Inequality and Social Comparisons

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Abstract

I present a model in which people develop certain traits (or skills) and also assign worth to those traits. The worth a person assigns to different traits are the person’s values. These values can be used to evaluate one’s own traits and the traits of others. I show that when people are incentivized to place more value on skills for which they are comparatively advantaged, people with lower opportunities for success may be led to adopt more extreme values, to perform below their own capabilities, and to perpetually experience cognitive dissonance by developing skills that are inconsistent with their own values. Attempts to induce low-status individuals to invest in certain skills by increasing the marginal productivity of those skills can backfire, leading to increased inequality.

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1 Introduction

Different individuals interviewing the same job candidate, watching the same debate, or listening to the same lecture often have markedly different perceptions of the people they are evaluating. People take an active role in interpreting the abilities of others, and may hold very different opinions about the skills and traits that others possess. These differences can be positive; they can potentially broaden individuals’ conception of what constitutes a successful or correct way of being. At the same time, these differences may pose particular challenges to democratic governance. Groups with values that differ from those of society at large may fail to develop trust in legal, educational, and civic institutions. In general, a government seeking to promote the general welfare of its citizens may face obstacles if people have dramatically different views of what constitutes a good life.

As the populations of many large metropolitan areas have become increasingly diverse, governments have faced the challenge of balancing multicultural accommodation with social integration, realizing that some degree of integration is necessary to enable universal access to economic and political opportunities. For example, differences across states in policies regarding headscarves and veils represent different approaches to achieving this balance. In Turkey and France the wearing of headscarves (and of any attire serving as an expression of religious affiliation) is banned in schools, reflecting, in part, an attempt to integrate Muslims into the broader community by emphasizing the secular nature of the state. Other countries, such as Canada, have pursued policies that accommodate the observance of religious and cultural norms, arguing that these policies actually further the integration of minority groups into mainstream society.

In this paper I am interested in how individuals with limited resources and varying capabilities choose to develop their own skills or traits, and in how they choose to value their own skills and the skills of others. I use the term values to represent a person’s belief that a certain collection of traits—a skill set representing how one has chosen to devote his or her energies—merits worth.¹ Thus, people with different values judge the success of people with different skills differ-

¹This idea is developed in (Bruner, 1990, p. 22).
ently. From this standpoint, a person’s values serve as a heuristic, or what Jerome Bruner terms a “communal tool kit,” that is used by individuals to both inform their own decisions about how to live their lives and to make sense of the decisions of others. By assigning more or less worth to different skills, people construct a narrative that explains the differences in the abilities of others.

In the model I develop, people simultaneously invest in various skills that yield an economic payoff, and place values on those skills. Similar to Rabin (1994), these values represent a belief that the mixture of skills invested in was good (appropriate, worthy, etc.), and individuals experience cognitive dissonance when their values conflict with the skills they have actually acquired.² Importantly, I assume that people are not only limited by their own capabilities in developing their skill sets, but also limited by a desire to protect their own self-esteem. This desire manifests itself in a “self-evaluation threshold” below which individuals will alter their skill sets, values, or both in an effort to improve their evaluation of themselves relative to their evaluation of others.

The idea that individuals and groups “selectively value” certain domains as an ego-defense mechanism has been well-supported in psychological studies, and the phenomenon can be derived from a number of theories of self-protection.³ At the individual level, numerous scholars have argued that perceived shortcomings on a particular domain cause people to describe that domain as less relevant to their concept of self.⁴ At the group level, studies have shown that people selectively value domains on which their in-group has fared better relative to an out-group; this “selective valuation” is characterized by values that are driven by comparison motivations, as opposed to

²Well-known in psychology and the social sciences, cognitive dissonance is the distress people experience when their behavior and beliefs conflict with each other. The theory originated with Festinger (1962), and was first incorporated into a formal model by Akerlof and Dickens (1982).

³James (1890) has been credited with originating this theory. He wrote:

“[O]ur self-feeling in this world...is determined by the ratio of our actualities to our supposed potentialities; a fraction of which our pretensions are the denominator and the numerator our success ... To give up pretensions is as blessed a relief as to get them gratified; and where disappointment is incessant and the struggle unending, this is what men will always do.” Quoted in (Schmader et al., 2001).

simple dissonance reduction. Moreover, many theories of intergroup relations, such as social identity theory, argue that an individual’s self-esteem is derived in part from his or her group membership, and that a desire to enhance self-esteem drives members of low status groups to devalue domains on which their group is at a relative disadvantage.

Given these assumptions, the model is used to examine several questions that are relevant to the goal of promoting desirable social outcomes, such as reducing social polarization and inducing individuals to fulfill their potential. The first question I consider concerns the effect of inequality on the values of individuals. One robust finding is that as the opportunities for success become more limited for low-status individuals, the process of selective valuation those individuals engage in must become more extreme; there is a level of inequality beyond which low-status individuals can, at most, place worth on only one trait. Too much inequality leads these individuals to adopt beliefs that only one type of skill merits value. Thus, inequality leads to polarization. As inequality increases further, in the limit, low-status individuals can place value on no domain. For such individuals to believe that any domain merits value imposes too high a burden on the individual, as it requires him or her to experience negative feelings of self worth. A consequence of this scenario is that when inequality is great, low-status individuals will invest in skills at levels below their own capabilities and will be unresponsive to increased economic incentives for skills acquisition. Thus, inequality generates substantial social inefficiencies.

This leads to the question of how economic incentives can shape the skills people cultivate and the values they hold. Becker’s famous model of human capital investment predicts that people will invest more heavily in skills as the economic benefits to those skills increase. Becker argues, for example, that fluctuations in the fraction of students attending college over time are a consequence of

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5 It has been argued that selective devaluation of a domain may be more apt to occur when status differences across groups are perceived as illegitimate by the low-status group. See Schmader and Major (1999); Schmader et al. (2001).

6 Tajfel and Turner (1979); For related work in political science, see Dickson and Scheve (2006); Dickson and Scheve (2010); Eguia (2013); Schnakenberg (2014); Shayo (2009), although the model presented in this paper is distinguished from some of these by not being one of endogenous social identity, but rather of the endogenous development of values.
of changes in the economic benefits to a college degree (Becker, 2009). Counter to this intuition, I find that increasing the rate of return to a particular skill may prompt a low-status type to invest even less in that skill. If people, in part, choose to invest in skills on domains that they are comparatively advantaged on, then incentivizing the acquisition of a particular skill for everyone may lead a comparatively disadvantaged person to shift his or her attention to a different, less productive domain in order to face comparative success on that domain, and to protect his or her self-esteem. The process is not symmetric—in equilibrium, comparatively advantaged individuals do not face these same self-esteem considerations.

My ultimate aim with this project is to present a model of politics consistent with the observation that individuals often evaluate people, policies, and institutions in terms of their perceived support for, or opposition to, different ways of life. This is reflected in the fact that questions concerning lifestyle and behavior choices are particularly salient to people, and that policy debates are frequently couched in terms that reflect these choices. Often these issues often concern behaviors that are protected versus those that are not. More specific examples include debates involving religion and science curricula in public schools, the role of women in the workforce, the legality of corporeal punishment in schools and the home, the death penalty, gay marriage and reproductive rights. In each of these instances an important aspect of the debate involves a tension between the social acceptability of behaviors and beliefs held by individuals versus behaviors and beliefs that are legitimized by the state. Of course this tension exists in part because political decisions can alter peoples’ values. For example, it is generally acknowledged that European welfare states in the late nineteenth century succeeded in bridging a vast class and ethnic divide on that continent, and that policies regarding female labor force participation have played an important part in changing gender-role attitudes.7

In describing the relationship between individual values and political outcomes, Aaron Wildavsky writes, “Preferences in regard to political objects are not external to political life; on the contrary, they constitute the very internal essence, the quintessence of politics: the construction

7See Crepaz and Damron (2008) and Sjöberg (2004), respectively.
and reconstruction of our lives together.”

