 6 states that the ideological extremity of policies is uniformly distributed over $\left[0, \frac{x}{\alpha}\right]$. It is thus immediately obvious that either an increase in polarization (as measured by x) or a decrease in costs (as measured by α) leads to more extreme policies being both developed and adopted in a first-order stochastic sense. Both of these factors can therefore be thought of as contributing to observable polarization of outcomes in the model. However, although the decisionmaker is worse off in the sense of ideology, he is *better off* overall – both in an expected utility sense and a first-order stochastic sense.

Proposition 3 *The ideological extremity of the policy outcome, and the decisionmaker's utility, are first-order stochastically increasing in polarization x and decreasing in the cost of quality α.*

The effect of changing α and x on equilibrium strategies is depicted in Figures 5a and 5b. We previously showed in Corollary 1 that for fixed parameters, more ideologically extreme policies in the support of an entrepreneur's strategy are better for the decisionmaker (i.e., $s(|y|)$ is increasing in $|y|$). Proposition 3 is stronger – it states that factors that induce the entrepreneurs to develop ideologically extreme policies also induce them to develop better policies overall. Mathematically, the result is easiest to see by considering the effect of decreasing α, which does not enter the score function $s(|y|)$ in Lemma 6 and therefore only stretches the range of uniformly distributed ideologies $\left[0, \frac{x}{\alpha}\right]$. Decreasing α thus shifts probability weight towards policies that are ideologically more extreme, but higher score. The result extends to increasing polarization x as well, but the proof is less straightforward.

Proposition 3 shows that in organizations where competing factions seek influence by making productive investments, factors that increase their incentive and ability to do so result in greater quality investments, to the benefit of a centrist decisionmaker. This holds even though the entrepreneurs exploit their investments to some extent to achieve more ideologically extreme outcomes.

Note that the presence of a competing faction that can generate policies is crucial for the result – were one entrepreneur to have a monopoly on the ability to generate quality, she would always extract its benefits in the form of ideological concessions. In the next section, we show that the benefits that accrue to the decisionmaker from competitive entrepreneurship have more to do with the mere *presence* of competition than the fact that the model is symmetric with two entrepreneurs competing on equal footing.

Application: Polarization Proposition 3 provides a novel lens for analyzing the effects of political polarization. The voluminous literature on this topic (e.g., Brady and Volden 1998, Krehbiel 1998) generally features two arguments: polarization causes non-centrist policy outcomes, and it is bad for centrists. In our model, polarization leads to non-centrist outcomes, but it is actually good for centrists. The key difference is that most existing work on polarization takes as given the set of available policies, whereas we consider incentives for polarized entrepreneurs to make productive investments in their proposals. An additional difference is that previous work focuses on polarization of the preferences of various actors (pivots or veto players) whose approval is necessary for policy enactment, whereas in our model decisionmaking authority remains in the hands of a single centrist.[7]

The literature on signaling games includes some single-decisionmaker models in which all actors have shared interests, in the sense that they benefit from variance reduction. The model most directly comparable to ours is Gilligan and Krehbiel (1989), in which two privately informed experts located symmetrically around a decisionmaker make policy recommendations. In Gilligan and Krehbiel, polarization harms the decisionmaker because the experts do not engage in confirmatory signalling in extreme states.[8] Our model is fundamentally different – more extreme entrepreneurs

[7] In a companion paper (Hirsch and Shotts 2013) we examine how decentralized decisionmaking authority affects competitive policy entrepreneurship.

[8] Krishna and Morgan (2001) show the Gilligan and Krehbiel model with two committees also has a fully-revealing

place a greater *marginal* value on shifting ideological outcomes toward their ideal point, magnifying their incentive to invest in quality. Our model thus demonstrates that polarization can be beneficial in political organizations when it induces competing factions to make productive investments to gain influence.

Beyond comparative statics on the effects of preference polarization, the symmetric version of our model with fixed parameters also provides surprising predictions about polarization of policy outcomes. Traditional spatial models of political economy (starting with Hotelling 1929, Downs 1957, and Black 1958) feature policy convergence because decision makers prefer policies close to their ideal points. However, a more general version of the assumptions in these models is that decision makers, cetris paribus, prefer policies close to their ideal points. Our model points out that there are good reasons to believe that other things are not, in fact, equal. In particular, from Corollary 1 we know that within an entrepreneur's strategy, extreme proposals are better for the decision maker than moderate ones, because they are sufficiently high quality to overcompensate him for his ideological losses. In the symmetric case of our model, the two entrepreneurs' equilibrium strategies are mirror images of each other, which means that when facing two policy proposals, the decision maker strictly prefers the one *farther* from his own ideal point. Thus our model suggests that for empirical applications of spatial models of policy choice it is crucial to assess policies' quality as well as their ideological locations.

Entrepreneur Utility We now analyze how the marginal cost α of developing quality affects the entrepreneurs' equilibrium utility. The effect is nonobvious, because lower costs make it cheaper to persuade the decisionmaker to accept the same degree of ideological change, but also increase the intensity of competition. The following result summarizes basic patterns.

Lemma 7 *The marginal cost of developing quality α has the following effects on the entrepreneurs' equilibrium utility.*

1. *As $\alpha \to 1$, an entrepreneur's utility converges to $U_i(x_i, 0) = 0$ (her utility from her ideal*

equilibrium, which is criticized by Krehbiel (2001) for being implausible and by Battaglini (2003) for being non-robust.

ideology with no quality). As $\alpha \to \infty$, an entrepreneur's utility converges to $U_i(b_0) = -x_i^2$ (her utility from the reservation policy).

2. *There exists an $\hat{\alpha}$ such that utility is decreasing in α when $\alpha < \hat{\alpha}$, and increasing otherwise.*

3. *There exists an $\bar{\alpha} < \hat{\alpha}$ such that the entrepreneurs benefit from competitive entrepreneurship when $\alpha < \bar{\alpha}$, and are harmed otherwise.*

The effect of α on the entrepreneurs' equilibrium utility is depicted in Figure 6. The limiting results are not surprising. As $\alpha \to 1$ – i.e., as the marginal cost of developing quality approaches its marginal benefit to the entrepreneurs – it is as if each entrepreneur can get her ideal ideological outcome at no cost. Thus, her utility is $U_i(x_i, 0) = 0$. Conversely, as $\alpha \to \infty$ – i.e., as the cost of developing quality becomes arbitrarily large – entrepreneurship collapses, the outcome is the reservation policy with no quality, and each entrepreneur's utility approaches $U_i(b_0) = -x_i^2$.

However, an interesting nonmonotonicity emerges between the limits. At low cost levels, competition is most intense but also least costly because the cost of developing quality approaches its value. Consequently, the entrepreneurs' expected utilities approach what they could achieve absent competition. In this region, higher costs thus *harm* the entrepreneurs by making it more difficult to engage in entrepreneurship. However, once the marginal cost crosses the threshold $\hat{\alpha}$, further cost increases benefit the entrepreneurs by diminishing the intensity of competition. Competitive entrepreneurship is thus most harmful to the entrepreneurs at intermediate cost levels, where competition is intense yet genuinely costly.

The proposition also reveals when the ability to engage in competitive entrepreneurship benefits the entrepreneurs relative to simply accepting the reservation policy. In common agency models of lobbying and influence, e.g., Dixit, Grossman, and Helpman (1997), opposing interest groups are harmed by the ability to lobby – their counteractive influence has no effect on policy outcomes, but each group must pay to prevent the decisionmaker from colluding with her competitor. In contrast, with influence via all-pay productive investments, the entrepreneurs can benefit from competition even when the average ideological policy outcome is unchanged, as long as the cost of improving policy quality is sufficiently low ($\alpha < \bar{\alpha}$). The reason is that each entrepreneur's investments, while sometimes wasted, are productive and therefore also benefit her competitor. For costs above $\bar{\alpha}$, however, the entrepreneurs are worse off with competition, as in counteractive lobbying.

Overall, symmetric competitive entrepreneurship is always beneficial to the decisionmaker. However, it can either benefit or harm the entrepreneurs, depending on how the cost of developing quality interacts with the overall intensity of competition.

6 Asymmetric Competition

Asymmetric competition is common feature of political organizations; often, one faction has more extreme ideological preferences and/or greater expertise and resources to develop high-quality policy alternatives. Because general comparative statics of the game with asymmetric competition can be relatively complex, in this section we consider particular special cases of interest.

We first consider the case in which there is a dominant entrepreneur in the sense of having both (weakly) more extreme preferences ($|x_k| \leq |x_{-k}|$) and lower costs of developing quality ($\alpha_{-k} \leq \alpha_k$). Within this case, we focus on the subcase of an entrepreneur who is cost dominant but equally extreme ($|x_k| = |x_{-k}|$ and $\alpha_{-k} < \alpha_k$), though we also comment on what happens when one is ideologically dominant but has no cost advantage ($|x_k| < |x_{-k}|$ and $\alpha_{-k} = \alpha_k$). Finally, we consider what happens if the entrepreneurs are equally engaged ($\epsilon_k = \epsilon_{-k}$) and thus always active, but their motive for engagement is distinct – one has a cost advantage ($\alpha_k < \alpha_{-k}$) whereas the other is more ideologically extreme ($x_{-k} > x_k$).

6.1 A Dominant Entrepreneur

As in Section 4.0.2, we say that entrepreneur i *score dominates* the policy contest i.f.f. she develops policies that are first-order stochastically better for the decisionmaker, i.e., $F_i(s) \leq F_{-i}(s)$, $\forall s \in [0, \bar{s}]$ with a strict inequality for some scores. Additionally, we say that entrepreneur i is more *ideologically aggressive* i.f.f. the policies that she develops are first-order stochastically more extreme, i.e., $G_i(|y|) \leq G_{-i}(|y|)$, $\forall y$ with a strict inequality for some ideologies.

The characteristic feature of competition when there is a dominant entrepreneur ($|x_k| \leq |x_{-k}|$ and $\alpha_{-k} \leq \alpha_k$ with at least one strict inequality) is that she is more engaged, score-dominates the policy contest, and is more ideologically aggressive.

Corollary 5 *If $\alpha_{-k} \leq \alpha_k$ and $|x_{-k}| \geq |x_k|$, with at least one inequality strict, then entrepreneur $-k$ is more engaged, score dominant, and more ideologically aggressive.*

Greater engagement and score dominance follow immediately from Lemma 4, which states that greater engagement at every probability p, i.e., $\left(\frac{\alpha_{-k}-p}{\alpha_k-1}\right)^{|x_{-k}|} > \left(\frac{\alpha_k-p}{\alpha_k-1}\right)^{|x_k|}$ $\forall p$, is a necessary and sufficient condition for score dominance. This clearly holds when $-k$ is both more extreme and has lower costs. First order stochastic dominance of ideologies is then an implication. Applying score dominance, entrepreneur $-k$ develops more extreme policies at every score, i.e., $\left|y^*_{-k}(s)\right| = \left|\frac{x_{-k}}{\alpha_{-k}}\right| F_k(s) > \left|\frac{x_k}{\alpha_k}\right| F_{-k}(s) = |y^*_k(s)|$ $\forall s$, which combined with score dominance implies that she is more ideologically aggressive in a first order stochastic sense.

6.1.1 A Cost Dominant Entrepreneur ($|x_k| = |x_{-k}|$ and $\alpha_{-k} < \alpha_k$)

The special case of a cost-dominant entrepreneur with equally balanced ideological preferences ($x_k = x_{-k}$) has a natural interpretation. The entrepreneurs may represent two competing factions within a firm or agency, each of which leans in favor of one particular approach to an organizational problem, yet one of whom has more staff and a greater budget to develop new policy proposals.

Under such circumstances, the cost-advantaged entrepreneur *exploits* her advantage to develop policies that are more reflective of her ideological preferences. Interestingly, despite the greater ideological extremism of her policies, she invests sufficiently in quality to make the decisionmaker probabilistically favor them; she does not overexploit her advantage. Unsurprisingly (from Corollary 5) her expected utility in the contest is also higher. Comparative statics in each entrepreneur's costs follow straightforwardly from this basic intuition.

Corollary 6 *As either entrepreneur i's costs α_i decrease,*

1. *her probability of winning the contest increases*

2. *her policies become more ideologically extreme, while her opponent's policies become more ideologically moderate*

3. *her opponent's expected utility decreases.*

   Moreover, the probability that the high-cost entrepreneur k is active decreases if her own costs α_k increase or her opponent's costs α_{-k} decrease.

The first two comparative statics, as well as the final observations about equilibrium activity, are simply restatements of Corollaries 2-4. Lower costs increase an entrepreneur's relative strength in the contest, which increases the extremism of her policies and moderates those of her opponent. The third is nonobvious because when entrepreneur $i's$ costs decrease it is easier for her to produce quality that her opponent $-i$ values The reason $-i's$ utility decreases is that i is sufficiently aggressive in exploiting this cost advantage to obtain ideological gains to offset the benefits $-i$ sees in terms of higher policy quality.

In addition, as engagement becomes increasingly imbalanced – either through increases in the costs of the high-cost entrepreneur or decreases in those of the low-cost entrepreneur – the disadvantaged entrepreneur becomes increasingly likely to drop out of the contest. In the limit, it is simple to verify that the high cost entrepreneur's probability of being active $1 - p_{-k}(\epsilon_k) = (\alpha_{-k} - 1)\left(\epsilon_k^{\frac{1}{|x_{-k}|}} - 1\right)$ converges to 0 either as she faces extremely high costs ($\alpha_k \to \infty$), or as the low-cost entrepreneur's cost of developing quality approaches its value ($\alpha_{-k} \to 1$). It is therefore natural to ask whether the benefits from competitive entrepreneurship to the decisionmaker vanish under extreme imbalances – that is, whether his utility approaches $s = 0$ from the reservation policy. The following result demonstrates that the answer depends on the source of the asymmetry.

Corollary 7 *The decisionmaker's utility converges to zero as $\alpha_k \to \infty$, but is bounded away from zero as $\alpha_{-k} \to 1$.*

Thus, when the absence of observable competition results from very high costs to one entrepreneur, the decisionmaker is indeed essentially no better off than in the absence of competition. The intuition is straightforward – for any positive score $\hat{s} > 0$, there exists a cost α_k sufficiently high that entrepreneur k would prefer staying out of the contest to developing a policy with that score. Thus, no such score can be in the support of either entrepreneur's strategy in the limit. This scenario matches Londregan's (2000) characterization of Chilean policymaking, where both the legislature and the president have formal proposal power, but the legislature has essentially no resources for policy development. The predictions of our model in this empirical domain are therefore similar to Londregan's model, in which only the president can develop high-quality policies.

However, when the absence of observable competition results from the extremely high efficiency of the low–cost entrepreneur $-k$, the decisionmaker is *strictly better off* with the possibility of competition, in the sense that her utility is bounded away from zero in the limit. This is true even though the high-cost entrepreneur's probability of developing a policy converges to zero. The reason is simple – the *threat* of entry by the high cost entrepreneur prevents the low-cost entrepreneur from developing policies that are little better than the reservation policy. If she did so, the higher-cost entrepreneur would prefer to develop strictly better policies and win the contest. Potential competition therefore prevents even an arbitrarily dominant entrepreneur from extracting all the benefits of quality in the form of ideological gains. This observation is crucial for empirical analysis of competitive policy development – in situations where only one actor or faction routinely develops proposals, it cannot be concluded that her actions are unaffected by potential activity from other interested groups.

It is also worth noting that Corollary 7 does not require $|x_k| = |x_{-k}|$. Moreover, a similar result holds for ideological extremism; holding fixed the cost parameters α_i and α_{-i}, the probability of active competition converges to zero as $x_k \to 0$ or $|x_{-k}| \to \infty$. In the former case, the decision maker's utility converges to zero but in the latter case it is bounded away from zero, even though the less-engaged entrepreneur is extremely unlikely to be active.

6.2 Equally-Engaged Entrepreneurs with Different Motives

Finally, we consider what happens entrepreneurs are equally engaged ($\epsilon_k = \epsilon_{-k}$) but one of them has a cost advantage whereas the other is more ideologically extreme ($\alpha_k < \alpha_{-k}$ and $|x_k| < |x_{-k}|$). For example k may be a corporate interest group whereas $-k$ is an environmentalist group with limited resources. The groups' equal engagement implies that both of them are always active in developing policies. However, their patterns of policy development differ.

By the reasoning underlying Lemma 4, the cost-advantaged entrepreneur k is score dominant; the entrepreneurs are equally engaged at $p = 0$ so for any $p > 0$ the cost-advantaged entrepreneur is more engaged than her opponent. As a straightforward consequence of score dominance, we also know that the cost-advantaged entrepreneur wins the contest more than half of the time.

Moreover, Lemma 2 enables us to characterize the entrepreneurs' policy offerings for any given score $s \in [0, \bar{s}]$. Their equal engagement means that $\left(\frac{\alpha_k}{\alpha_k - 1}\right)^{|x_k|} = \left(\frac{\alpha_{-k}}{\alpha_{-k} - 1}\right)^{|x_{-k}|}$, which implies (after a bit of algebra) that $\frac{|x_k|}{\alpha_k} < \frac{|x_{-k}|}{\alpha_{-k}}$ because $|x_k| < |x_{-k}|$. Thus, at any score s the cost-advantaged entrepreneur's proposal is more moderate than the proposal of the more ideologically-motivated entrepreneur. Combined with the observation (Corollary 1) that within an entrepreneur's strategy, more extreme policies have higher scores, this implies the following additional patterns of policy making. First, when the two entrepreneurs propose policies that are equally ideologically distant from the decision-maker, the cost-advantaged one wins the contest. Second, the cost-advantaged entrepreneur always wins when her proposal is more extreme than her opponent's proposal. However, the more ideologically-motivated entrepreneur sometimes loses when she makes a proposal that is more extreme than her opponent's proposal.

These patterns of winning and losing are consistent with stylized facts about interest group competition between cost-advantaged entreprenuers (e.g., firms) and ideologically-motivated ones (e.g., environmental advocacy groups). However our results are not driven by factors such as backdoor dealings or quid pro quo lobbying expenditures and campaign contributions that could enable corporate interest groups to dominate policy making. Nor do our results stem from irrational behavior by idealistic activists who insist on maintaining ideological purity. Of course, such factors may well contribute to observed patterns of behavior. But our model shows that these patterns can also arise simply due to preference and cost asymmetries among rational actors who make productive investments that improve the quality of their policy proposals.