Viewed in this way, my aim is to take a step toward formalizing a process by which the skills people develop and the values they hold jointly emerge, and in which institutions may potentially shape both.

## 2 The Model

People can dedicate energy toward cultivating various skills or traits of themselves. For simplicity I assume that there are two possible skills a person could cultivate, \( x \), and \( y \), and two types of people, \( i \) and \( j \). The “level” of each skill obtained by a person of type \( i \) is denoted \( x^i = (x^i, y^i) \).

Skills are valuable, and people receive a direct payoff from cultivating any skill. Skills could be the direct result of formal education—such as mathematics and writing—but could also include things such as being pious, physically beautiful, eloquent, creative, hardworking, or a team player. Each of these skills (or traits) is a type of competency that requires some time and energy to cultivate, and the “level” of such a skill that one obtains may be observable by others. The payoff that individual \( i \) receives by cultivating \( x^i \) is represented by \( M(x^i) \), or \( i \)'s material payoff. Each person faces the same \( M \), which can be thought of as the mechanism by which society rewards cultivation of the various traits. I assume that \( M \) is increasing, continuously differentiable and strictly concave in \( x^i \) and \( y^i \).

The “quantity” of skill that a person could cultivate is dependent on his or her ability. Thus, individuals are constrained by the equation \( p^i \cdot x^i \leq C^i \), where \( p^i = (p_{x}^i, p_{y}^i) \) is a vector representing the relative cost to an individual of type \( i \) to cultivating a unit of each skill, and \( C^i \) captures an over-

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8Wildavsky (1987, p. 5).

9As I discuss later, the analysis that follows could be readily extended to a setting with multiple types and a multitude of skills.

10Again, the setup could accommodate type-specific material payoff functions. I let \( M \) be the same across individuals because I think of it as an institutional lever that could potentially be used by policymakers to make certain skills more or less economically viable. My focus on the economic value of various skills is intended to make the analysis more easily comparable to standard models of human capital investment; the material payoff function could just as easily be used to capture psychological payoffs to particular skills or traits.
all endowment of ability of the individual. I refer to this constraint as \(i\)'s capability constraint.\(^{11}\)

The preceding few paragraphs set up a very typical consumer budgeting problem in which the individual (consumer) chooses a bundle of skills to cultivate (goods) dependent on his own capabilities (budget constraint), which may vary by skill (prices). The following equations define this baseline problem.

\[
\max_{\mathbf{x}^i} M(\mathbf{x}^i) \text{ subject to } \mathbf{p}^i \cdot \mathbf{x}^i \leq C^i. \quad \text{(Problem 1)}
\]

Let \(\mathbf{x}_o^i\) represent the baseline solution to the optimization problem presented in Problem 1, for each type \(i\).\(^{12}\)

### Adding values to the baseline model

Now consider a slight variation on the above problem in which people additionally receive a payoff from believing that they are behaving in an appropriate, correct or worthwhile way by having cultivated a “good” mixture of skills. I refer to these beliefs as a person’s values, \(\mathbf{v}^i = (v^i_x, v^i_y) \in \mathbb{R}^2_+\), with the ratio of \(v^i_x\) to \(v^i_y\) representing the optimal tradeoff between skills \(x\) and \(y\) according to person \(i\)’s values. The payoff to behaving in accordance with a person’s values is \(-D(\mathbf{x}^i, \mathbf{v}^i)\).

\(D\) can be thought of as the cognitive dissonance an individual experiences from investing in a mixture of skills that is inconsistent with \(\mathbf{v}^i\). I assume that \(D\) is convex, continuously differentiable, additively separable in \((x, v_x)\) and \((y, v_y)\), and strictly increasing in \(d(\mathbf{x}^i, \mathbf{v}^i)\), where \(d(a, b)\) is the distance between vectors \(a\) and \(b\). Utility now equals the sum of these two components:

\[
U^i(\mathbf{x}^i, \mathbf{v}^i) \equiv M(\mathbf{x}^i) - D(\mathbf{x}^i, \mathbf{v}^i).
\]

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\(^{11}\)This term is inspired by Sen’s capability approach (Sen, 1985), which defines an individual’s capability as the effective freedom of an individual to pursue various ends in his or her life. A person’s capability is distinct from his or her actual achievements, or functionings in Sen’s language. The notion of functionings as derived from capabilities is consistent with the acquisition of skills in this model.

\(^{12}\)We know \(\mathbf{x}_o^i\) is unique because the objective function in Problem 1 is strictly concave and the capability constraint is linear.
Suppose that people face the following new problem:

\[
\max_{x^i, v^i} U^i(x^i, v^i) \text{ subject to } p^i \cdot x^i \leq C^i. \tag{Problem 2}
\]

Given Problem 1, the solution to Problem 2 is trivial: everyone chooses \( x^i = x^i_o \) and sets \( v^i = x^i_o \). Thus, at an optimal solution to Problem 2 each person chooses to cultivate a collection of skills that maximizes his or her material payoff, and maintains values that perfectly justify those choices. Thus, each person believes that his or her skills are best. That values have no effect on behavior in this case distinguishes the model from some prior work in behavioral decision theory that assumes values are static, and thus affect behavior in the absence of interpersonal comparisons. It also distinguishes the model from other approaches that assume the existence of “true” disinterested beliefs, and that changes in these true beliefs to reduce dissonance incur a cost.\(^{13}\) In this model there is no direct cost to changing one’s values in the absence of interpersonal comparisons.

**Adding values and interpersonal comparisons to the baseline model**

Now consider a final problem, which is the problem that this paper is primarily concerned with. Again suppose that people receive a payoff from believing that they have cultivated worthwhile traits, but now additionally suppose that people are motivated by a desire for a positive self-image; a belief that they are successful human beings. The tension between these two desires—to both behave in accordance with one’s own beliefs and to believe that one’s behavior is successful—drives the model.

To capture this tension I assume that person \( i \)'s evaluation of any set of skills \( x \) is the sum of how much they value each skill times the level of each skill acquired, or \( v^i \cdot x = v_x x + v_y y \). Thus, in addition to the capability constraint, people also face a constraint that their relative self-evaluation be greater than or equal to some exogenous threshold \( T^i \). I term this second constraint a *status constraint*; it is formally defined as

\[
v^i \cdot x^i - v^i \cdot x^j \geq T^i, \text{ for each } i \neq j. \tag{13}
\]

\(^{13}\)See Rabin (1994) who proposes such a model and provides a brief overview of this assumption.
which will be developed in more detail in following sections, requires that a person \( i \) must hold values \( v^i \) that evaluate the difference between her own skills and another type’s skills, \( x^i - x^j \), as greater than threshold \( T^i \). Throughout, I assume that \( T^i \leq 0 \), so that individuals never need to evaluate themselves as strictly superior to others in order to satisfy this constraint. However, if an individual’s relative evaluation of herself dips below threshold \( T^i \), she will begin changing her values and/or developing different skills in order to protect her self-esteem. Variation in \( T^i \) could capture differences in how legitimate a type perceives its own status shortfall to be. For example, Schmader et al. (2001) find that when group status differences are believed to be legitimate by a low status group, then that group does not devalue certain skills in order to preserve its own self-esteem. Thus, \( T^i \) may be lower for historically low-status types than it is for historically high-status types. Regardless of the size of \( T^i \), however, I assume that there is some threshold of relative self-evaluation below which status considerations trigger changes in behavior and/or values.