7 Conclusion

This paper develops a model of political organizations in which individuals or factions have different ideologies or preferences regarding organizational priorities, yet also agree on certain common objectives. Competing entrepreneurial policy developers within such an organization can appeal to decision makers by making productive, policy-specific investments to improve the quality of their proposals. Rather than being tailored narrowly to any one specific institution, our model is designed to capture key features of a variety of different political organizations, including legislatures, NGOs, firms, militaries, democratic polities, political parties, and executive branch agencies.

We characterize the unique equilibrium of the all pay contest played by two competing entrepreneurs as they generate proposals comprised of two dimensions: ideology and quality. In the course of the analysis, we also develop techniques that can be applied to other environments in which individuals or groups compete to have their preferred spatial policies enacted by exerting costly up-front effort; examples include valence competition in elections (e.g. Wiseman 2006, Meirowitz 2008, Ashworth and Bueno de Mesquita 2009) and expenditures in lobbying contests (Meirowitz and Jordan 2012). In many such models it would be very natural to analyze simultaneous choices of ideology and policy by two competing actors, but to the best of our knowledge no model has either formulated or solved the resulting all-pay contest.

8 References

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9 Appendix

Proof of Lemma 1

Player i's utility for developing a policy (s, y_i) at any score s where her opponent does not have an atom (and also if a tie would be broken in her favor) is,

$$\Pi_i(s, y_i; \sigma_{-i}) = -\alpha_i\left(s + y_i^2\right) + F_{-i}(s) \cdot V_i(s, y_i) + \int\limits_{s(y_{-i}, q_{-i}) > s} U_i(y_{-i}, q_{-i})\, d\sigma_{-i}.$$

Let $G_i(y_i; s)$ denote i's probability distribution over ideologies conditional on producing a policy with score s, let $w_i(y_i, y_{-i}; s)$ denote the probability that i's policy is selected when the players develop policies (s, y_i) and (s, y_{-i}), and let $\bar{y}_t(s)$ denote the expected ideological outcome conditional on a tie at score s (i.e. $\int\int (w_i(y_i, y_{-i}; s)\, y_i + (1 - w_i(y_i, y_{-i}; s))\, y_{-i})\, dG_i|s \cdot dG_{-i}|s$.

Part 1

Consider an equilibrium $(\sigma, w(\mathbf{b}))$ where the first part of the statement fails, so that with strictly positive probability a player i develops policies other than the reservation policy with score $s(b_0) = 0$; all such policies must have strictly positive quality. If σ_{-i} doesn't generate an atom at $s = 0$, then i's utility for developing one such policy $(0, y_i)$ is $\Pi_i(b_0; \sigma_{-i}) - \alpha_i y_i^2$, and she is strictly better off developing the reservation policy b_0. So suppose σ_{-i} also generates an atom at $s = 0$ of size p_{-i}; then i's utility for playing according her strategy conditional on generating score 0 is

$$-\alpha_i E\left[y_i^2\right] + p_{-i} V_i(0, \bar{y}_t(0)) + \int\limits_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i})\, d\sigma_{-i} = U_i^*$$

But since she can also achieve utility arbitrarily close to $V_i(0,0) + \int\limits_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i})\, d\sigma_{-i}$ simply by developing the reservation policy b_0 with ε-quality, it must be the case that

$$p_{-i}(V_i(0, \bar{y}_t) - V_i(0,0)) = 2x_i \cdot \bar{y}_t(0) \geq \alpha_i E\left[y_i^2\right] > 0$$

But again this cannot be true for both players since $sign(x_i) \neq sign(x_{-i})$ so we have a contradiction. Intuitively, playing the tie is costly for both players, both could achieve the reservation policy effectively for free instead, and the policy that results from a tie cannot be on average better for both players than the reservation policy due to linearity and opposing ideologies.

Part 2

Consider an equilibrium where the second part fails, so that each player i's strategy generates an atom at some common $s > s(b_0) = 0$ of size p_i. It is straightforward to verify (exploiting the linearity of $V_i(s, y_i)$) that player i's utility for playing according to her strategy conditional on generating score s can be written as both

$$-\alpha_{-i} Var[y_i|s] + \lim_{s_i \to s^-} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} + 2x_i \cdot p_{-i}(\bar{y}_t(s) - E[y_{-i} \mid s]) \quad \text{and}$$
$$-\alpha_{-i} Var[y_i|s] + \lim_{s_i \to s^+} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} + 2x_i \cdot p_{-i}(\bar{y}_t(s) - E[y_i \mid s])$$

Now $\lim_{s_i \to s+} \{\Pi_i(s_i, E[y_i|s]; \sigma_{-i})\} \leq \lim_{s_i \to s+} \left\{ \max_{y_i} \{\Pi_i(s_i, y_i; \sigma_{-i})\} \right\} \leq U_i^*$, and the same holds true for $\lim_{s_i \to s-} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\}$. Additionally, we have that $\lim_{s_i \to s+} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\} \neq \lim_{s_i \to s_-} \{\Pi_i(s_i, E[y_i]; \sigma_{-i})\}$ because $-i$ has an atom at s. So one of these terms must be strictly less than U_i^*. Since $-\alpha_i Var[y_i|s] \leq 0$, both of the third terms must then be weakly positive and at least one must be strictly positive - hence their sum must be strictly positive. Consequently, $\forall i$

$$x_i p_{-i} \left(\bar{y}_t(s) - \left(\frac{E[y_i \mid s] + E[y_{-i} \mid s]}{2} \right) \right) > 0,$$

i.e., the expected ideological outcome conditional on a tie must be better for i than the midpoint between the expected ideologies of each player's strategy at s. But this cannot be true for both players since since $sign(x_i) \neq sign(x_{-i})$, so we have a contradiction. ■

Proof of Lemma 2 Entrepreneur i's utility from developing policy (s_i, y_i) where $s_i > 0$ and σ_{-i} has no atom is,

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int\limits_{s(y_{-i}, q_{-i}) > s_i} U_i(y_{-i}, q_{-i})\, d\sigma_{-i}. \tag{3}$$

The derivative of eqn. (3) with respect to ideology y_i is linear and equal to $-2\alpha_i y_i + 2F_{-i}(s_i) x_i$, which generates the result.■

Proof of Lemma 3

Sufficiency

We first show that ideological optimality and no ties jointly imply that every policy delivers utility $\leq \Pi_i^*(s, y_i^*(s); F)$ for some s, which furthermore implies that i's utility for developing any feasible policy is $\leq \max_s \{\Pi_i^*(s, y_i^*(s); F)\}$. We then show that the three conditions imply i's utility for playing her strategy is $= \max_s \{\Pi_i^*(s, y_i^*(s); F)\}$, which means she has no profitable deviation and we have an equilibrium.

Subpart 1

First, note that i can achieve utility equal to $\Pi_i^*(s, y_i^*(s); F)$ with policy $(s, y_i^*(s))$ for any $s > 0$ where her opponent $-i$ has no atom, and utility arbitrarily close to $\Pi_i^*(s, y_i^*(s); F)$ for $s \geq 0$ where her opponent does have an atom using policy $(s+\varepsilon, y_i^*(s+\varepsilon))$ for arbitrarily small ε.

Second, i's exact utility for developing any policy (s, y_i) with $s \geq 0$ is,

$$\Pi_i^*(s, y_i; F) - p_{-i}(s) \cdot \left(1 - w_i\left(y_i, y_{-i}^*(s)\right)\right) 2x_i\left(y_i - y_{-i}^*(s)\right), \tag{4}$$

where $p_{-i}(s)$ denotes the size of $-i$'s atom at s and $w_i(y_i, y_{-i}; s)$ is as previously defined. Note that we are applying the no-ties property in the case of $s = 0$; no ties implies that $p_{-i}(0) > 0 \rightarrow F_i(0) = 0 \rightarrow y_{-i}^*(0) = 0$, which implies that whenever $s = 0$ and $p_{-i}(0) > 0$ and i's policy is not selected, the reservation policy – which is equal to $\left(0, y_{-i}^*(0)\right)$ – is the outcome.

Now, if $p_{-i}(s) = 0$ (i.e. $-i$ has no atom at s) or $p_{-i}(s) > 0$ but $w_i\left(y_i^*(s), y_{-i}^*(s)\right) = 1$ (i wins for sure in a tie between ideologically-optimal policies at s), then i achieves utility $\Pi_i^*(s, y_i^*(s); F) \geq \Pi_i^*(s, y_i; F)$ by developing $(s, y_i^*(s))$ and the property holds. If instead $p_{-i}(s) > 0$ ($-i$ has an atom at s) and $w_i\left(y_i^*(s), y_{-i}^*(s)\right) < 1$ (i will not win in a tie for sure) then $x_i y_i^*(s) > 0 \geq x_i y_{-i}^*(s)$ (winning at s is strictly beneficial) $\rightarrow \Pi_i^*(s, y_i^*(s); F) >$ eqn. (4). And i since can achieve utility arbitrarily close to $\Pi_i^*(s, y_i^*(s); F)$ by developing some $(s+\varepsilon, y_i^*(s+\varepsilon))$, the property again holds. Finally, i's utility for developing a policy (s, y_i) with $s < 0$ is $-\alpha_i(s + y_i^2) + \Pi_i^*(0, 0; F)$ (again applying the no ties property), which is weakly worse than $\Pi_i^*(0, 0; F)$. Since $\Pi_i^*(0, 0; F)$ is i's exact utility from developing the reservation policy, the preceding arguments apply.

Subpart 2

Suppose a strategy profile satisfies no ties, ideological optimality, and score optimality. Then every $s \in \text{supp}\{F_i\}$ satisfies $\Pi_i^*(s, y_i^*(s); F) = \max_s \{\Pi_i^*(s, y_i^*(s); F)\}$ by score optimality, at all such s where $-i$ has no atom i's utility for developing policy $(s, y_i^*(s))$ is in fact $\Pi_i^*(s, y_i^*(s); F)$, and by no ties the set of $s \in \text{supp}\{F_i\}$ where $-i$ has an atom is probability 0; thus i's utility from playing her strategy is equal to $\max_s \{\Pi_i^*(s, y_i^*(s); F)\}$.

Necessity

Necessity of no ties is just Lemma 1. We now argue that no ties and equilibrium jointly imply ideological optimality. Suppose not, and we have an equilibrium where no ties holds and ideological optimality fails. Because negative-score policies with positive quality are strictly dominated by developing the reservation policy, some player i must be placing strictly positive probability on policies (y_i, q_i) with scores $s(y_i, q_i) \geq 0$ that satisfy $y_i \neq y_i^*(s(y_i, q_i); F_{-i})$. By no ties, at least one such policy $(\hat{y}_i, \hat{s}_i)$ must deliver i's equilibrium utility and not generate a score tie with $-i$. But then Lemma 2 implies that developing $(s(y_i, q_i), y_i^*(s(y_i, q_i); F_{-i}))$ would deliver strictly higher utility, a contradiction.

We now argue that no ties, ideological optimality, and equilibrium jointly imply the necessity of score optimality. First, when $-i$'s strategy satisfies ideological optimality then i can achieve utility arbitrarily close to $\Pi_i^*(s, y_i^*(s); F)$ for any s, so equilibrium utility must be $\geq \{\Pi_i^*(s, y^*(s); F)\}$. Second, if $-i$ has no atom at $\hat{s}_i$, then i's utility for developing policies $(s, y_i^*(s))$ in an $\varepsilon-$ball around $\hat{s}_i$ approaches $\Pi_i^*(\hat{s}_i, y_i^*(\hat{s}_i); F) < \max_{s_i} \{\Pi_i^*(s_i, y_i^*(s_i); F)\}$, and since the probability is strictly positive for any ε we have a contradiction. Third, if $-i$ has an atom at $\hat{s}_i$, then i cannot be developing policies with scores below $\hat{s}_i$ within a sufficiently small neighborhood, her probability of developing policies in an $\varepsilon-$half ball $[\hat{s}_i, \hat{s}_i + \varepsilon]$ must be strictly positive for any ε, her utility for doing so again approaches $\Pi_i^*(\hat{s}_i, y_i^*(\hat{s}_i); F) < \max_{s_i} \{\Pi_i^*(s_i, y_i^*(s_i); F)\}$ by right-continuity of F_{-i}, and we again have a contradiction. All cases are covered, which completes the proof. ■

Proof of Proposition 1

Part 1

We show that in any equilibrium, the support of both player's score CDFs must be a common interval $[0, \bar{s}]$. We first argue that each player's support must be bounded. Boundedness from below is assumed w.l.o.g. To see that i's support is bounded from above, first observe that $|y_i^*(s_i; F_i)| \leq \left|\frac{x_i}{\alpha_i}\right|$ – in words i will never play an ideology beyond the weighted midpoint between herself and the decisionmaker. Thus, i's utility from developing the reservation policy $\Pi_i^*(0, 0; F) = \int V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_i$ is $\geq V_i\left(E[s_{-i}], \frac{x_{-i}}{\alpha_{-i}}\right)$ and is thus bounded from below. Conversely, it is easy to verify that $\lim_{s \to \infty} \Pi_i^*(s; F) \to -\infty$ for any F; thus unbounded support would require scores that cannot satisfy score optimality.

We next argue that the players' strategies must have common support. Suppose not. Then there $\exists \hat{s}$ in the support of i and an interval $[\hat{s} - \varepsilon, \hat{s}]$ over which $F_{-i}(s)$ is constant. But this contradicts score optimality since $\Pi_i^*(\hat{s}; F) - \Pi_i^*(\hat{s} - \varepsilon; F) = -\alpha_i \varepsilon$. Intuitively, increasing the score from $\hat{s} - \varepsilon$ to $\hat{s}$ has no policy benefits because $-i$ has no support and is thus all (net) cost. Finally, the common support must be the full interval $[0, \bar{s}]$ between score 0 and the (common) maximum score $\bar{s}$. Suppose not. Then there exists two scores $s', s'' \in [0, \bar{s}]$ such that $F_i(s)$ is constant over $[s', s'')$ $\forall i$. Score optimality would be violated if $F_i(s'') = F_i(s')$ for any i by the argument in the previous paragraph; so we require $F_i(s'') > F_i(s')$ $\forall i$. But this is the same as both players having an atom at s'' which violates no ties.

Part 2:

F_i must be continuous $\forall i$ over the the common support $[0, \bar{s}]$ – otherwise $\Pi_{-i}^*(s; F)$ would have a discontinuity and score optimality could not be satisfied. Continuity and score optimality imply

$$\begin{aligned} \frac{\partial}{\partial s} \Pi_i^*(s; F) &= \left.\frac{\partial \Pi_i^*(s, y_i; F)}{\partial s}\right|_{\left(s, y_i^*(s)\right)} \quad \text{(by envelope theorem)} \\ &= -\alpha_i + F_{-i}(s) + f_{-i}(s)\left(V_i\left(s, y_i^*(s)\right) - V_i\left(s, y_{-i}^*(s)\right)\right) = 0 \ \forall i, s \in [0, \bar{s}], i \end{aligned}$$

which is equivalent to the differential equation in the statement. Finally, the boundary condition at the bottom that $F_i(0) > 0$ for at most one i follows from no ties – otherwise $y_i(0) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(0) \neq$

0, i would be developing $(0, y_i(0))$ with strictly positive probability by ideological optimality which violates it. ■

Proof of Proposition 2:

Part 1

We seek a solution to the differential equation

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(\left(\frac{x_i}{\alpha_i} \right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}} \right) F_i(s) \right) \tag{5}$$

that satisfies the necessary boundary conditions. We can transform the equation as,

$$\frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)} = 2x_i \left(\left(\frac{x_i}{\alpha_i} \right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}} \right) F_i(s) \right)$$

which implies that $\frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)} = -\frac{\alpha_{-i} - F_i(s)}{x_{-i} f_i(s)}$. Now letting $s_i(F_i)$ denote the inverse of $F_i(s)$, observing that $s_i'(F_i) = \frac{1}{f_i(s_i(F_i))}$, substituting in $s_i(F_i)$ for s, and rearranging yields

$$\frac{\alpha_i - F_{-i}(s_i(F_i))}{x_i} = -\left(\frac{\alpha_{-i} - F_i}{x_{-i}} \right) \cdot \frac{\partial}{\partial F_i} \left(F_{-i}(s_i(F_i)) \right)$$

This is a differential equation on the composite function $F_{-i}(s_i(F_i))$ giving entrepreneur $-i$'s probability of developing a policy with score less than the score $s_i(F_i)$ associated with F_i. It is simple to verify that the following function with an arbitrary constant c solves the differential equation:

$$F_{-i}(s_i(F_i)) = \alpha_i + c\left(\alpha_{-i} - F_i\right)^{-\frac{x_{-i}}{x_i}}$$

Now, from Proposition 1 we know that the boundary condition $F_{-i}(s_i(F_i)) = 1$ must be satisfied, since $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$. Imposing this boundary condition implies that $c = -(\alpha_i - 1)(\alpha_{-i} - 1)^{\frac{x_{-i}}{x_i}}$; substituting and rearranging then yields,

$$\left(\frac{\alpha_i - F_{-i}(s_i(F_i))}{\alpha_i - 1} \right)^{x_i} = \left(\frac{\alpha_{-i} - F_i}{\alpha_{-i} - 1} \right)^{-x_{-i}}$$

Finally, substituting $F_i(s)$ for F_i yields

$$\left(\frac{\alpha_i - F_{-i}(s)}{\alpha_i - 1} \right)^{|x_i|} = \left(\frac{\alpha_{-i} - F_i(s)}{\alpha_{-i} - 1} \right)^{|x_{-i}|} \iff \epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))$$

– i.e., at every score the probabilities of victory must be such that the entrepreneurs are equally engaged.