Again defining utility as
\[
U^i(x^i, v^i) \equiv M(x^i) - D(x^i, v^i),
\]
this new problem is represented by the following:
\[
\max_{x^i, v^i} U^i \quad \text{subject to} \quad p^i \cdot x^i \leq C^i \quad \text{and subject to} \quad v^i \cdot x^i - v^i \cdot x^j \geq T^i. \quad \text{(Problem 3)}
\]

As the status and capability constraints are linearly independent, any solution to Problem 3 must satisfy the Kuhn-Tucker conditions. These conditions are presented in the Appendix, along with a discussion of the fact that there exists a solution to Problem 3 for \( i \) when \( x^j \) is fixed. However, because the choices of types \( i \) and \( j \) are potentially interdependent, it remains to prove the existence of a set of optimizing skills and values for the two types, \( x^{i*}, v^{i*} \) and \( x^{j*}, v^{j*} \). First, suppose that such a set exists that solves Problem 3 for the two types simultaneously. The following result characterizes an important property of any such equilibrium: it will always be the case that the status constraint only binds for, at most, one type, and therefore that status considerations can only affect the behavior or values of this type. All proofs are in the Appendix.
Proposition 1 Suppose that $x_i^*, v_i^*$ and $x_j^*, v_j^*$ simultaneously solve Problem 3 and that $x_i^* \neq x_i$. Then the status constraint will not bind for the type $j$ with $||x_j|| \geq ||x_i^*||$.

The following corollary states that if the addition of the status constraint to the capability constraint results in a change in skill acquisition or values from the baseline optimum $x_i^o, v_i^o$ (which, recall, is the optimum in which skills simply maximize material payoffs and values equal those skills), then then the status constraint must bind for type $i$.

Corollary 1 Suppose that an equilibrium exists and that $(x_i^*, v_i^*) \neq (x_i^o, v_i^o)$ for type $i$. Then the status constraint must bind for type $i$ at a solution to Problem 3.

Along with Proposition 1, the above corollary can be used to show that there exists an equilibrium to Problem 3. In equilibrium, the type $j$ for which $||x_j|| \geq ||x_i||$ always sets $x_j^* = v_j^* = x_j^o$, its baseline optimum. Thus, this type behaves as if status considerations do not matter, and it experiences no cognitive dissonance as its values and skills are the same. The remaining type $i$ chooses skills and values to solve the Kuhn-Tucker conditions, conditional on the actions of $j$. Thus, this type simultaneously chooses a bundle of skills $x_i$ and values $v_i$ to maximize its utility (which is a function of its economic payoff to $x_i$ and the cognitive dissonance it experiences from differences in $x_i$ and $v_i$), subject to its own capabilities (the capability constraint) and its status considerations (the status constraint). The Extreme Value Theorem tells us that there exists a solution to $i$’s problem.

Proposition 2 An equilibrium solution to Problem 3 exists.

Proposition 2 simply establishes that the problem I consider is solvable; there always exists an equilibrium, although it may not be unique. In the Appendix I discuss the status constraint in more detail through a geometric interpretation of how values are used to assess individuals’ abilities. The next section presents some general properties of equilibria of the model. The final part of the paper works through several examples detailing comparative statics that can emerge.
Properties of equilibria

Propositions 1 and 2 simplify our problem considerably because they imply that we can restrict attention to the values and behavior of a single type; the type for which status considerations (potentially) bind. As the actions of the other type are pinned down, for the remainder of the paper I will refer to type \( j \) with \( ||x_j^o|| > ||x_i^o|| \) as type “\( h \)” for “high.” I drop the superscript for type \( i \) (the “low” type), as this is the only type whose behavior is substantively interesting.

**Observation 1** *In every equilibrium the high type faces no binding status considerations; it always chooses the optimal traits and values for its status-unconstrained problem. The low type chooses its status-unconstrained skills and values only if its status constraint is satisfied at this choice.*

This observation is consistent with ethnographic accounts of differences between mainstream and underclass behavior, and in part motivates my use of a constraint to capture individuals’ status considerations (as opposed to directly incorporating such considerations into the utility function).\(^\text{14}\)

Given the above observation, one might be interested in the situations in which status considerations play no role in the decisions of either type. The next observation shows that the more different the types are, in terms of their values in the absence of a status constraint, the more likely it is that in equilibrium each type remains at this baseline optimum. I define “value differences” as the angle between vectors \( v_h^o \) and \( v_o \), or \( \theta_{v_h^o, v_o} \). The biggest such difference occurs when this angle is \( 90^\circ \), which corresponds to a situation in which one type only places strictly positive value


“[M]ainstream culture dictates not merely that men should support their families as best they can, but requires good husbands to support their families at a *socially acceptable level*...” (italics in original).

Because these men have inadequate resources to provide for their families at such a level, Montgomery argues that they reduce cognitive dissonance by altering the belief that they are good husbands. In both Liebow’s account and Montgomery’s model (and, of course, this model), only the actions of individuals who fail to meet a threshold of acceptability are affected by self-esteem considerations.
on one skill domain and the other type on the other domain. In this stark circumstance the status constraint is always satisfied for both types at their unconstrained optima.

**Observation 2** *The greater the value differences are across types, the less likely it is that status considerations will matter to the low type.*

A different (and more precise) way of putting this point is that for any possible disparity between the norms of $x_h^o$ and $x_o$—which informally capture the “levels of skills” obtained by the two types—there is a problem of the form of Problem 3, along with some distribution of skills and values that each type could have, that would render $x_h^o = v_h^o, x_o = v_o$ an equilibrium.

Recall that at the unconstrained optimum, each type values its own bundle of skills as best; thus, when values are very different, so are traits. Observation 2 says that if the two types place very little value on the skills of others and very high value on their own skills, then each person’s relative evaluation of him or herself is more likely to be high than if the types shared similar values and traits. While this may be desirable from the standpoint of the low type, as—all else equal—this type is better off when its status constraint does not bind than when it does, I do not consider such a situation to be normatively desirable. This is because when types share very different values it implies that one or both types must be placing little weight on the skills of others. As I will discuss later in the paper, I think of this as a scenario in which such a type is necessarily *extreme* in its values, and cannot see worth in certain traits of others. Observation 2 says that this type of extreme heterogeneity can characterize a stable, if perhaps undesirable, outcome.

The next observation describes the relationship between equilibrium skills and values when $x^* \neq x_o$, or when the low type changes its behavior in response to binding status considerations.

**Observation 3** *Suppose that $x^* \neq x_o$ (i.e. the low type must change its own behavior and values in an effort to preserve status). Then when $x^*$, the equilibrium level of skill $x$, is strictly greater than $x^h$, $v^*_x$ must be strictly greater than $x^*$. When $x^*$ is strictly less than $x^h$ then $v^*_x$ must be strictly less than $x^*$, provided that $x^* > 0$. (The same holds for trait $y$). Thus, the low type generically experiences some cognitive dissonance when its status constraint binds.*

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\(^{15}\)This observation follows immediately from the third and fourth Kuhn-Tucker conditions describing $\frac{\partial L}{\partial v_x}$ and $\frac{\partial L}{\partial v_y}$.
Observation 3 says that if, in equilibrium, the low type outperforms the high type on some skill $z$ then the low type overvalues skill $z$ relative to the optimal distribution of skills simply characterized by its own behavior. Similarly, if the low type underperforms relative to the high type on that domain, then it undervalues the skill relative to its own behavior. This implies that when status considerations matter for the low type, the type’s skills and values will, in general, not coincide. Consequently, the low type will always experience some cognitive dissonance while the high type will never experience cognitive dissonance.

**Observation 4** If the low type’s capability, $C$, becomes too low, its capability constraint may not bind in equilibrium: individuals will invest in skills at levels below their own capabilities. Moreover, those individuals will choose to undervalue every domain.

Observation 4 says that when a type is incapable of cultivating some minimum level of any skill, then the type chooses to cultivate even less of each skill than it potentially could. Although this statement seems ironic, the intuition for the result is clear: if a person’s self-evaluation cannot reach some minimum threshold with respect to any domain, then the person will choose to believe that all domains are unimportant. Believing otherwise imposes too high a cost on the person, because it requires him or her to experience negative feelings of self worth.

The clearest example of this is for $T = 0$. In this case, if the low type cannot outperform the high type on any skill, then it must set $v = 0$ in order to satisfy its status constraint. This implies that the low type faces the optimization problem

$$\max_x M(x) - D(x, 0), \text{ subject to } p \cdot x \leq C. \quad (1)$$

While $M$ is strictly increasing in $x$, $D$ is strictly decreasing in $x$. It is easy to construct examples in which the solution to this problem results in the low type choosing to invest in skills at levels below its capabilities (and such an example will be discussed in the following section).\(^{16}\)

\(^{16}\)For $T < 0$ the same intuition holds with the recognition that as $C$ decreases beyond the point that $x < x^h$ and $y < y^h$, $||v||$ must get arbitrarily small for the status constraint to be satisfied. Note that for this situation to emerge
3 Consequences of Inequality

In this model interpersonal considerations can affect how individuals choose to value different skills. Sometimes it will be the case that status considerations are not binding for any type; that at the baseline solution $x_o = v_o$ each type believes its own skills to be sufficient relative to the outgroup. In this section I consider the other scenario in which status considerations are binding for the low type. Note that a necessary condition for the constraint to bind is that $||x_o|| < ||x_h||$: the norm of the baseline solution for the low type is smaller than it is for the high type, or the low type has cultivated “fewer skills.” While this does not in itself imply that the low type’s capability term is less than the high’s ($C < C^h$), for any $p = (p_x, p_y)$ (where $p_z$ is the cost of cultivating a unit of skill $z$) there is a $C$ low enough for the constraint to bind and a $C$ high enough for the constraint to not bind. Moreover, if the two types share the same costs to cultivating skills, then $||x_o|| < ||x_h||$ if and only if the low type has less capability than the high. Thus—while it is a slight abuse of the terminology I have used so far—I will refer to a binding status constraint for the low type as representing a scenario in which there is inequality in capabilities across the two types. Inequality of this form represents situations in which the low type is induced to change its behavior and/or values in order to improve its self-evaluation.

I explore several comparative statics that emerge as a consequence of this type of inequality. While most of these comparative statics hold for the more general functional form of utility considered to this point, the examples that follow focus on a specific functional form of utility: status threshold $T = 0$, material payoff is represented by a Cobb-Douglas function of $x$ and $y$, and cognitive dissonance stemming from choices $(x, v)$ is simply the squared Euclidean distance between

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it does not have the be the case that $C$ is so low that $x$ must be less than $x^h$ and $y$ must be less than $y^h$. Situations can arise in which the low type could potentially choose $x > x^h$ but chooses not to, because doing otherwise yields a greater payoff. However, if in equilibrium $x^* > x^h$ or $y^* > y^h$ then the capability constraint must bind.
Thus,

\[ U(x, v) = \alpha \log x + (1 - \alpha) \log y - (x - v_x)^2 - (y - v_y)^2. \] (2)

I chose these functions because they are well-known and easy to interpret; \( \alpha \) (respectively \( 1 - \alpha \)) represents the responsiveness of the material payoff function to a change in the level of skill \( x \) (respectively \( y \)). In the absence of a binding status constraint, this utility function is maximized at

\[ (x_0, y_0) = (v_{x_0}, v_{y_0}) = \left( \frac{\alpha C}{p_x}, \frac{(1 - \alpha) C}{p_y} \right). \]

Reducing a type’s capability

Consider the following thought experiment: Initially there is just one type of person who faces a capability constraint of \( x + y = 10 \) and whose utility function is defined as in Equation 2 with \( \alpha = .4 \). Thus, everyone dedicates 60% of their efforts to \( y \) and 40% to \( x \), choosing \( x = v = (4, 6) \). Suppose that some people of this type face a negative shock to their capability term \( C \), so that they are no longer capable of acquiring skills equal to \( (4, 6) \). How do values and skills change for this newly “low” type? The changes in the low type’s skills are pictured in Figure 1.

Initially, at \( C = 10 \), the low and high types are identical; each chooses \( x = x_0 = (4, 6) \). At this point the low type’s capability constraint is represented by the line connecting \( (10, 0) \) to \( (0, 10) \). As capability decreases the line representing this constraint line moves inward, remaining parallel to the original line. The following paragraphs detail several features of this example that I wish to highlight.

As capability decreases, the primary domain of value changes

The dotted arrows in Figure 1 trace out the changes in the low type’s equilibrium skills \( (x = (x, y)) \) as its capability decreases. For a small reduction in capability the low type initially invests in the

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17These functions clearly satisfy the requirements specified earlier in the paper: \( M \) is strictly concave, increasing and continuously differentiable; \( D \) is convex, continuously differentiable, increasing in the distance between \( x \) and \( v \), and additively separable in \( (x, v_x) \) and \( (y, v_y) \).
more highly-valued skill $y$ even more than it previously did, represented by the arc linking $x_0$ to $x_1$, which equals approximately $(.85, 6.4)$. However, when capability decreases to approximately $C = 7.25$ the low type can no longer utilize this dimension to maintain a sufficiently high relative self-evaluation, because the level of $y$ attained by the high type is simply too high. At this point the low type maintains its status by competing on the less desirable $x$ dimension, and switches to $x_2$ which equals approximately $(5.35, 1.9)$.

The shift from over-investment on the more materially productive domain $y$ to over-investment on domain $x$ is a consequence of the interpersonal nature of the low type’s self-evaluation and the type’s overall reduction in capability. It is a general feature of the model that is not dependent on this specific functional form. To understand why this occurs, note that in the absence of a status constraint the low type is indifferent between a small decrease in $x$ and $y$; the baseline solution requires that the marginal utility of $x$ equals the marginal utility of $y$ in equilibrium. However, when the status constraint binds this logic no longer holds. This is because if $y$ is valued more highly than $x$ then a small increase in $y$ has a larger (positive) effect on a person’s own status.
evaluation than a small increase in $x$ does. For this reason—and if the person is able—he or she will over-invest in the more valued skill in response to a small decrease in capability. For a large reduction in capability, however, this will not be possible, and the only feasible response is over-investment in $x$ relative to the high type.

This type of dynamic could be used to explain why inequality may cause some individuals to invest more in an economically productive skill than a mainstream (high) type, and others to invest less. Examples of such over- and under-investment (relative to the mainstream) are racial and ethnic differences in academic identification and schooling outcomes, and gender differences in math education. Asian American college students, for example, disproportionately concentrate in quantitative and scientific fields while, in college, women abandon these fields at a strikingly high rate.\textsuperscript{18} Significant and persistent achievement gaps between African American and White students are deeply concerning to policymakers, educators, and the public.

Clearly all of the groups described above have faced historical barriers to educational access that were not typically faced by native-born White males, including inadequate resources, a lack of role models, and preparational disparities (Steele, 1997, p. 613). What has perplexed academics and policymakers however, is the fact that these differences in achievement remain even after controlling for socioeconomic class; they are not simply a function of the present-day opportunities for success available to these groups. Two possible explanations for this persistence (that are both compatible with this model and with each other) are first, that large differences in the types of skills people acquire can stem from small differences in capabilities—differences that may appear insignificant. In Figure 1 a person with capability 7.3 will invest in a starkly different collection of skills than a person with capability 7.2; these individuals face nearly identical costs to investing in the different skills.