Part 2

Part 1 proves that there is a unique equilibrium engagement $\epsilon(s)$ associated with every score, where $\epsilon(s) = \epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))$. It is simple to verify by taking logs and differentiating that $-\frac{\epsilon(s)}{\epsilon'(s)} = \frac{\alpha_i - F_{-i}(s)}{x_i f_{-i}(s)}$, and hence that

$$-\frac{\epsilon(s)}{\epsilon'(s)} = 2\left(\left(\frac{x_i}{\alpha_i}\right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)\right)$$

Now letting $p_i(\epsilon) = \epsilon_i^{-1}(p) = \alpha_i - (\alpha_i - 1)\,\epsilon^{\frac{1}{|x_i|}}$, we can rewrite the differential equation in terms of the inverse function $s(\epsilon)$, which yields

$$s'(\epsilon) = -2\sum_i \frac{(|x_i|/\alpha_i)\cdot p_i(\epsilon)}{\epsilon}.$$

It is then easily verified that $\int \frac{(|x_i|/\alpha_i)\cdot p_i(\epsilon)}{\epsilon} = |x_i|\left(\ln(\epsilon) + (|x_i|/\alpha_i)\cdot p_i(\epsilon)\right)$; thus,

$$s(\epsilon) = 2\sum_i |x_i|\left(-\ln(\epsilon) - \left(\frac{|x_i|}{\alpha_i}\right)\cdot p_i(\epsilon)\right) + C.$$

Finally, we must set the constant. We know that the score ranges from $[0, \bar{s}]$, and that score is a decreasing function of engagement; so the maximum engagement $\epsilon^*(0) = \bar{\epsilon}$ is associated with the minimum score $s = 0$. We argue that $\bar{\epsilon} = \min_i\{\epsilon_i(0)\} = \epsilon_k(0)$. If the maximum engagement were lower, then $F_i(0) = p_{-i}(\bar{\epsilon}) > p_{-i}(\epsilon_k(0)) \geq 0\ \forall i$ and the boundary condition at the bottom score would not be satisfied. If the maximum engagement were higher, then for entrepreneur $-k$, $F_{-k}(0) = p_k(\bar{\epsilon}) < p_k(\epsilon_k(0)) = 0$, a contradiction. Hence, C must be such that $s(\epsilon_k) = 0$. The unique solution can be divided up among the four additive subterms so that:

$$s(\epsilon) = 2\sum_i |x_i|\left(\ln\left(\frac{\epsilon_k}{\epsilon}\right) - \left(\frac{|x_i|}{\alpha_i}\right)\cdot\left(p_i(\epsilon) - p_i(\epsilon_k)\right)\right).$$

Entrepreneur i's score CDF at s must then be the unique probability such that her opponent $-i$'s engagement is equal to $\epsilon(s)$ (the inverse of $s(\epsilon)$), i.e. $F_i(s) = p_{-i}(\epsilon(s))$.

Proof of Lemma 4

First sufficiency: $\epsilon_{-k}(F_k(s)) = \epsilon_k(F_{-k}(s))$ and $\epsilon_{-k}(p) > \epsilon_k(p)\,\forall p \rightarrow F_k(s) > F_{-k}(s)$ since $\epsilon_i(p)$ is decreasing in p. Now necessity. $\epsilon_{-k}(F_k(s)) = \epsilon_k(F_{-k}(s))$ and $F_k(s) > F_{-k}(s) \rightarrow \epsilon_{-k}(F_{-k}(s)) > \epsilon_k(F_{-k}(s))$. Since $F_{-k}(s)$ maps one to one to $[0,1]$ (since $-k$ is always active) we must have $\epsilon_{-k}(p) > \epsilon_k(p)\,\forall p$.

Proof of Lemma 5

We first show that the equilibrium score function $s^*(\epsilon)$ is increasing in either x_i and decreasing in either α_i $\forall\epsilon$. Expressing the dependence of equilibrium quantities on an arbitrary structural parameter $q \in \{x_L, x_R, \alpha_L, \alpha_R\}$, the function can be written as $s(\epsilon; q) = \int_1^{\epsilon} s'(\varepsilon; q)\, d\varepsilon + C(q)$. Thus

$$\frac{\partial s(\epsilon; q)}{\partial q} = \int_1^{\epsilon} \frac{\partial s'(\varepsilon; q)}{\partial q} d\varepsilon + C'(q) \tag{6}$$

Since the constant is chosen so that $s(\epsilon_k; q) = \int_1^{\epsilon_k(q)} s'(\varepsilon; q)\, d\varepsilon + C(q) = 0$ (where $\epsilon_k(q)$ is shorthand for $\epsilon_k(0; q)$), we then have

$$C'(q) = -\int_1^{\epsilon_k} \frac{\partial s'(\varepsilon; q)}{\partial q} d\varepsilon - \frac{\partial \epsilon_k(q)}{\partial q} s'(\epsilon_k; q)$$

Combining with (6) yields,

$$\begin{aligned}
\frac{\partial s(\epsilon; q)}{\partial q} &= -\int_{\epsilon}^{\epsilon_k} \frac{\partial s'(\varepsilon; q)}{\partial q} d\varepsilon - \frac{\partial \epsilon_k(q)}{\partial q} s'(\epsilon_k; q) \iff \\
\left(\frac{1}{2}\right) \frac{\partial s(\epsilon; q)}{\partial q} &= \int_{\epsilon}^{\epsilon_k} \frac{\partial}{\partial q} \left(\sum_i \frac{(|x_i| / \alpha_i) \cdot p_i(\varepsilon)}{\varepsilon} \right) d\varepsilon + \frac{\partial \epsilon_k(q)}{\partial q} \cdot \left(\frac{|x_{-k}|}{\alpha_{-k}} \frac{p_{-k}(\epsilon_k)}{\epsilon_k} \right)
\end{aligned}$$

It is then straightforward to see that $s(\epsilon; q)$ is strictly increasing in x_i for either i and strictly decreasing in α_i; the functions $\epsilon_i(p)$ satisfy the desired comparative statics (and hence $\epsilon_k(0)$ does), the inverse functions $p_i(\varepsilon)$ inherit the same comparative statics in the structural parameters, and hence the quantity $(|x_i| / \alpha_i) \cdot p_i(\varepsilon)$ in the integral also satisfies the desired comparative statics, so the overall expression does as well.

Now $\bar{s} = s(1)$ inherits the previously proved comparative statics that hold for all levels of engagement ϵ; thus the comparative statics stated in the Lemma hold. The effects of an opponent's parameters (x_{-i}, α_{-i}) on entrepreneur i's expected utility then follow immediately from the expression in the main Lemma.

Proof of Lemma 6

In symmetric case ($|x_i| = x$ and $\alpha_i = \alpha$), Corollary 4 implies that the ideological extremism of policies is uniformly distributed over $\left[0, \frac{x}{\alpha}\right]$, score CDFs are identical ($F_i(s) = F_{-i}(s)$), and policies are symmetric ($y_i(s) = -y_{-i}(s)$). Noting that $p_i(\epsilon_k) = 0\ \forall i$ from symmetry (both entrepreneurs are always active), we can write the equilibrium score function $s^*(\epsilon)$ as

$$s^*(\epsilon) = 4x\left(\ln\left(\frac{\epsilon_k}{\epsilon}\right) - \frac{x}{\alpha}p_k(\epsilon)\right) \tag{7}$$

Now, every score s is associated with both a unique level of engagement and a unique degree of ideological extremism y. Since $\epsilon(s) = \epsilon_k(F(s)) = \epsilon_k\left(\frac{y(s)}{x/\alpha}\right)$, the level of engagement associated with each degree of ideological extremism must be $\epsilon(y) = \epsilon_k\left(\frac{y}{x/\alpha}\right)$. So we can write score as a function of ideological extremism $s^*(y)$ which is,

$$\begin{aligned}
s^*\left(\epsilon_k\left(\frac{y}{x/\alpha}\right)\right) &= 4x\left(\ln\left(\frac{\epsilon_k}{\epsilon_k\left(\frac{y}{x/\alpha}\right)}\right) - \frac{x}{\alpha}p_k\left(\epsilon_k\left(\frac{y}{x/\alpha}\right)\right)\right) \\
&= 4x\left(x\ln\left(\frac{x}{x-y}\right) - y\right)
\end{aligned}$$

Note that the score as a function of ideological extremism does not depend on α. The maximum score is then

$$\begin{aligned}
\bar{s} &= s^*\left(\frac{x}{\alpha}\right) = 4x\left(x\ln\left(\frac{x}{x-x/\alpha}\right) - \frac{x}{\alpha}\right) \\
&= 4x^2\left(\ln\left(\frac{\alpha}{\alpha-1}\right) - \frac{1}{\alpha}\right)
\end{aligned}$$

and expected utilities of the entrepreneurs are straightforward to derive.

The expected utility of the decisionmaker is,

$$\int_0^{\bar{s}} \frac{\partial\left(F^2(s)\right)}{\partial s} s \cdot ds \tag{8}$$

since $F^2(s)$ is the CDF of the maximum score. We can derive the inverse function $F^{-1}(p)$ by observing that $F(s) = p_k(\epsilon^*(s)) \rightarrow F^{-1}(p) = s^*(\epsilon_k(p))$. Substituting into (7) yields,

$$F^{-1}(p) = 4x^2 \left(\ln\left(\frac{\alpha}{\alpha - p} \right) - \frac{p}{\alpha} \right)$$

Using this we perform a change of variables on (8) so the DM's expected utility is,

$$\int_0^1 \frac{\partial}{\partial p}\left(p^2\right) F^{-1}(p)\, dp = 4x^2 \int_0^1 2p \left(\ln\left(\frac{\alpha}{\alpha - p} \right) - \frac{p}{\alpha} \right)$$

Integration by parts and algebra verifies that the definite integral in the expression above equals the term inside the parentheses in the Lemma, yielding the result.