A second explanation for the persistence of differences in the level and type of educational attainment people acquire is that the “costs” faced by individuals to acquire different skills may be non-monetary and difficult to measure. \textit{Stereotype threat}, a social-psychological cost that arises

\textsuperscript{18}See Woo (2000) and Steele (1997).
when an individual engages in an activity for which a negative stereotype about their group applies, causes people to fear being reduced to that stereotype, and thus, to invest in and value that domain less. This phenomenon is particularly troublesome for individuals who do identify with the domain in question (e.g. math-identifying women and school-identifying African Americans), as stereotype threat can lead to self-defeating behavior. In the context of this framework, a psychological cost to investing in a particular skill domain $x$ lowers capability on that domain (manifested in an increase $p_x$).

Both of the above arguments could help explain why certain groups may over- or under-invest in particular skills even in the absence of observable socio-economic differences. These explanations are distinct from the “ability-motivation hypothesis”—the argument that immigrants are observed to invest more highly in skills acquisition because high-ability, high-motivation individuals are more likely to migrate to the United States (Chiswick, 1978). In the context of this example such a group may be observed to over-invest in high productivity skills relative to a mainstream group even when its relative ability on the various skill domains is the same as that of the mainstream group. When feasible, this type of over-investment yields greater status “bang for the buck” than a shift to the less desirable skill domain.

**For low enough capabilities all domains are devalued**

In the general framework described by this model, sufficiently low capabilities render the low-type incapable of maintaining its self-esteem by valuing any domain. Thus, the only possible equilibrium values are $v = 0$; no value is placed on investment in any skill domain. At some capability threshold values must jump to this point, with investment in skills jumping toward the

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19Steele (1997) presents a theory of domain identification, discusses the role of stereotype threat in shaping outcomes for women and African Americans on certain domains, and proposes practices that could reduce the negative effects of stereotype threat.

20Strictly speaking, when $T = 0$ the only equilibrium values in the face of sufficiently low capabilities are $v = 0$. When $T < 0$ then $v < x$ for all sufficiently low capabilities, which implies that $v$ ultimately approaches (or reaches) 0 as capability approaches zero.
origin in response, in the low type’s effort to reduce cognitive dissonance.

In Figure 1 this phenomenon occurs when capability dips below approximately $C = 4.4$. At this point the low type switches from having skills $x_3$ equal to approximately $(4, 4)$ to $x_4$, which approximately equals $(.45, .55)$.\footnote{The low type remains at $x_4$ until $C$ becomes so low that this choice cannot be sustained, which I do not picture in the figure.} This switch is an example of Observation 4, in that when this shift occurs the low type’s capability constraint clearly does not bind: the constraint requires that $x + y \leq 4.4$ and we instead see $x + y = 1$.

The phenomenon of underperformance relative to one’s capability can be interpreted as a self-handicapping behavior: by believing that no domain matters, the individual’s relative shortcomings on every domain are made less painful. Reducing effort on every domain is the only type of behavior consistent with this belief. This type of defensive self-handicapping has been used to explain a variety of anti-social and negative behaviors such as underachievement and drug and alcohol abuse. It arises principally in situations where individuals are concerned with perceptions (and importantly, self-perceptions) of competence.\footnote{See Berglas and Jones (1978).}

As capability decreases the high type’s evaluation of the low type drops discontinuously; the low type’s evaluation of itself does not

In this example a reduction in capability continuously reduces the utility of the low type, although the equilibrium choices of this type change discontinuously at a two points. The first discontinuous jump is the low type’s shift from $x_1$ to $x_2$, representing a shift in over-investment in skill $y$ to over-investment in $x$. The second is the shift from $x_3$ to $x_4$, the point at which the low type changes from placing some positive value on the skills to placing zero value on each skill. While each of these jumps represents an infinitesimal change in the low type’s utility it represents a substantial, negative change in the high type’s \textit{evaluation of the low type}, with this evaluation represented as $v^h \cdot x$. Moreover, for a range of capabilities the low type’s equilibrium skills are invariant to its capabilities, $C$: when $C$ drops below approximately 4.4, skills remain at $x_4$ until the capability
constraint once again binds, at \( C = 1 \).

In this model, individuals’ relative self-evaluations cannot fall below a threshold \( T \) in equilibrium. This is consistent with widespread evidence that overall self-esteem among stigmatized groups is not persistently lower than it is among the non-stigmatized (Osborne, 1995). This example provides another—perhaps more pernicious—explanation for why the stigmatized may not negatively internalize dramatic disparities in skill acquisition that may be perceived by outgroups, such as the unmistakable and significant move from \( x_1 \) to \( x_2 \). This change in skill acquisition and values may register as merely a blip in the type’s overall experience of well-being.

Figure 1 also illustrates why attempts to induce investment in skills by either increasing overall capability or by reducing the cost of skills acquisition (\( p_x \) or \( p_y \)) may be ineffective. At the point \( x_4 \), for example, increasing capabilities through either of these means will have no effect on the behavior of the low type in a wide range of circumstances.\(^{23}\) In this case, the possibility of relative success on any domain is so remote that the low-type is insensitive to improvements in its own capabilities. This finding suggests that, when considering highly stigmatized individuals, one should be wary of assuming that people will naturally respond to direct incentives to alter their skill sets.

**Reductions in capability lead to “value extremism”**

The final feature of this example that I wish to highlight is that along each of the two arcs representing the behavior of the low type in Figure 1 a drop in capability shifts \( x \) toward one of the two axes; it leads the low type to invest in a more unbalanced collection of skills. Thus, with the exception of the discontinuous jumps from \( x_1 \) to \( x_2 \) and \( x_3 \) to \( x_4 \), a reduction in capability makes the low type more extreme in its choice of skills and, consequently, in its choice of values.\(^{24}\)

This point can be made somewhat generally. Figure 2 shows how the feasible set of \( x \) varies as the low type’s capability decreases. When \( C = 10 \) and status threshold \( T = 0 \) the feasible set of \( x \) is the interval \([0, 10]\). For \( C < 10 \), and when values are nonzero, so that positive value is placed on

\(^{23}\)In particular, for \( C \in [1, 4.4] \) and (holding \( C \) fixed), for similarly-derived intervals for \( p_x \) and \( p_y \).

\(^{24}\)This is because \( v_x \geq x \) and \( v_y < y \) when \( x \geq x^h \) and vice versa when \( x < x^h \) (Observation 3) and so values are, in a sense, more extreme than skills.
at least one domain, the feasible set consists of the union of two disjoint intervals corresponding to either \( x \geq x^h \) or \( y \geq y^h \).

These intervals become smaller, and ultimately disappear, as \( C \) decreases. At \( C = 4.4 \), for example, the feasible set of \( x \) is represented by the single dark triangle below the low type’s capability constraint. For any feasible mixture of skills in the region, values must place nearly all relative weight on domain \( x \) and virtually no weight on \( y \). In this sense, as capability decreases, behavior and values are increasingly constrained to “extreme” distributions of \( x \) and \( y \), with either most value and investment being placed on a single domain, or, finally, with no domain valued.

This particular implication of the model is related to recent work arguing a connection between inequality and extremism (and in particular, terrorism).\(^{25}\) A recurring theme in this literature is that socio-economic inequality breeds, either directly or indirectly, indignation. This indignation can facilitate ethnic mobilization for a variety of reasons. By contrast, in this model these types of

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explicit grievances play no role in the low type’s increasing value extremism in the face of rising
inequality. Rather, as an individual’s capability is reduced that individual self-defensively reacts by
investing more in the trait that positively differentiates it from the outgroup. As inequality becomes
more pronounced, these attempts become increasingly desperate—the individual must ultimately
dedicate nearly all of his or her energy to maintaining positive differentiation on that one trait,
leading to the value extremism observed here.

**Altering economic payoffs**

Here I consider the question of how changes in the material rewards to skills acquisition affect
the equilibrium behavior of individuals. This is related to the “headscarves” discussion in the
introduction of this paper: if we consider material payoff $M$ to be a matter of public policy or
institutional design, how might increasing or decreasing the rewards of possessing certain traits
(e.g. religiosity) affect the traits that people choose to cultivate? Consistent with the observations
presented so far, the answer to this question is that it depends on the initial degree of inequality
between low and high types.

Consider again the “Cobb-Douglas / quadratic” utility function given in Equation 2 and suppose
that skills $x$ and $y$ are equally costly to obtain for both types (e.g. $p_x = p_y = 1$). Set the capability
of the high type at $C^h = 10$. How does varying $\alpha$—the parameter of the Cobb-Douglas function
describing the responsiveness of material payoff to a change in $x$—alter each type’s investment
in $x$? For the status-unconstrained problem the optimal level of $x$ is $\frac{\alpha C}{p_x}$, so in this example we
know that the high type always sets $x^h = 10\alpha$. Thus, as $\alpha$ increases from 0 to 1 skill $x$ becomes
increasingly profitable; the high type engages in more of $x$ and less of $y$.

Figure 3 shows the low type’s investment in $x$ as $\alpha$ ranges from 0 to 1—in other words, as
$x$ becomes increasingly profitable relative to $y$. In this figure, the capability of the low type is
$C = 6$. The diagonal dotted line is the low type’s baseline investment in $x$ in the absence of the
status constraint. With the exception of the point $\alpha = .5$, this type’s equilibrium investment in
$x$ is always increasing in $\alpha$. However, if the goal of increasing the profitability of $x$ is to induce
increased investment in $x$ by the low type, this strategy will backfire if the increase pushes $\alpha$ over the .5 mark: at that point an increase in $\alpha$ causes a substantial and negative decrease in investment in $x$ by the low type. The reason for this is clear: if a policy to make $x$ more profitable affects both high and low types, then the (status-unconstrained) high type will invest in $x$ more. This makes it increasingly difficult for the low type to maintain a positive self-evaluation by investing in that same skill. In this case, as in the previous example in Figure 1, the low type is reduced to over-investing in the less productive skill ($x$ when $\alpha < .5$ and $y$ when $\alpha > .5$) in order to positively differentiate itself from the high type.

The situation reverses itself when the low type’s capability is sufficiently high (i.e. when inequality is sufficiently low). Figure 4 depicts the same scenario but increases the low type’s capability to $C = 9.5$ (and recall, the high type’s capability is 10). In this case, as in the previous Figure 1, the low type out-performs the high type on the most productive domains for most values of $\alpha$. Again, this is because these domains are more highly valued by the low type in the absence of the status constraint, and over-investment in them consequently makes the status constraint easier to satisfy.

These two examples illustrate the idea that policies intended to make certain behaviors more
or less costly to individuals or groups can have a variety of consequences. These consequences depend critically on the effect of these policies on other groups (groups that are not, perhaps, the intended targets of the policy). The ultimate effect of policies stigmatizing or embracing public displays of religion, for example, will depend on both the status quo policy and the relative capabilities and costs to skill acquisition of the types comprising the society in question. In some instances a ban on headscarves will induce individuals of a particular type to value secular traits more, and in other instances it may induce individuals to value secularism less. The latter observation would be seen in this example in Figure 3, with trait $x$ being “secularism.” Increasing the material benefit of this trait in the form of raising $\alpha$ from .45 to .55, for example, would result in a dramatic decrease in secularism for the low type and a small increase in secularism for the high type.\footnote{Dickson (2013) presents a very different model that generates some similar insights regarding the consequences of a ban on symbols of social identity.}

### Changing the relative costs of acquiring skills

This final example considers how changing the cost of acquiring a certain skill—making acquisition of that particular skill easier or harder for an individual—affects the ultimate bundle of skills an individual chooses to cultivate. The question is of relevance to numerous debates concerning the
effect of social programs designed to induce people to invest in a targeted set of skills. Examples abound, including Pell Grants (designed to help low-income students obtain a college education), vocational training programs designed to reintroduce the unemployed back into the workforce, and even Head Start, which in part aims to ease the transition from preschool to elementary school for low-income children.

Using the same utility function we have considered to this point, this example sets the capabilities of the two types to be the same ($C = C^h = 10$), but varies the cost of obtaining skill $y$ for the low type from $p_y = 1$ to $p_y = 1.5$. For both types the cost of obtaining $x$ is the same: $p_x = p^h_x = 1$. For the high type the cost of obtaining $y$ is also the same: $p^h_y = 1$. However, as the low type’s cost of obtaining a “unit” of skill $y$ increases from 1 to 1.5, this skill becomes increasingly difficult for the low type to obtain.

![Figure 5: Increasing the cost of acquiring $y$](image)

Suppose that a policymaker is interested in increasing the low type’s investment in skill $y$, and is considering easing the acquisition of $y$ by the low type by reducing $p_y$. While for many values of $p_y$ in Figure 5 a small reduction in $p_y$ will lead to a small increase in $y$, the policy will meet with the most success in a scenario where $p_y$ shifts from being above the discontinuous jump in $y$ that occurs at $p_y = 1.125$ to below it. This shift represents a simultaneous change in the beliefs and behavior of the low type. In particular, this type changes from being incapable of relative success on the $y$ domain, thus reducing the value placed on the $y$ domain and the corresponding
investment in the $y$ domain, to being capable of comparative success, and thus increasing both value and investment in the domain. If the reduction of $p_y$ is very costly, then it may be the case that the policy is not desirable unless it can push $p_y$ below this threshold. Moreover, if $p_y$ is already below this threshold then a reduction in the cost of $y$ will actually reduce investment in $y$. This occurs because for these values the low type is investing in an artificially high level of $y$, purely to maintain its own self-esteem in the face of a disparity in capabilities. As this disparity decreases, so does investment in $y$.

4 Conclusions and Extensions

People are more likely to value skills that they are comparatively successful at. At the same time, a person’s success on a particular domain is, in part, a consequence of his or her investment on that domain. This investment, in turn, can depend on how much the individual values that domain. The preceding model represents my attempt to capture this logic. It identifies a mechanism by which the skills a person invests in and the worth assigned to those skills develop simultaneously. These skills and values emerge as a consequence of the person’s own abilities and how they perceive their strengths or shortcomings relative to others. While numerous accounts exist showing that people selectively value domains that they expect to be viewed positively on, by and large this work assumes that outcomes on a given domain are fixed. One contribution of this project is to model the process by which people invest in their own success on a given domain, and to show that these investments may vary unexpectedly as a function of a person’s own abilities, the abilities of others, and economic incentives.

The model leaves a variety of questions open for future research. While the paper focuses exclusively on a setting with two types of individuals, the model could straightforwardly be extended to allow for a richer society consisting of many individuals of many types. Of course, this type of extension would require an explicit assumption about how a person assesses his or her own relative status when faced with multiple “others.” In a similar vein, the model could readily accommodate
type-specific material payoff functions; with a richer variety of types, one might be interested in allowing those types to potentially engage in type-specific economies, and receive type-specific payoffs that are separate from the psychological payoff people receive from behaving consistently with their values. For example, it might be economically productive for a person belonging to an identifiable group to behave in accordance with certain group norms in the absence of any inherent belief about the worth of such norms.