Proof of Lemma 3

First order stochastic changes in ideology $|y_i|$ are obvious since the CDF of $|y_i|$ is $\frac{|y_i|}{x/\alpha}$ (i.e. it is uniform).

Now since $F(s) = \frac{y(s)}{x/\alpha}$ and $y(s)$ is unaffected by α (since $s(y)$ is unaffected), first order stochastic decreasing in α is straightforward. To see that $F(s)$ is first order stochastically increasing in x, note that $F^{-1}(F(s;x);x) = s \rightarrow \frac{\partial F}{\partial x} = -\frac{\partial F^{-1}/\partial x}{\partial F^{-1}/\partial p}$. Clearly $F^{-1}(p)$ is increasing in p and from the previous proof $F^{-1}(p)$ is obviously increasing in x, hence $\partial F/\partial x < 0$ and F is first-order stochastically increasing in x.

Proof of Lemma 7

To be written.

Proof of Corollary 7

Part (i). The maximum score from Corollary 5 can be rewritten as,

$$2\left((x_k + x_{-k}) \cdot \ln(\epsilon_k) - \frac{x_k}{\alpha_k} - x_{-k}\left(1 - \frac{1}{\alpha_{-k}}\right)\left(\epsilon_k^{\frac{1}{\lceil x_{-k} \rceil}} - 1\right) \right)$$

which clearly converges to 0 as $\alpha_k \rightarrow \infty$ since $\frac{x_k}{\alpha_k} \rightarrow 0$ and $\epsilon_k \rightarrow 1$. Thus the decisionmaker's utility must converge to zero.

Part (ii). Suppose not, i.e., the decisionmaker's utility converges to zero. Then for any score $s > 0$, $F_{-k}(s) \to 1$. Consider $\hat{s} = \frac{x_k^2}{2\alpha_k}$. Because $-k$ only develops policies on her side of the decisionmaker, $k's$ utility from any policy $-k$ develops at a score $\leq \hat{s}$ is less than $-x_k^2 + \hat{s}$. Thus, rather than staying out (which k does with strictly positive probability) she would be strictly better off offering policy $\left(\frac{x_k}{\alpha_k}, \left(\frac{x_k}{\alpha_k}\right)^2 + \hat{s}\right)$, which improves her utility by at least

$$
\begin{aligned}
& F_{-k}\left(\hat{s}\right)\left[\left[-\left(x_k - \frac{x_k}{\alpha_k}\right)^2 + \left(\frac{x_k}{\alpha_k}\right)^2 + \hat{s}\right] - \left[-x_k^2 + \hat{s}\right]\right] - \alpha_k\left[\left(\frac{x_k}{\alpha_k}\right)^2 + \hat{s}\right] \\
= \; & F_{-k}\left(\hat{s}\right)\left[\frac{2x_k^2}{\alpha_k} + x_k^2\right] - \frac{x_k^2}{\alpha_k} - \alpha_k \hat{s} \\
= \; & \frac{x_k^2}{\alpha_k}\left(2F_{-k}\left(\hat{s}\right) - 1\right) + x_k^2\left[F_{-k}\left(\hat{s}\right) - \frac{1}{2}\right]
\end{aligned}
$$

which is strictly greater than zero because $F_{-k}(\hat{s}) \to 1$.