A different extension of the model that is farther from the scope of the current project would allow material payoffs to be the product of one’s own skills and the skills of others. As the model stands, individuals are only affected by the actions of others through a desire to maintain positive self-esteem. A natural extension would allow for non-psychological interdependencies, as would be observed in a competitive labor market, for example.

This paper fits into a literature that aims to explain underclass behavior as the product of rational action (Montgomery, 1994). My hope is that the type of analysis developed here—by clarifying the motivations of individuals and deducing the consequences of those motivations—can serve as a part of a useful dialogue to further desirable social outcomes. In particular, the model provides some insight into why individuals may choose to self-handicap and under-invest in their own skills, and into why polarized beliefs about the worth of certain activities may emerge as a consequence of economic inequality. That these potentially self-defeating behaviors can be deduced from a model of rational choice—and that in many circumstances these behaviors may be insensitive to economic incentives—could shed light on important policy debates.
References


Dickson, E. S. (2013). Struggles over symbols and endogenous group norms. *Mimeo*.


## 5 Appendix

### The Kuhn-Tucker Conditions

The Langrangean and first-order conditions are below (note the $i$ superscript for each type has been omitted to simplify notation). The non negativity constraints on $x$ and $v$ yield first-order conditions
that are inequalities; if the solution to any of these variables is non-zero then the corresponding first-order condition must hold with equality.

\[
L = M(x) - D(x^i, v^i) - \lambda_1(p \cdot x) - \lambda_2(v \cdot x^i - v \cdot x)
\]

\[
\frac{\partial L}{\partial x} = \frac{\partial}{\partial x} M(x, y) - \frac{\partial}{\partial x} D(x, v_x) - \lambda_1 p_x + \lambda_2 v_x \leq 0,
\]

\[
\frac{\partial L}{\partial y} = \frac{\partial}{\partial y} M(x, y) - \frac{\partial}{\partial y} D(y, v_y) - \lambda_1 p_y + \lambda_2 v_y \leq 0,
\]

\[
\frac{\partial L}{\partial v_x} = -\frac{\partial}{\partial v_x} D(x, v_x) - \lambda_2 (x^i - x) \leq 0, \text{ and}
\]

\[
\frac{\partial L}{\partial v_y} = -\frac{\partial}{\partial v_y} D(y, v_y) - \lambda_2 (y^i - y) \leq 0.
\]

(Kuhn-Tucker conditions)

We additionally know that

\[
\lambda_i \geq 0 \text{ for all } i,
\]

and that the following complementary slackness conditions hold:

\[
\lambda_1 (p \cdot x - C) = 0,
\]

\[
\lambda_2 (v \cdot x^i - v \cdot x + T) = 0.
\]

Any solution to Problem 3 must satisfy the above conditions because the utility function and constraints are continuously differentiable and the constraints are linearly independent of each other (linear independence follows if each type’s vector of costs is strictly positive and each type chooses a strictly positive bundle of traits). However, while satisfaction of the above conditions is necessary it is not sufficient for the solution to be a global optimum; this is because the status constraint is not quasiconvex in \(x\) and \(v\).

Holding the actions of type \(j\) fixed, there is a solution to Problem 3 for type \(i\). This is because the feasible region of \(x^i\) is nonempty, closed and bounded, and because there is a \(v^i\) that effectively bounds \(i\)'s choice of \(v^i\). In particular, \(i\) will always choose \(0\) over any \(v^i\) that is farther from \(x^i\) than \(0\); this is because \(0\) is feasible (always satisfies \(i\)'s status constraint) and \(i\) chooses \(v^i\) solely to minimize distance to \(x^i\) subject to satisfying \(i\)'s status constraint. The upper bound on \(x^i\) thus places an effective upper bound on \(v^i\) and it follows that \(i\)'s feasible set is nonempty, closed and
bounded. By the Extreme Value Theorem there exists a solution to Problem 3 for \( i \) when \( x^i \) is fixed.

**The Geometry of Values and Status Preservation**

In this model, a type’s values represent a shared belief among individuals of that type that having a certain collection of skills—or being a certain way—merits worth. These values serve as a group heuristic for evaluating all individuals on the basis of how they have chosen to live their lives. In this sense, values are tools that individuals can use to make order out of the highly multidimensional characteristics of others, by deeming certain types of people to be more or less valuable (capable, successful, or worthy) than others. People derive utility from cultivating skills that comport with their own values and also derive utility from cultivating skills that are of economic or material value.

Recall that an individual with values \( v \) evaluates the worth of a set of skills \( x \) as \( v \cdot x \), which can be rewritten as \( ||v|| ||x|| \cos \theta_{x,v} \), where \( \theta_{x,v} \) is the angle between vectors \( x \) and \( v \). Thus, the status constraint that \( v \cdot x - v \cdot x^j \geq T \) can be rewritten as

\[
||x|| \cos \theta_{x,v} - ||x^j|| \cos \theta_{x^j,v} \geq \frac{T}{||v||}.
\]  

Equation 3

There are two things to note about Equation 3. First is that the cosine term ranges from 1 (if \( x \) and \( v \) are collinear) to 0 (if they are orthogonal) and the norm of \( x \) represents the level of skills achieved by a type cultivating bundle \( x \). Thus, an evaluation of \( x \) according to values \( v \) is increasing in the norm of \( x \), or levels of skills attained, and decreasing in \( \theta_{x,v} \), a measure of the difference between the ratio of skills attained and the ratio considered ideal by values \( v \). The second thing to note about Equation 3 is that \( ||x|| \cos \theta_{x,v} \) is the norm (or distance to the origin) of the orthogonal projection of \( x \) onto the ray passing through \( v \).

Given that only the direction of \( v \) matters in making relative status evaluations, a type’s values \( v = (v_x, v_y) \) can be thought of as a ray in the positive quadrant, originating at the origin, that individuals of the type consider to represent the optimal distribution of skills \( x \) and \( y \). The norm of \( v \) is immaterial to the status calculation; all that matters is its direction, and so \( v = (1, 2) \) represents...
the same values as \( v = (6, 12) \), and both value \( y \) as twice as important as \( x \). As an example, suppose that someone with values \( v = (1, 2) \) is evaluating three bundles of skills, \( x_1 = (4, 5) \), \( x_2 = (7, 3) \), and \( x_3 = (13, 1) \). This scenario is pictured in Figure 6. The difference between the individual’s evaluation of \( x_1 \) and \( x_2 \) is \( v_x \times x_1 + v_y \times y_1 - v_x \times x_2 - v_y \times y_2 = 1 \times 4 + 2 \times 5 - 1 \times 7 - 2 \times 3 = 1 \), and between \( x_1 \) and \( x_3 \) is \(-1\). Thus, \( v \) evaluates \( x_1 \) as better than \( x_2 \); even though the norm of \( x_2 \) is greater than the norm of \( x_1 \)—“more” skills are cultivated at \( x_2 \)— its greater distance to \( v \), represented by \( \theta_2 \) in the figure, lowers its relative evaluation. At the same time, \( v \) evaluates \( x_3 \) as better than \( x_1 \); although \( \theta_3 > \theta_1 \), this difference is offset by the greater norm of \( x_3 \).

![Figure 6: Evaluating \( x_1, x_2, \) and \( x_3 \) with values \( v \).]

Suppose that an individual holds values \( v \) and has attained skills \( x_1 \), while the other type has attained skills \( x_3 \). If \( T = 0 \) the individual’s status considerations are triggered whenever the projection of the other type’s skills onto \( v \) are farther from the origin than the projection of \( x_1 \) onto \( v \). In this example the individual would be induced to change his or her values, behavior, or both to improve status. If \( T < 0 \) then the individual’s status considerations are only triggered if the difference between these evaluations falls below \( \frac{T}{\|v\|} \). Thus, as \( \|v\| \) becomes smaller (keeping the direction of \( v \) fixed), the constraint becomes easier to satisfy; the individual is devaluing both domains in order to preserve his or her self-evaluation. When \( T < 0 \) there is always a strictly
positive vector of values that can satisfy the status constraint, as the individual can always devalue both domains to such an extent that his or her relative self evaluation exceeds $T$. This is not the case when $T = 0$; if a type $i$ cannot outperform the outgroup on either domain then the only values that can satisfy the status constraint are $v^i = 0$. In this case the individual believes neither domain is of any value.

**The Cobb-Douglas / Quadratic Utility Example**

In this section I briefly describe some technical aspects of the functional form of utility used in the examples:

$$U(x, v) = \alpha \log x + (1 - \alpha) \log y - (x - v_x)^2 - (y - v_y)^2.$$  \hspace{1cm} (4)

A solution to Problem 3 requires that $v$ minimize $D(x, v)$ subject to the status constraint, which (assuming utility is of the form given above) implies that

$$v = \left( \frac{(y - y^h)(x^h y - x y^h)}{(x - x^h)^2 + (y - y^h)^2}, \frac{(x - x^h)(-x^h y + x y^h)}{(x - x^h)^2 + (y - y^h)^2} \right),$$  \hspace{1cm} (5)

if this vector is weakly positive, and $v = 0$ otherwise. By inspection, $v$ is weakly positive if and only if $x \geq x^h$ and $y < y^h$ or $x < x^h$ and $y \geq y^h$ (the low type is weakly better than the high type on one domain and strictly worse on the other). And if $x = x^h$ then $y < y^h$ and $v = (x, 0)$.

As discussed in Observation 4 (and its accompanying footnote), if $v = 0$ then utility is uniquely maximized at the solution to Equation 1. Let this solution be $\underline{x}$. We know $\underline{x}$ is unique because utility is strictly concave and the feasible set of $x$ defined by the capability constraint is compact and convex. If utility is not maximized for the low type at $x^* = \underline{x}$ and $v^* = 0$ then the capability constraint must bind, so that $y = \frac{C}{p_y} - \frac{p_x}{p_y} x$ and $v$ is as in Equation 5. Substituting these terms into the utility function, the problem now reduces to maximizing $U(x)$—a continuous function of skill $x$—over the domain $[0, \frac{C}{p_x} - \frac{p_y}{p_x} y^h] \cup [x^h, \frac{C}{p_x}] \cup \underline{x}$.

Equilibria are not guaranteed to be unique because, in general, the utility function defined in Equation 2 is not single-peaked after the above substitutions are made. This is because the status constraint is not quasiconvex as a function of $v$ and $x$. In all of the examples that follow, however, utility is strictly concave on the sets $[0, \frac{C}{p_x} - \frac{p_y}{p_x} y^h]$ and $[x^h, \frac{C}{p_x}]$ and so finding solutions simply
involves finding the unique maximum of the utility function on each of the three sets that comprise
the domain of $x$ and choosing the best of the three (which is generically unique in these examples).
I have not shown that $U(x)$ is strictly quasiconcave on this particular domain in general, nor have
I found an example where a function satisfying the earlier concavity conditions I have specified is
not strictly quasiconcave on this domain. In the examples I provide $U(x)$ is multi-peaked off the
two intervals.

**Proofs to Numbered Results**

**Proposition 1:** Suppose that $x^i^*$, $v^i^*$ and $x^j^*$, $v^j^*$ simultaneously solve Problem 3 and that $x^j^* \neq x^i^*$. Then the status constraint will not bind for the type $j$ with $||x^j^*|| \geq ||x^i^*||$.

**Proof:** Suppose by way of contradiction that the status constraint *does* bind for both types, so
that $v^j^*(x^j^* - x^i^*) = T^j \leq 0$ and $v^i^*(x^i^* - x^j^*) = T^i \leq 0$. Without loss of generality, let $||x^i^*|| \geq ||x^j^*||$. We know that $v^j^* \neq x^j^*$, because if these vectors were equal the left side of $j$’s status constraint could be rewritten $||x^j^*||^2 - ||x^j^*|| \cdot ||x^i^*|| \cdot \cos \theta_a, \theta_b$, with $\theta_a, \theta_b$ being the angle between the vectors $a, b$. Since both vectors lie in the positive quadrant, angle $\theta$ (subscripts omitted) is strictly greater than 0° and weakly less than 90°, and thus $\cos \theta \in [0, 1]$. This, along with the assumption that $x^j^* \neq x^i^*$, implies that $||x^j^*||^2 - ||x^j^*|| \cdot ||x^i^*|| \cdot \cos \theta > 0$, contradicting our assumption that the constraint was binding.

However, since $\tilde{v}^j^* = x^j^*$ uniquely maximizes type $j$’s utility for all possible $x^j^*$ in the absence
of a status constraint, $v^j^*$ cannot be optimizing for $j$, because $j$ could switch to $\tilde{v}^j^*$ and strictly
improve its payoff while still satisfying both its status constraint and its capability constraint. It
follows that this constraint cannot be binding for both groups if $x^j^* \neq x^i^*$. □

**Corollary 1:** Suppose that an equilibrium exists and that $(x^i^*, v^i^*) \neq (x^i_o, v^i_o)$ for type $i$. Then the status constraint must bind for type $i$ at a solution to Problem 3.

**Proof:** This follows from Proposition 1 and the necessity of the Kuhn-Tucker conditions holding
at a solution to Problem 3. □
Proposition 2: An equilibrium solution to Problem 3 exists.

Proof: Suppose that there is a type \( j \) for which \( ||x_j^o|| > ||x_i^o|| \). Let \( x_i^* = v_i^* = x_j^o \). Holding this choice fixed, I first show that when \( i \) best responds to \( j \)'s choice, \( j \) does not have an incentive to change its choice. By Corollary 1 \( i \)'s choice of \( x_i^*, v_i^* \) satisfies

\[
||x_i^*|||v_i^*||\cos\theta_{x_i^*,v_i^*} = ||x_i^*|||v_i^*||\cos\theta_{x_j^*,v_i^*} + T_i, \text{ or}
\]

\[
||x_i^*||\cos\theta_{x_i^*,v_i^*} \leq ||x_j^*||\cos\theta_{x_j^*,v_i^*}.
\]

If \( \cos\theta_{x_i^*,v_i^*} < \cos\theta_{x_j^*,v_i^*} \) then \( v_i^* \) is not optimal for \( i \), as \( i \) could reduce this angle (moving \( v_i^* \) in the direction of \( x_i^* \)), decrease its cognitive dissonance and still satisfy its status constraint. It follows that \( \cos\theta_{x_i^*,v_i^*} \geq \cos\theta_{x_j^*,v_i^*} \) and thus that \( ||x_i^*|| \leq ||x_j^*|| \). By previous arguments, \( j \)'s status constraint is satisfied and \( j \) remains at \( x_j^o, v_j^o \).

To see that there is no equilibrium in which \( i \) setting \( x_i^* = v_i^* = x_j^o \) causes \( j \) to change its choice of \( x_j^* = v_j^* = x_j^o \), note that \( j \)'s status constraint is always satisfied at this pair of points. Thus, an equilibrium exists and it always involves \( j \) setting \( x_j^* = v_j^* = x_j^o \). \( \square \)