Figure 1: Game

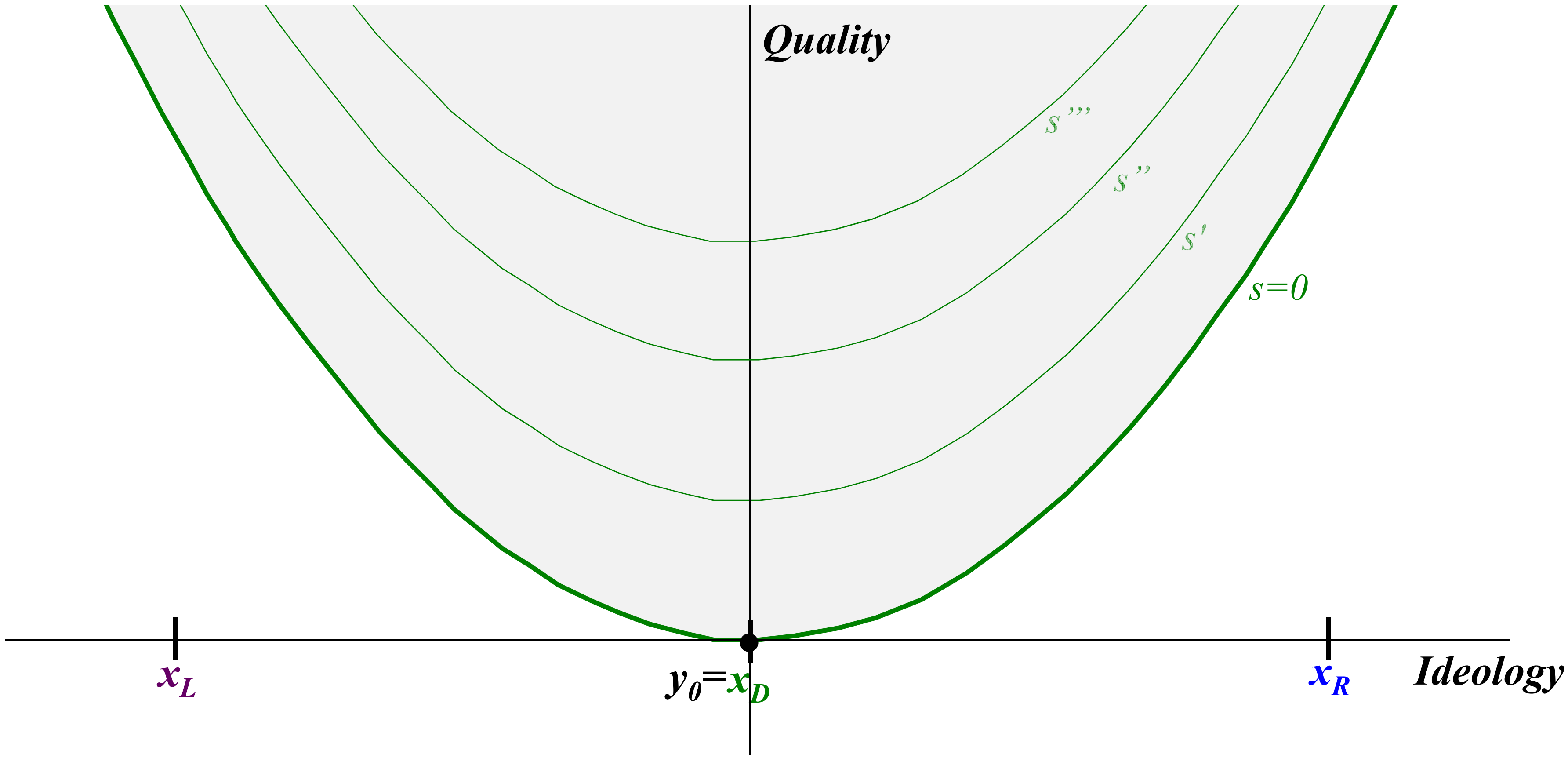


Figure 2a: Equilibrium Score CDFs

$-x_L = x_R,\ \alpha_L > \alpha_R$

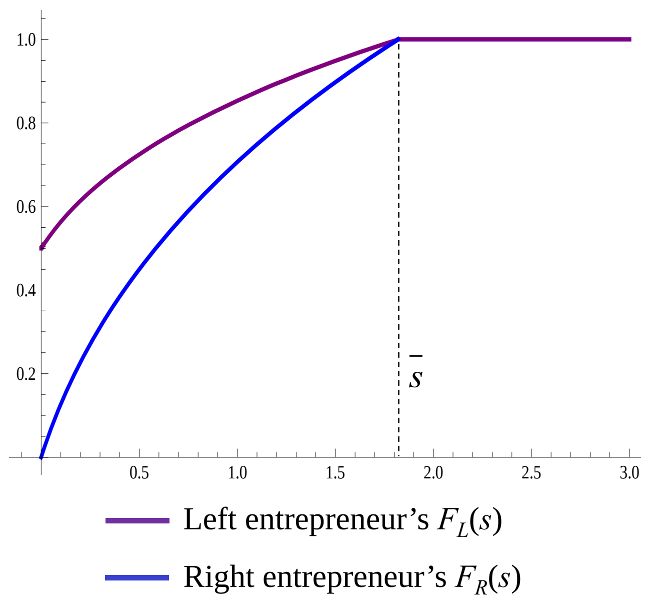


Figure 2b: Equilibrium Policies

$-x_L = x_R,\ \alpha_L > \alpha_R$

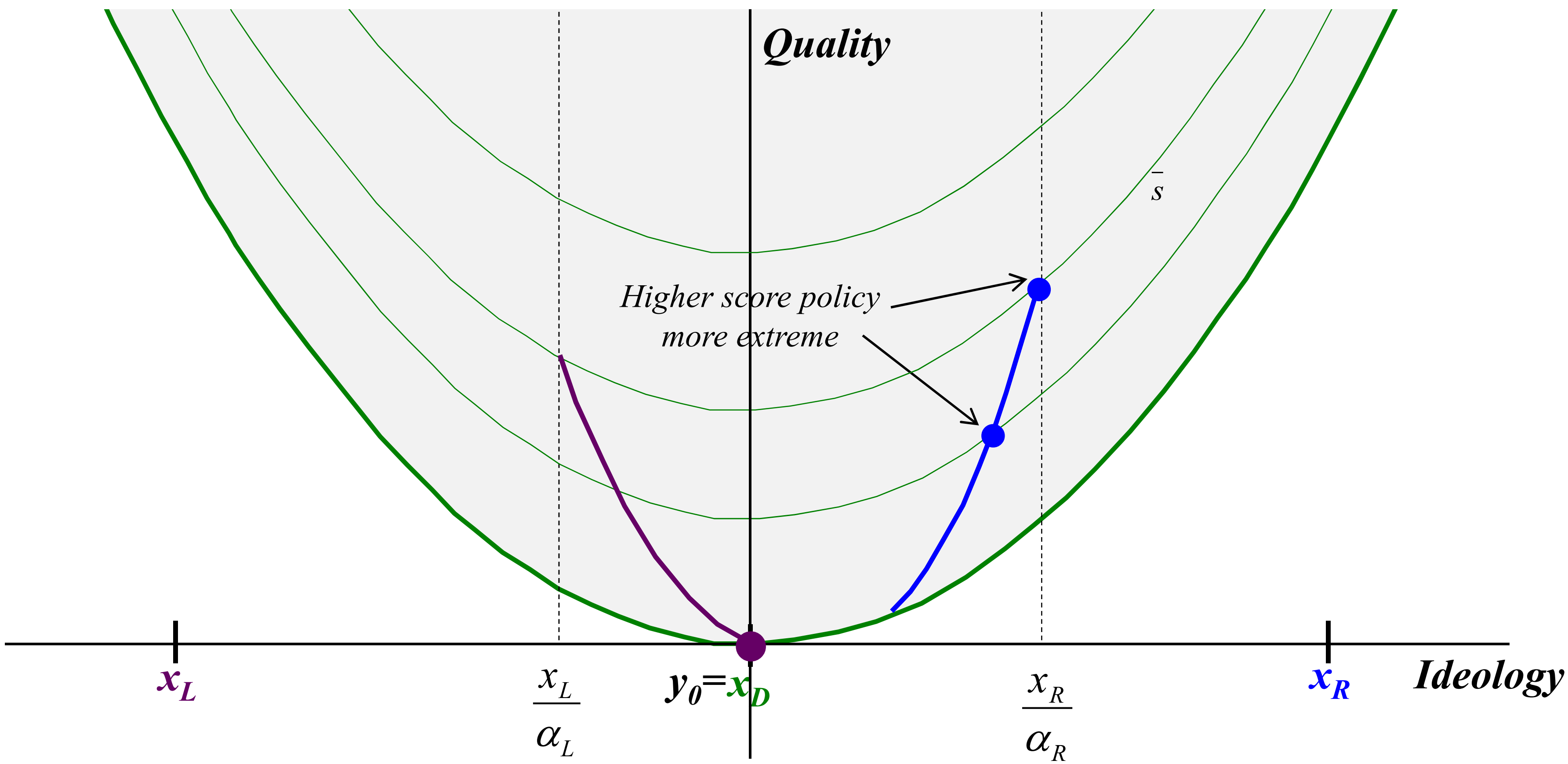
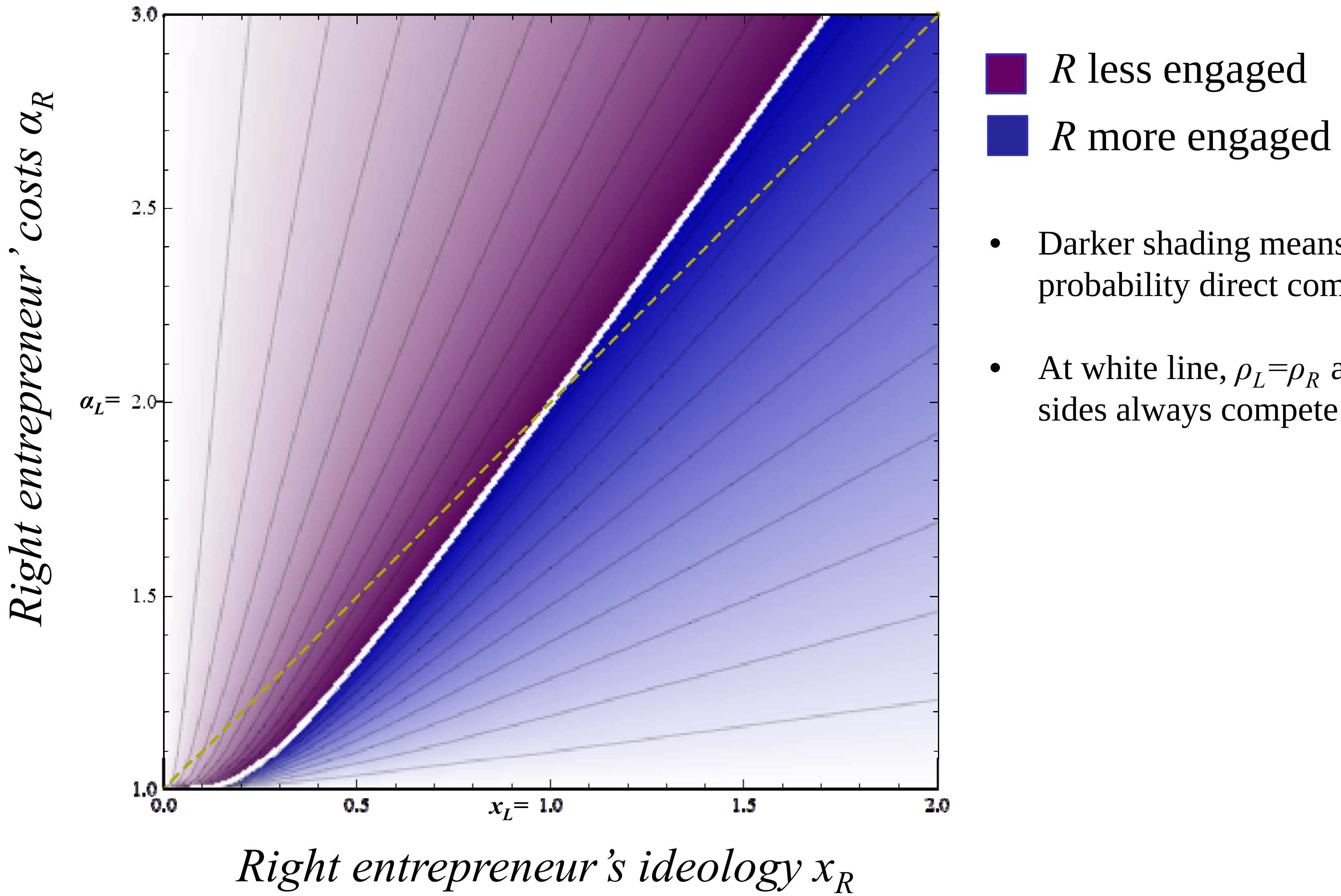


Figure 3: Effect of Right Entrepreneur's Parameters on Probability of Direct Competition

$x_L = 1$, $\alpha_L = 2$



- Darker shading means higher probability direct competition
- At white line, ρ_L=ρ_R and both sides always compete

Figure 4a: Equilibrium $F(s)$

$-x_L = x_R,\ \alpha_L = \alpha_R$

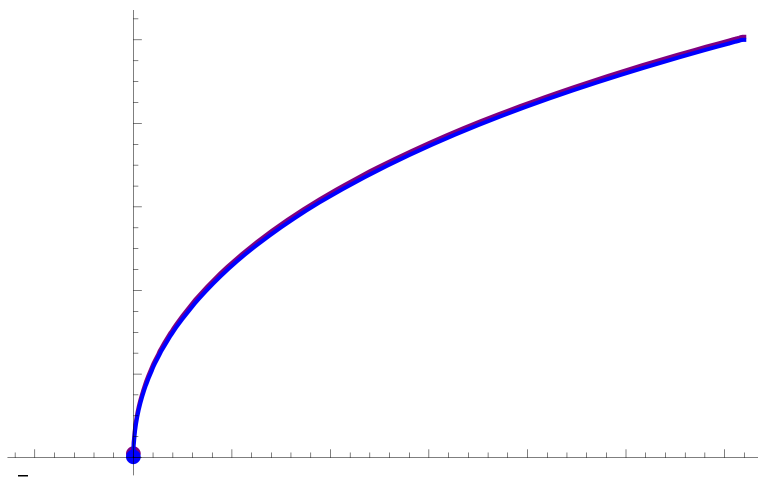


Figure 4b: Equilibrium Policies

$-x_L = x_R,\ \alpha_L = \alpha_R$

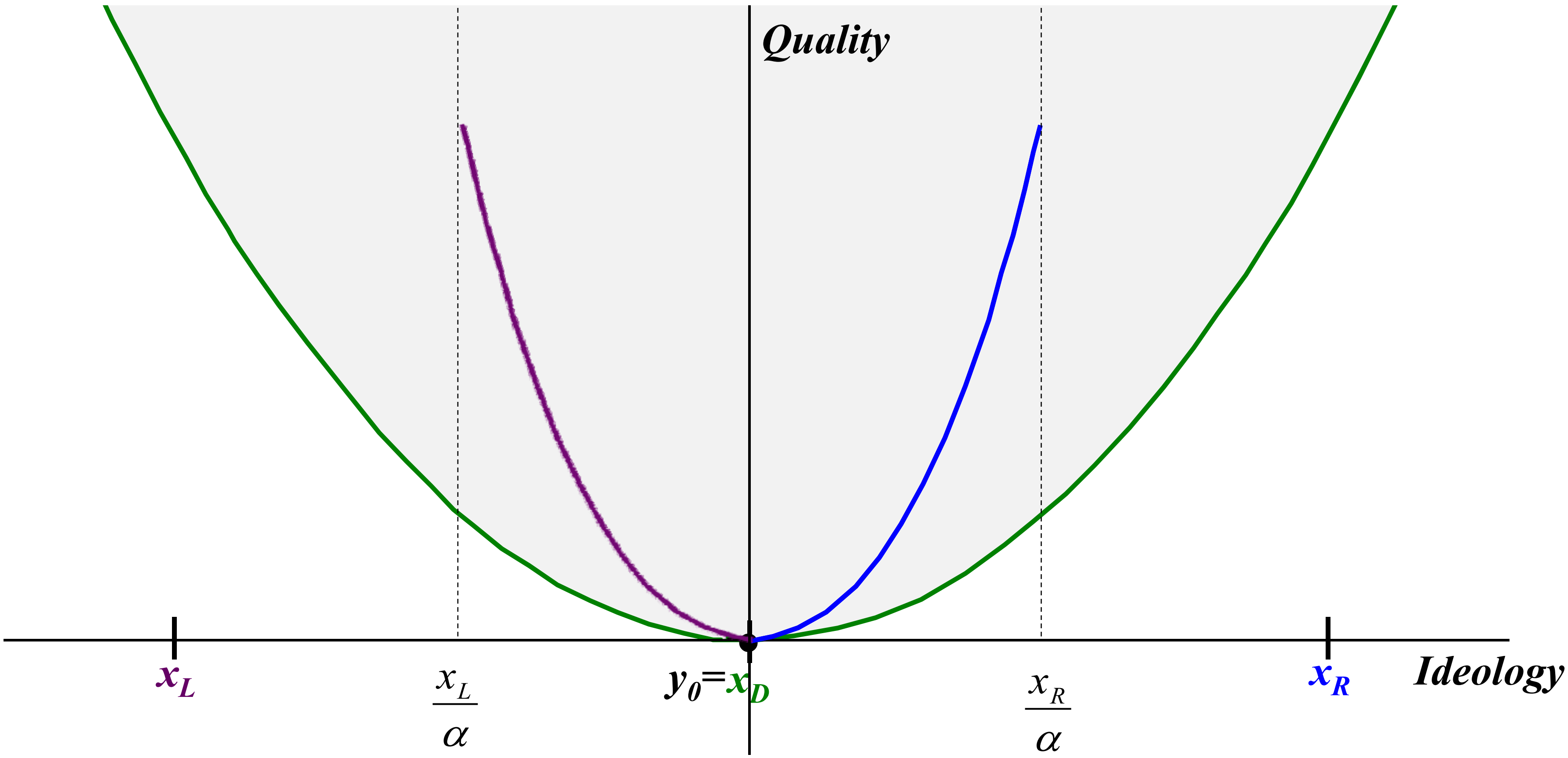


Figure 5a: Effect of increasing α on policies, $\alpha' > \alpha$

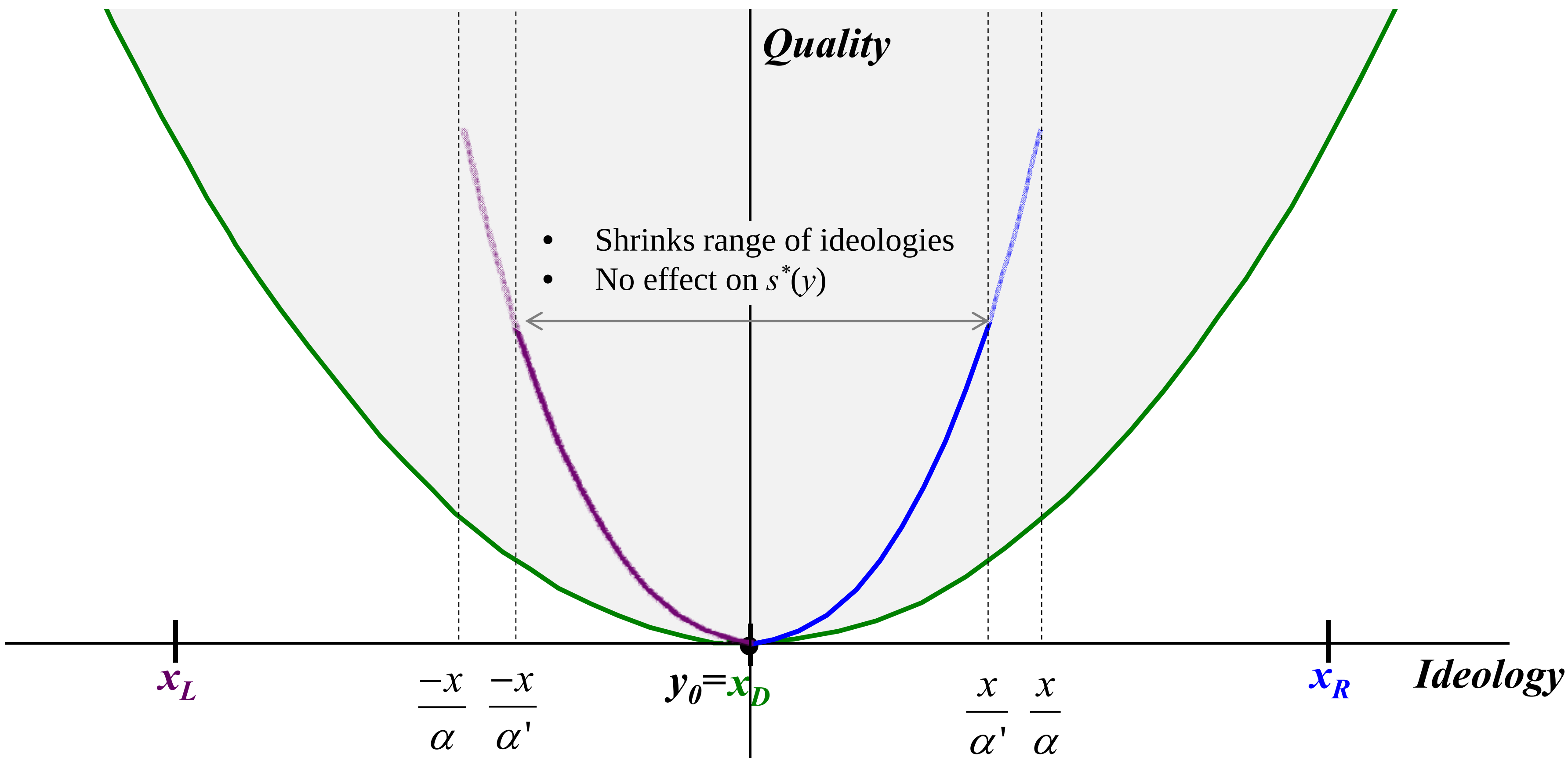


Figure 5b: Effect of increasing α on score CDFs, $\alpha' > \alpha$

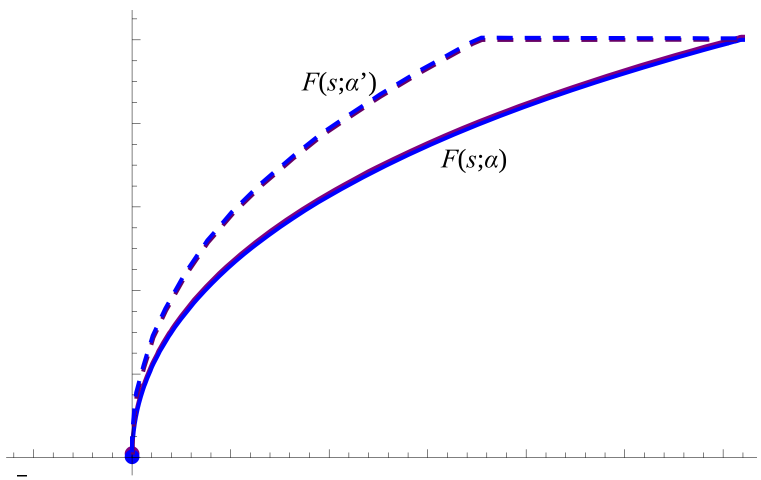


Figure 6: Effect of α on Entrepreneur Utility